1. $$\frac{\partial u_m}{\partial t} = L[u_m] + u_m f(t, u_1 - b_1 u_n, \dots, u_{n-1} - b_{n-1} u_n) + g_m(t, u_1 - b_1 u_n, \dots, u_{n-1} - b_{n-1} u_n), \qquad m = 1, \dots, n.$$

The system involves $n+1$ arbitrary functions f, $g_1, \dots, g_n$ dependent on n arguments; L ia an arbitrary linear differential operator in the space variables $x_1, \dots, x_n$ (of any order in derivatives), whose coefficients can depend on $x_1, \dots, x_n, t$. It is assumed that $L[\mathrm{const}] = 0$.

Solution:

$$u_m = \varphi_m(t) + \exp\left[\int f(t, \varphi_1 - b_1\varphi_n, \dots, \varphi_{n-1} - b_{n-1}\varphi_n)\,dt\right]\theta(x_1, \dots, x_n, t).$$

Here, the functions $\varphi_m = \varphi_m(t)$ are determined by the system of ordinary differential equations

$$\varphi_m' = \varphi_m f(t, \varphi_1 - b_1\varphi_n, \dots, \varphi_{n-1} - b_{n-1}\varphi_n) + g_m(t, \varphi_1 - b_1\varphi_n, \dots, \varphi_{n-1} - b_{n-1}\varphi_n),$$

where $m = 1, \dots, n$, the prime denotes a derivative with respect to t, and the function $\theta = \theta(x_1, \dots, x_n, t)$ satisfies the linear equation

$$\frac{\partial \theta}{\partial t} = L[\theta].$$

Reference

Polyanin, A. D., Exact solutions of nonlinear systems of reaction-diffusion equations and mathematical biology equations, *Theor. Found. Chem. Eng.*, Vol. 37, No. 6, 2004.

