5. $\dfrac{\partial^2 u}{\partial t^2} = L[u] + au \ln u + uf\left(t, \dfrac{u}{w}\right), \quad \dfrac{\partial^2 w}{\partial t^2} = L[w] + aw \ln w + wg\left(t, \dfrac{u}{w}\right).$

Here, L is an arbitrary linear differential operator in the variables $x_1, \dots, x_n$ (of any order in derivatives), whose coefficients can depend on the space variables.

Solution:

$$u = \varphi(t)\theta(x_1, \dots, x_n),$$
$$w = \psi(t)\theta(x_1, \dots, x_n),$$

where the functions $\varphi = \varphi(t)$ and $\psi = \psi(t)$ are determined by the nonlinear system of second-order ordinary differential equations

$$\varphi''_{tt} = a\varphi \ln \varphi + b\varphi + \varphi f(t, \varphi/\psi),$$
$$\psi''_{tt} = a\psi \ln \psi + b\psi + \psi g(t, \varphi/\psi),$$

b is an arbitrary constant, and the function $\theta = \theta(x_1, \dots, x_n)$ satisfies the stationary equation

$$L[\theta] + a\theta \ln \theta - b\theta = 0.$$

