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Analytical capabilities of the DeepSeek neural network: A review of results from solving test problems in applied mathematics and mathematical physics

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This article describes the characteristic features and practical use of the DeepSeek neural network for analytical computations and the construction of exact solutions in various fields of applied mathematics and mathematical physics. The results of applying DeepSeek to numerous test problems, primarily of high difficulty level, are presented and analyzed. Special attention is given to the construction of exact analytical solutions for diverse mathematical equations (including algebraic, functional, integral, ordinary differential, and partial differential equations). A number of nonlinear equations of mathematical physics possessing complex non-invariant solutions are considered. Almost all test equations contain one or more free parameters or arbitrary functions, which significantly complicates the solution procedure. To adequately assess the potential capabilities of DeepSeek, test equations and problems were selected whose solution is based on the application of various methods and ideas (more than 60 equations and problems were considered in total). In some cases, the results obtained using DeepSeek are compared with those of specialized computer algebra systems, such as Mathematica and Maple. Overall, the results of the conducted testing show that the DeepSeek neural network possesses broad analytical capabilities and is useful for the work and education of researchers, university professors, postgraduate students, and engineers specializing in various fields of applied mathematics and mathematical physics.

Keywords: artificial intelligence, neural networks, DeepSeek, algebraic equations, functional equations, integral equations, ordinary differential equations, partial differential equations, nonlinear equations of mathematical physics, test problems.

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Permanent web address of the article: https://eqworld.ipmnet.ru/Arts_Polyanin/Polyanin_Shingareva_2026.02.04.pdf

1 Introduction

Artificial Intelligence (AI) is an interdisciplinary field of computer science that aims to create intelligent systems capable of performing tasks that traditionally require human cognition. These tasks include learning, reasoning, perception, natural language understanding, and decision-making. Modern AI is built on complex algorithms and methods, such as machine learning and deep learning (based on artificial neural networks), which enable machines to extract knowledge from huge datasets and improve their performance without explicit programming. Key technological directions in AI include computer vision, natural language processing (NLP), and cognitive computing. Today, AI is actively transforming almost every sphere of human activity: from healthcare (where it assists in diagnostics) to science, education, transportation, finance, and customer service, enhancing accuracy, efficiency, and automation levels. The potential of AI lies not only in optimizing existing processes but also in unlocking fundamentally new opportunities (for instance, in drug discovery or combating climate change). Thus, AI is not merely a tool, but a driving force that profoundly transforms and dramatically accelerates scientific and technological progress.

For clarity, the development path of AI can be conveniently represented as a logical chain (see, for example, [1]):

$$\text{Artificial Intelligence} \Longrightarrow \text{Machine Learning} \Longrightarrow$$
$$\Longrightarrow \text{Deep Learning} \Longrightarrow \text{Generative AI} \Longrightarrow \text{Reasoning Models.}$$

Machine Learning focuses on recognizing patterns in data. *Deep Learning*, which uses multi-layer *neural networks*, can work with unstructured data (e.g., texts, images). *Generative AI*, utilizing machine learning and deep learning, focuses on the dynamic generation of new content (e.g., text, images, or anything else) from large volumes of training data [2]. Unlike AI, which is often designed for pattern recognition and decision-making based on existing data, generative AI creates new content [3]. This capability opens up a world of possibilities for creating realistic images and videos, composing music, and writing text [4].

There are eight types of generative AI [5]: text-to-text (e.g., GPT-4, Claude), text-to-audio (e.g., Google Tacotron), text-to-image (e.g., OpenAI DALL-E), text-to-video (e.g., InvideoAI), text-to-music (e.g., Suno, Udio), audio-to-text (e.g., Amazon Alexa), text-to-code (e.g., GitHub Copilot), and image-to-video (e.g., RunwayML).

