2. $\dfrac{\partial u}{\partial t} = L_1[u] + uf\left(\dfrac{u}{w}\right), \quad \dfrac{\partial w}{\partial t} = L_2[w] + wg\left(\dfrac{u}{w}\right).$

Here, L_1 and L_2 are arbitrary constant-coefficient linear differential operators (of any order) in the coordinate x.

1°. Solution:

$$u = e^{kx-\lambda t}y(\xi), \quad w = e^{kx-\lambda t}z(\xi), \quad \xi = \beta x - \gamma t,$$

where k, λ, β, and γ are arbitrary constants, and the functions $y = y(\xi)$ and $z = z(\xi)$ are determined by the system of ordinary differential equations

$$M_1[y] + \lambda y + yf(y/z) = 0, \quad M_2[z] + \lambda z + zg(y/z) = 0,$$
$$M_1[y] = e^{-kx}L_1[e^{kx}y(\xi)], \quad M_2[z] = e^{-kx}L_2[e^{kx}z(\xi)].$$

To the special case $k = \lambda = 0$ there corresponds a traveling-wave solution.

2°. If the operators L_1 and L_2 involve only even derivatives, there are solutions of the form

$$u = [C_1\sin(kx) + C_2\cos(kx)]\varphi(t), \quad w = [C_1\sin(kx) + C_2\cos(kx)]\psi(t);$$
$$u = [C_1\exp(kx) + C_2\exp(-kx)]\varphi(t), \quad w = [C_1\exp(kx) + C_2\exp(-kx)]\psi(t);$$
$$u = (C_1x + C_2)\varphi(t), \quad w = (C_1x + C_2)\psi(t),$$

where C_1, C_2, and k are arbitrary constants (the third solution is degenerate).

Reference

Polyanin, A. D., Exact solutions of nonlinear systems of reaction-diffusion equations and mathematical biology equations, *Theor. Found. Chem. Eng.*, Vol. 37, No. 6, 2004.

