4. $\dfrac{\partial^2 u}{\partial t^2} = L[u] + uf\left(\dfrac{u}{w}\right) + g\left(\dfrac{u}{w}\right), \quad \dfrac{\partial^2 w}{\partial t^2} = L[w] + wf\left(\dfrac{u}{w}\right) + h\left(\dfrac{u}{w}\right).$

Here, L is an arbitrary linear differential operator in the variables $x_1, \ldots, x_n$ (of any order in derivatives), whose coefficients can depend on $x_1, \ldots, x_n, t$.

Solution:

$$u = k\theta(x_1, \ldots, x_n, t), \quad w = \theta(x_1, \ldots, x_n, t),$$

where k is a root of the algebraic (transcendental) equation $g(k) = kh(k)$ and the function $\theta = \theta(x, t)$ satisfies the linear equation

$$\frac{\partial^2 \theta}{\partial t^2} = L[\theta] + f(k)w + h(k).$$