Generative AI has rapidly become an accessible tool in scientific research. Currently, generative AI is experiencing rapid growth and an increasingly prominent presence in scientific publications. Generative AI (which includes large language models, e.g., ChatGPT) has attracted significant attention from both the scientific community and the wider public due to its potential to accelerate the pace of scientific progress, stimulate the discovery of new materials, transform experimental design and hypothesis generation, and reshape approaches to a scientific publication [6]. A survey conducted by the journal Nature in 2023 revealed a substantial proliferation of generative AI tools (e.g., ChatGPT, Gemini, Claude, Perplexity) among researchers. A significant portion of them use these technologies both for generating ideas and conducting research [7], as well as for writing code and literature reviews, and for preparing presentations and manuscripts [8, 9].

The next evolutionary stage has given rise to *Reasoning Models*, which are focused not merely on generating responses, but on deep reasoning, causal analysis, and multi-step planning. Mechanisms enabling these processes are embedded in their architecture: internal dialogue (chain-of-thought reasoning), memory manipulation, and the capacity for autonomous self-learning. This advancement moves AI closer to the concept of artificial thinking, allowing systems not only to answer but also to reason, justify decisions, demonstrate cognitive resilience, and adapt to novel tasks.

DeepSeek neural networks are next-generation advanced generative AI language models (LLMs) developed in China, possessing sophisticated capabilities for deep reasoning and multi-step logical inference [10].

Currently, two main DeepSeek neural network models (DeepSeek-V3 and DeepSeek-R1) are available to users via the official API, along with a third specialized version (DeepSeek-V3.2-Special) designed for maximum reasoning performance. There is also a highly specialized research model, DeepSeekMath-V2, intended exclusively for mathematical reasoning and rigorous theorem proving. DeepSeekMath-V2 successfully solved 5 out of 6 problems at the 2025 International Mathematical Olympiad (IMO), which is a performance at the gold medal level. Furthermore, this model scored 118 out of 120 points on the Putnam competition, a prestigious mathematics contest for undergraduates, surpassing the best human score (90 points). More detailed information on these achievements can be found on The-decoder.com, a website dedicated to AI research.

This paper discusses the results of the practical application of the universal model DeepSeek-V3 (henceforth referred to as DeepSeek) in applied mathematics and mathematical physics.

The key advantages and disadvantages of DeepSeek are listed below.

Advantages of DeepSeek:

- Free access. The service is free of charge and available via web browser and mobile app.
- Full support for 12 languages (specifically, provides higher-quality Russian-English and English-Russian translation than Google and Yandex translators). Partial support for approximately 100 languages.
- High performance. Comparable to GPT-4 in response speed and answer quality.
- Versatile applications. Suitable for text generation, programming, data analysis, and solving various mathematical equations and problems.
- Extreme ease of use. No knowledge of specific codes or specialized expertise is required. Excellent for self-study as all intermediate reasoning and calculations are provided.
- Context understanding. Thanks to its transformer architecture, DeepSeek effectively understands the context and meaning of text.
- Successfully processes texts written in LaTeX format, including those containing equations and formulas. Also supports formats such as: txt, doc, docx, pdf, ppt, and others.
- Self-verification of solutions — DeepSeek-R1 can recalculate results using multiple methods and identify errors in computations.
- Open source. Companies can modify and adapt it for their specific tasks.
- Up-to-date knowledge. Like many other models, DeepSeek is regularly updated, and its capabilities are continuously expanded.

Disadvantages of DeepSeek:

- Occasionally makes mistakes and hallucinations (it is necessary to monitor DeepSeek and, if needed, ask it to double-check results).
- Possible operational disruptions. Sometimes, due to high user traffic, DeepSeek temporarily stops generating responses.
- Limited functionality. DeepSeek currently cannot generate images and videos. Additionally, the neural network cannot process links or recognize voice queries.
- According to its guidelines, DeepSeek does not create images directly but can provide textual descriptions and instructions on how to construct graphs. It does not retain previous context (if used without registration or when starting a conversation in a new window).

