3. $\dfrac{\partial^2 u}{\partial t^2} = \dfrac{a}{x^n}\dfrac{\partial}{\partial x}\left(x^n \dfrac{\partial u}{\partial x}\right) + uf\left(\dfrac{u}{w}\right), \quad \dfrac{\partial^2 w}{\partial t^2} = \dfrac{b}{x^n}\dfrac{\partial}{\partial x}\left(x^n \dfrac{\partial w}{\partial x}\right) + wg\left(\dfrac{u}{w}\right).$

1°. Periodic multiplicative separable solution:

$$u = [C_1 \cos(kt) + C_2 \sin(kt)]y(x), \qquad w = [C_1 \cos(kt) + C_2 \sin(kt)]z(x),$$

where C_1, C_2, and k are arbitrary constants, and the functions $y = y(x)$ and $z = z(x)$ are determined by the system of ordinary differential equations

$$ax^{-n}(x^n y'_x)'_x + k^2 y + yf(y/z) = 0,$$
$$bx^{-n}(x^n z'_x)'_x + k^2 z + zg(y/z) = 0.$$

2°. Multiplicative separable solution:

$$u = [C_1 \exp(kt) + C_2 \exp(-kt)]y(x), \qquad w = [C_1 \exp(kt) + C_2 \exp(-kt)]z(x),$$

where C_1, C_2, and k are arbitrary constants, and the functions $y = y(x)$ and $z = z(x)$ are determined by the system of ordinary differential equations

$$ax^{-n}(x^n y'_x)'_x - k^2 y + yf(y/z) = 0,$$
$$bx^{-n}(x^n z'_x)'_x - k^2 z + zg(y/z) = 0.$$

3°. Degenerate multiplicative separable solution:

$$u = (C_1 t + C_2)y(x), \qquad w = (C_1 t + C_2)z(x),$$

where the functions $y = y(x)$ and $z = z(x)$ are determined by the system of ordinary differential equations

$$ax^{-n}(x^n y'_x)'_x + yf(y/z) = 0,$$
$$bx^{-n}(x^n z'_x)'_x + zg(y/z) = 0.$$

4°. Multiplicative separable solution:

$$u = x^{\frac{1-n}{2}}[C_1 J_\nu(kx) + C_2 Y_\nu(kx)]\varphi(t), \qquad \nu = \tfrac{1}{2}|n-1|,$$
$$w = x^{\frac{1-n}{2}}[C_1 J_\nu(kx) + C_2 Y_\nu(kx)]\psi(t),$$

where C_1, C_2, and k are arbitrary constants, $J_\nu(z)$ and $Y_\nu(z)$ are the Bessel functions, and the functions $\varphi = \varphi(t)$ and $\psi = \psi(t)$ are determined by the system of ordinary differential equations

$$\varphi''_{tt} = -ak^2\varphi + \varphi f(\varphi/\psi),$$
$$\psi''_{tt} = -bk^2\psi + \psi g(\varphi/\psi).$$

5°. Multiplicative separable solution:

$$u = x^{\frac{1-n}{2}}[C_1 I_\nu(kx) + C_2 K_\nu(kx)]\varphi(t), \qquad \nu = \tfrac{1}{2}|n-1|,$$
$$w = x^{\frac{1-n}{2}}[C_1 I_\nu(kx) + C_2 K_\nu(kx)]\psi(t),$$

where C_1, C_2, and k are arbitrary constants, $I_\nu(z)$ and $K_\nu(z)$ are the modified Bessel functions, and the functions $\varphi = \varphi(t)$ and $\psi = \psi(t)$ are determined by the system of ordinary differential equations

$$\varphi''_{tt} = ak^2\varphi + \varphi f(\varphi/\psi),$$
$$\psi''_{tt} = bk^2\psi + \psi g(\varphi/\psi).$$

6°. Solution with $b = a$:

$$u = k\theta(x,t), \qquad w = \theta(x,t),$$

where k is a root of the algebraic (transcendental) equation $f(k) = g(k)$, and the function $\theta = \theta(x,t)$ satisfies the linear Klein–Gordon equation

$$\frac{\partial^2\theta}{\partial t^2} = \frac{a}{x^n}\frac{\partial}{\partial x}\left(x^n \frac{\partial\theta}{\partial x}\right) + f(k)\theta.$$

