2. $\dfrac{\partial^2 u}{\partial t^2} = L_1[u] + uf\left(\dfrac{u}{w}\right), \quad \dfrac{\partial^2 w}{\partial t^2} = L_2[w] + wg\left(\dfrac{u}{w}\right).$

Here, L_1 and L_2 are arbitrary linear differential operators (of any order) in x with constant coefficients. It is assumed that $L_1[\text{const}] = 0$ and $L_2[\text{const}] = 0$.

1°. Solution in the form of the product of two waves traveling at different velocities:

$$u = e^{kx-\lambda t} y(\xi), \quad w = e^{kx-\lambda t} z(\xi), \quad \xi = \beta x - \gamma t,$$

where k, λ, β, and γ are arbitrary constants, and the functions $y = y(\xi)$ and $z = z(\xi)$ are determined by the system of ordinary differential equations

$$\gamma^2 y''_{\xi\xi} + 2\lambda\gamma y'_\xi + \lambda^2 y = M_1[y] + yf(y/z), \quad \gamma^2 z''_{\xi\xi} + 2\lambda\gamma z'_\xi + \lambda^2 z = M_2[z] + zg(y/z),$$
$$M_1[y] = e^{-kx} L_1[e^{kx} y(\xi)], \qquad M_2[z] = e^{-kx} L_2[e^{kx} z(\xi)].$$

To the special case $k = \lambda = 0$, there corresponds a traveling-wave solution.

2°. Periodic multiplicative separable solution:

$$u = [C_1 \sin(kt) + C_2 \cos(kt)]\varphi(x), \quad w = [C_1 \sin(kt) + C_2 \cos(kt)]\psi(x),$$

where C_1, C_2, and k are arbitrary constants and the functions $\varphi = \varphi(x)$ and $\psi = \psi(x)$ are determined by the system of ordinary differential equations

$$L_1[\varphi] + k^2\varphi + \varphi f(\varphi/\psi) = 0,$$
$$L_2[\psi] + k^2\psi + \psi g(\varphi/\psi) = 0.$$

3°. Multiplicative separable solution:

$$u = [C_1 \sinh(kt) + C_2 \cosh(kt)]\varphi(x), \quad w = [C_1 \sinh(kt) + C_2 \cosh(kt)]\psi(x),$$

where C_1, C_2, and k are arbitrary constants and the functions $\varphi = \varphi(x)$ and $\psi = \psi(x)$ are determined by the system of ordinary differential equations

$$L_1[\varphi] - k^2\varphi + \varphi f(\varphi/\psi) = 0,$$
$$L_2[\psi] - k^2\psi + \psi g(\varphi/\psi) = 0.$$

4°. Degenerate multiplicative separable solution:

$$u = (C_1 t + C_2)\varphi(x), \quad w = (C_1 t + C_2)\psi(x),$$

where C_1 and C_2 are arbitrary constants and the functions $\varphi = \varphi(x)$ and $\psi = \psi(x)$ are determined by the system of ordinary differential equations

$$L_1[\varphi] + \varphi f(\varphi/\psi) = 0, \quad L_2[\psi] + \psi g(\varphi/\psi) = 0.$$

Remark 1. The coefficients of the operators L_1, L_2 and the functions f, g in Items 2°–4° can depend on x.

Remark 2. If L_1 and L_2 contain only even derivatives, there are solutions of the form

$$u = [C_1 \sin(kx) + C_2 \cos(kx)]U(t), \qquad w = [C_1 \sin(kx) + C_2 \cos(kx)]W(t);$$
$$u = [C_1 \exp(kx) + C_2 \exp(-kx)]U(t), \quad w = [C_1 \exp(kx) + C_2 \exp(-kx)]W(t);$$
$$u = (C_1 x + C_2)U(t), \qquad w = (C_1 x + C_2)W(t),$$

where C_1, C_2, and k are arbitrary constants (the third solution is degenerate).

