10. $\dfrac{\partial u}{\partial t} = \dfrac{\partial}{\partial x}\left[f\left(t, \dfrac{u}{w}\right)\dfrac{\partial u}{\partial x}\right] + ug\left(t, \dfrac{u}{w}\right), \quad \dfrac{\partial w}{\partial t} = \dfrac{\partial}{\partial x}\left[f\left(t, \dfrac{u}{w}\right)\dfrac{\partial w}{\partial x}\right] + wh\left(t, \dfrac{u}{w}\right).$

Solution:

$$u = \varphi(t)\exp\left[\int h(t,\varphi(t))\,dt\right]\theta(x,\tau), \quad w = \exp\left[\int h(t,\varphi(t))\,dt\right]\theta(x,\tau), \quad \tau = \int f(t,\varphi(t))\,dt,$$

where the function $\varphi = \varphi(t)$ is determined by the ordinary differential equation

$$\varphi_t' = [g(t,\varphi) - h(t,\varphi)]\varphi,$$

and the function $\theta = \theta(x,\tau)$ satisfies the linear heat equation

$$\frac{\partial\theta}{\partial\tau} = \frac{\partial^2\theta}{\partial x^2}.$$