More detailed information about the DeepSeek neural network and its extensive capabilities can be found, for example, in publications [10–13] and on the website `https://deepseekcom.ru`.

This paper describes the results of using the main DeepSeek-V3 model (specifically DeepSeek-V3.2) on numerous advanced-level mathematical test problems. The primary focus is on constructing exact analytical solutions for various types of mathematical equations (including algebraic, functional, integral, ordinary differential, partial differential, and some other types) and analyzing related associated problems. To adequately assess the potential capabilities of DeepSeek, the authors endeavored to select test equations whose solutions require the application of diverse methods. Almost all test equations contain one or more free parameters or arbitrary functions, which significantly complicates the solution procedure.

In some cases, the results obtained using DeepSeek are compared with those from Google's flagship AI model, Gemini 3 Pro (hereafter referred to as Gemini for brevity), as well as specialized computer algebra systems Mathematica and Maple (see, for example, [14, 15]).

2 Algebraic equations

The following five algebraic equations of higher degrees with one or two free parameters were proposed as test problems for DeepSeek:

$$(a - x^2)^3 = (b - x^3)^2, \tag{1}$$

$$(x^3 + a)^3 + a = x, \tag{2}$$

$$(x^3 + x + b)^3 + x^3 + 2b = 0, \tag{3}$$

$$2z^6 - 3\alpha z^4 - 2\beta z^3 + 3\alpha^2 z^2 + \beta^2 - \alpha^3 = 0, \tag{4}$$

$$x^5 - (a^4 - 3a^2 b + b^2)x + ab(a^2 - 2b) = 0, \tag{5}$$

where a, b, α, and β are free parameters.

For all five equations, DeepSeek was asked the same question: *Can the solutions of this equation be expressed in radicals?*

For the first four equations (1)–(4), DeepSeek gave the correct answer (providing detailed proofs that they are solvable). For the last equation (5), DeepSeek stated that it is generally unsolvable in radicals with respect to xx for arbitrary parameters a and b (this conclusion is incorrect, since the roots of the quadratic equation $x^2 - ax + b = 0$ are also roots of the given equation [16]).

For clarity, here and in what follows, equations that DeepSeek failed to solve (or solved incorrectly, either completely or partially) are highlighted in blue.

The test problems for algebraic equations formulated above fall into the category of advanced difficulty problems, since the solutions of algebraic equations of the fifth degree and higher generally (with arbitrary coefficients) cannot be expressed in radicals (the Ruffini–Abel theorem, see, e.g., [17]).

A complete report* (including the intermediate calculations) on the construction of solutions to the test equations (1)–(5) by DeepSeek can be viewed online at: https://chat.deepseek.com/share/ry48m2tnhhrtrwv89o

Remark 1. The first four equations possess hidden symmetries (the fourth equation differs from the first only in its expanded form and the renaming of variables/parameters). The methods for solving these equations and the proof of their solvability are presented in the article [18] (see also [19, 20]). The solvability in radicals of the equation (5) is proved in [16].

Remark 2. The Gemini correctly solved four test problems (2)–(5).

Remark 3. Table 1 presents the results of testing the specialized computer algebra systems Mathematica and Maple on the algebraic equations with parameters (1)–(5). It can be seen that neither Mathematica nor Maple can explicitly express in radicals all or even some of the roots of the sixth-degree polynomial equations with two free parameters, (1) and (4), and they were only able to find three roots of the two ninth-degree equations with one free parameter, (2) and (3). It should be noted that Mathematica and Maple were able to solve equation (5).

Table 1. Results of testing the Maple and Mathematica systems on five algebraic equations with parameters.

