2. $$\frac{\partial u_m}{\partial t} = L[u_m] + u_m f_m\Big(t, \frac{u_1}{u_n}, \dots, \frac{u_{n-1}}{u_n}\Big) + \frac{u_m}{u_n} g\Big(t, \frac{u_1}{u_n}, \dots, \frac{u_{n-1}}{u_n}\Big),$$
$$\frac{\partial u_n}{\partial t} = L[u_n] + u_n f_n\Big(t, \frac{u_1}{u_n}, \dots, \frac{u_{n-1}}{u_n}\Big) + g\Big(t, \frac{u_1}{u_n}, \dots, \frac{u_{n-1}}{u_n}\Big).$$

Here, $m = 1, \dots, n-1$, the system involves $n+1$ arbitrary functions $f_1, \dots, f_n$, g dependent on n arguments; L ia an arbitrary linear differential operator in the space variables $x_1, \dots, x_n$ (of any order in derivatives), whose coefficients can depend on $x_1, \dots, x_n, t$. It is assumed that $L[\text{const}] = 0$.

Solution:

$$u_m = \varphi_m(t) F_n(t)\Big[\theta(x_1, \dots, x_n, t) + \int \frac{g(t, \varphi_1, \dots, \varphi_{n-1})}{F_n(t)}\,dt\Big], \qquad m = 1, \dots, n-1,$$
$$u_n = F_n(t)\Big[\theta(x_1, \dots, x_n, t) + \int \frac{g(t, \varphi_1, \dots, \varphi_{n-1})}{F_n(t)}\,dt\Big],$$
$$F_n(t) = \exp\Big[\int f_n(t, \varphi_1, \dots, \varphi_{n-1})\,dt\Big],$$

where the functions $\varphi_m = \varphi_m(t)$ is determined by the nonlinear system of first-order ordinary differential equations

$$\varphi_m' = \varphi_m[f_m(t, \varphi_1, \dots, \varphi_{n-1}) - f_n(t, \varphi_1, \dots, \varphi_{n-1})], \qquad m = 1, \dots, n-1,$$

and the function $\theta = \theta(x_1, \dots, x_n, t)$ satisfies the linear equation

$$\frac{\partial \theta}{\partial t} = L[\theta].$$

Reference

Polyanin, A. D., Exact solutions of nonlinear systems of reaction-diffusion equations and mathematical biology equations, *Theor. Found. Chem. Eng.*, Vol. 37, No. 6, 2004.

