7. $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = uf(u^2+w^2) - wg(u^2+w^2), \quad \dfrac{\partial^2 w}{\partial x^2} + \dfrac{\partial^2 w}{\partial y^2} = wf(u^2+w^2) + ug(u^2+w^2).$

1°. Periodic solution in the coordinate y with phase shift in components:

$$u = r(x)\cos\bigl[\theta(x) + C_1 y + C_2\bigr], \qquad w = r(x)\sin\bigl[\theta(x) + C_1 y + C_2\bigr],$$

where C_1 and C_2 are arbitrary constants, and the functions $r = r(x)$ and $\theta = \theta(x)$ are determined by the system of ordinary differential equations

$$r''_{xx} = r(\theta'_x)^2 + C_1^2 r + rf(r^2),$$
$$r\theta''_{xx} = -2r'_x\theta'_x + rg(r^2).$$

2°. Solution (generalizes the solution of Item 1°):

$$u = r(z)\cos\bigl[\theta(z) + C_1 y + C_2\bigr], \qquad w = r(z)\sin\bigl[\theta(z) + C_1 y + C_2\bigr], \qquad z = k_1 x + k_2 y,$$

where C_1, C_2, k_1, and k_2 are arbitrary constants, and the functions $r = r(z)$ and $\theta = \theta(z)$ are determined by the system of ordinary differential equations

$$(k_1^2 + k_2^2)r''_{zz} = k_1^2 r(\theta'_z)^2 + r(k_2\theta'_z + C_1)^2 + rf(r^2),$$
$$(k_1^2 + k_2^2)r\theta''_{zz} = -2\bigl[(k_1^2 + k_2^2)\theta'_z + C_1 k_2\bigr]r'_z + rg(r^2).$$

 http://eqworld.ipmnet.ru/en/solutions/syspde/spde3107.pdf