No.	Test equation	Parameters	Maple: roots found	Mathematica: roots found
(1)	$(a-x^2)^3=(b-x^3)^2$	a and b are any	0	0
(2)	$(x^3+a)^3+a=x$	a is any	3	3
(3)	$(x^3+x+b)^3+x^3+2b=0$	b is any	3	3
(4)	$2z^6-3\alpha z^4-2\beta z^3+3\alpha^2z^2+\beta^2-\alpha^3=0$	α and β are any	0	0
(5)	$x^5-(a^4-3a^2b+b^2)x+ab(a^2-2b)=0$	a and b are any	5	5

Thus, based on the testing results, it can be concluded that DeepSeek and Gemini are better at solving algebraic equations (with parameters) than the specialized computer algebra systems Mathematica and Maple.

Remark 4. For equations (1), (2), and (5), Yandex's Alice AI provided an incorrect answer, stating that they are not solvable in radicals. Therefore, we did not proceed to test it further on other mathematical equations.

*Here and in what follows, all online DeepSeek reports detailing the solution of the test problems are provided in Russian (some reports contain descriptions of solutions to several test problems at once).

3 Functional equations

As test problems for DeepSeek, it was proposed to solve the following nine functional equations of various types (the first seven equations are taken from the handbook [16], the eighth equation is new, and the ninth equation was considered in [16, 21]):

$$y(x)y(x+1) + a[y(x+1) - y(x)] = 0, \tag{6}$$

$$y(\sin x) - y(\cos x) = 0, \tag{7}$$

$$y(x)y(a-x) = b^2, \tag{8}$$

$$f(x+y) + f(x-y) = 2f(x) + 2f(y), \tag{9}$$

$$f(x,y) = a^n f(x + (1-a)y, ay), \tag{10}$$

$$f(x+y) = f(x) + f(y) - af(x)f(y), \tag{11}$$

$$\left(\frac{f^2(x) + f^2(y)}{2}\right)^{1/2} = f\left(\left(\frac{x^2+y^2}{2}\right)^{1/2}\right), \tag{12}$$

$$f(x+y) = g(x) + f(y) - f(x)h(y), \tag{13}$$

$$f(t) + g(x)Q(z) + h(x)R(z) = 0, \qquad z = \varphi(x) + \psi(t), \tag{14}$$

where y, f, g, h, Q, and R are the sought functions of their arguments, $\varphi(x)$ and $\psi(t)$ are arbitrary functions, a and b are free parameters, n is a given number. Note that equations (6)–(8) involve one independent variable x, while equations (9)–(13) involve two independent variables x and y. Equation (13) contains three sought functions $f(x)$, $g(x)$, and $h(y)$, and equation (14) includes five sought functions $f(t)$, $g(x)$, $h(x)$, $Q(z)$, and $R(z)$, as well as two arbitrary functions $\varphi(x)$ and $\psi(t)$. It is important to note that solving these test functional equations requires the use of several different methods (see, for example, [19, 21]).

DeepSeek correctly solved seven equations (6), (7), (9)–(12), (14), employing methods for some equations that differed from those used in [16]. For equations (6) and (7), DeepSeek provided the correct answer in a different form than in the cited handbook.

For equation (8), DeepSeek found one solution but missed the second (the obtained and missed solutions differ only in sign). For equation (13), DeepSeek obtained the obvious solutions and missed the most important one. Following this, we additionally wrote: *Some solutions are missing. Check this by setting $f(y) = g(y)$ and $h(y) = af(y)$, where a is an arbitrary parameter (we considered such an equation earlier).* This referred to equation (11), which is a special case of equation (13). After that, upon thinking for 240 seconds, DeepSeek correctly solved equation (13).

A complete report (including the necessary intermediate calculations and our additional questions) on the construction of solutions to the test equations (6)–(14) by the DeepSeek neural network can be viewed online at:
https://chat.deepseek.com/share/bxbb4bbto9umwgou1u and
https://chat.deepseek.com/share/dvmwoyempfu4se2kso.

In summary, it can be concluded that DeepSeek handles functional equations quite successfully.

Remark 5. The Gemini correctly solved equation (13) using the method of differentiation with respect to independent variables (see, for example, [19, 21]).

