3. $\dfrac{\partial u}{\partial t} = L[u] + uf\left(t, \dfrac{u}{w}\right), \quad \dfrac{\partial w}{\partial t} = L[w] + wg\left(t, \dfrac{u}{w}\right).$

Here, L is an arbitrary linear differential operator in the variables $x_1, \ldots, x_n$ (of any order in derivatives), whose coefficients can depend on $x_1, \ldots, x_n$ and t:

$$L[u] = \sum A_{k_1 \ldots k_n}(x_1, \ldots, x_n, t) \frac{\partial^{k_1+\cdots+k_n} u}{\partial x_1^{k_1} \ldots \partial x_n^{k_n}}.$$

It is assumed that $k_1 + \cdots + k_n \geq 1$.

Solution:

$$u = \varphi(t) \exp\left[\int g(t, \varphi(t))\, dt\right] \theta(x_1, \ldots, x_n, t),$$

$$w = \exp\left[\int g(t, \varphi(t))\, dt\right] \theta(x_1, \ldots, x_n, t),$$

where the function $\varphi = \varphi(t)$ is determined by the nonlinear first-order ordinary differential equation

$$\varphi_t' = [f(t, \varphi) - g(t, \varphi)]\varphi,$$

and the function $\theta = \theta(x_1, \ldots, x_n, t)$ satisfies the linear equation

$$\frac{\partial \theta}{\partial t} = L[\theta].$$

Reference

Polyanin, A. D., Exact solutions of nonlinear systems of reaction-diffusion equations and mathematical biology equations, *Theor. Found. Chem. Eng.*, Vol. 37, No. 6, 2004.

