1. $\dfrac{\partial u}{\partial t} = L[u] + f_1(t)u + g_1(t)w, \qquad \dfrac{\partial w}{\partial t} = L[w] + f_2(t)u + g_2(t)w.$

Here, L is an arbitrary linear differential operator in the coordinates $x_1, \ldots, x_n$ (of any order in derivatives), whose coefficients can depend on $x_1, \ldots, x_n, t$. It is assumed that $L[\text{const}] = 0$.

Solution:

$$u = \varphi_1(t)U(x_1, \ldots, x_n, t) + \varphi_2(t)W(x_1, \ldots, x_n, t),$$
$$w = \psi_1(t)U(x_1, \ldots, x_n, t) + \psi_2(t)W(x_1, \ldots, x_n, t),$$

where the two pairs of functions $\varphi_1 = \varphi_1(t)$, $\psi_1 = \psi_1(t)$ and $\varphi_2 = \varphi_2(t)$, $\psi_2 = \psi_2(t)$ are linearly independent (fundamental) solutions of the system of first-order linear ordinary differential equations

$$\varphi'_t = f_1(t)\varphi + g_1(t)\psi,$$
$$\psi'_t = f_2(t)\varphi + g_2(t)\psi,$$

and the functions $U = U(x_1, \ldots, x_n, t)$ and $W = W(x_1, \ldots, x_n, t)$ satisfy the independent linear equations

$$\frac{\partial U}{\partial t} = L[U], \qquad \frac{\partial W}{\partial t} = L[W].$$