Remark 6. Mathematica and Maple could not solve the functional equations (8), (10), (13), and (14) (they were not tested on the remaining functional equations).

Thus, based on the testing results, it can be concluded that DeepSeek and Gemini are better at solving functional equations than the specialized computer algebra systems Mathematica and Maple.

4 Integral equations

As test problems for DeepSeek, it was proposed to solve the following twelve integral equations of various types (all equations, except one, are taken from the handbooks [16, 22]):

$$\int_0^x \frac{y(t)\,dt}{\sqrt{x-t}} = f(x) \qquad \text{([16], p. 396, Eq. 7)}, \tag{15}$$

$$\int_0^x \cosh[a(x-t)]\,y(t)\,dt = f(x), \qquad \text{([16], p. 398, Eq. 19)}, \tag{16}$$

$$\int_0^x \{\cosh[a(x-t)]+b\}\,y(t)\,dt = f(x) \qquad \text{([16], p. 398, Eq. 21)}, \tag{17}$$

$$\int_0^x y(t)y(x-t)\,dt = ax+b \qquad \text{([16], p. 403, Eq. 21)}, \tag{18}$$

$$y(x) + a\int_0^x (x-t)y(t)\,dt = f(x) \qquad \text{([16], p. 406, Eq. 2)}, \tag{19}$$

$$ay(x) + \int_0^x y(t)y(x-t)\,dt = bx \qquad \text{(new equation)}, \tag{20}$$

$$\int_{-\infty}^{\infty} e^{-\lambda|x-t|}y(t)\,dt = f(x) \qquad \text{([16], p. 431, Eq. 12)}, \tag{21}$$

$$\int_{-\infty}^{\infty} \sin(\lambda|x-t|)y(t)\,dt = f(x) \qquad \text{([16], p. 434, Eq. 27),} \tag{22}$$

$$y(x) + \lambda\int_{0}^{\infty} e^{-|x-t|}y(t)\,dt = f(x) \qquad \text{([16], p. 441, Eq. 7),} \tag{23}$$

$$y(x) - \lambda\int_{-\infty}^{\infty} e^{-|x-t|}y(t)\,dt = 0 \qquad \text{([16], p. 441, Eq. 8),} \tag{24}$$

$$y(x) - \lambda\int_{0}^{\infty} \sin(xt)y(t)\,dt = f(x) \qquad \text{([16], p. 444, Eq. 16),} \tag{25}$$

$$y(x) + \int_{a}^{b} g(t)y(x)y(t)\,dt = f(x) \qquad \text{([16], p. 450, Eq. 1),} \tag{26}$$

where $f(x)$ and $g(t)$ are arbitrary functions, a, b, and λ are free parameters (for some equations, additional restrictions were imposed on the defining functions and parameters).

DeepSeek correctly solved eleven integral equations (15)–(23), (25), (26). It should be noted that after trying different approaches, DeepSeek ultimately obtained the solution to the nonlinear equation (20) in the form of a convergent power series.

For equation (24), DeepSeek provided the correct solution for $\lambda > \frac{1}{2}$ and an incorrect one for $0 < \lambda < \frac{1}{2}$ (erroneously stating that the solution was zero). Subsequently, we requested it to additionally verify whether the functions $y = \exp(\pm x\sqrt{1-2\lambda})$, where $0 < \lambda < \frac{1}{2}$, are solutions to the considered equation. DeepSeek performed the verification and agreed that this is indeed the correct solution in the specified domain (noting that this solution could be multiplied by an arbitrary constant).

A complete report (including the necessary intermediate steps, calculations, and our additional questions) on the construction of solutions to the test equations (15)–(26) by the DeepSeek neural network can be viewed online at:
https://chat.deepseek.com/share/yvhpawyjnp4wm7amml

Overall, it can be concluded that DeepSeek handles integral equations quite successfully.

