8. $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}=uf(u^2-w^2)+wg(u^2-w^2),\qquad \dfrac{\partial^2 w}{\partial x^2}+\dfrac{\partial^2 w}{\partial y^2}=wf(u^2-w^2)+ug(u^2-w^2).$

Solution:

$$u=r(z)\cosh\big[\theta(z)+C_1y+C_2\big],\qquad w=r(z)\sinh\big[\theta(z)+C_1y+C_2\big],\qquad z=k_1x+k_2y,$$

where C_1, C_2, k_1, and k_2 are arbitrary constants, and the functions $r=r(z)$, $\theta=\theta(z)$ are determined by the system of ordinary differential equations

$$(k_1^2+k_2^2)r''_{zz}+k_1^2r(\theta'_z)^2+r(k_2\theta'_z+C_1)^2=rf(r^2),$$
$$(k_1^2+k_2^2)r\theta''_{zz}+2\big[(k_1^2+k_2^2)\theta'_z+C_1k_2\big]r'_z=rg(r^2).$$