Remark 7. Mathematica and Maple could not solve the integral equations (22) and (24) (they were not tested on the remaining integral equations). Thus, it appears that DeepSeek performs better at solving integral equations than the aforementioned specialized computer algebra systems.

Remark 8. DeepSeek was asked the following question: *Can it solve singular integral equations where the improper integral containing the unknown function is understood in the sense of the Cauchy principal value?*

Its answer: *DeepSeek can serve as a useful consultant-assistant when working with such equations, helping with the theoretical background and general methods, but it is not an autonomous solver for such equations. For actual computations, specialized mathematical packages (for example, the specialized Julia package: SingularIntegralEquations.jl) and your own expertise to verify correctness will be required.*

5 Ordinary differential equations

As test problems for DeepSeek, it was proposed to solve the following fifteen ordinary differential equations (ODEs) of various types (all equations, with the exception of the last one, are taken from the books [16, 19, 23–25]):

$$y''_{xx} + f(x)y'_x + a[f(x) - a]y = 0 \qquad ([16], \text{p. 72, Eq. 4}), \qquad (27)$$

$$xy''_{xx} + [xf(x) + a]y'_x + (a-1)f(x)y = 0 \qquad ([16], \text{p. 73, Eq. 6}), \qquad (28)$$

$$xy''_{xx} + [(ax+1)f(x) + ax - 1]y'_x + a^2xf(x)y = 0 \qquad ([16], \text{p. 73, Eq. 7}), \qquad (29)$$

$$x^4y''_{xx} + (ax^2 + bx + c)y = 0 \qquad ([16], \text{p. 64, Eq. 35}), \qquad (30)$$

$$(ax^2 + bx + c)^2y''_{xx} + y = 0 \qquad ([23], \text{p. 449, Eq. 2.396}), \qquad (31)$$

$$yy'_x - y = ax + b \qquad ([16], \text{p. 42, Eq. } 1^\circ), \qquad (32)$$

$$yy'_x - y = ax + b\exp(-2x/a) \qquad ([16], \text{p. 43, Eq. } 5^\circ), \qquad (33)$$

$$yy''_{xx} - (y'_x)^2 = ay^3e^{\lambda x} \qquad ([19], \text{p. 25, Eq. 1.3.4.7}), \qquad (34)$$

$$y''_{xx} = ay'_x + e^{2ax}f(y) \qquad ([16], \text{p. 78, Eq. 5}), \qquad (35)$$

$$y''_{xx} = [e^{ax}f(y) + a]y'_x \qquad ([16], \text{p. 79, Eq. 7}), \qquad (36)$$

$$y''_{xx} - a(y'_x)^2 = f(x)e^{ay} \qquad ([16], \text{p. 81, Eq. 27}, \qquad (37)$$

$$y''_{xx} - y'_x = b/y \qquad ([16], \text{p. 77, Eq. } 2^\circ), \qquad (38)$$

$$y''_{xx} = axy^{-1/2} \qquad ([24], \text{p. 309, Eq. 11}), \qquad (39)$$

$$yy'_x = (3ax + b)y - a^2x^3 - abx^2 + cx \qquad ([24], \text{p. 121, Eq. 2}), \qquad (40)$$

$$y''''_{xxxx} - cy''_{xx} = ae^{\lambda y} + be^{2\lambda y} \qquad ([19], \text{p. 257, Exercise 4}), \qquad (41)$$

where $f(x)$ and $f(y)$ are arbitrary functions, a, b, c, and λ are free parameters (in the last equation, it was necessary to find particular solutions).

DeepSeek correctly solved nine equations (27), (28), (30), (32), (34)–(37), (41) and failed to solve (or solved incorrectly) six equations (29), (31), (33), (38)–(40). Thus, DeepSeek's success rate on these test problems is 60%.

This is a very respectable result, given that four of the ODEs that DeepSeek failed to solve are either Abel equations (equations (33) and (40)) or can be reduced to Abel equations (equations (38) and (39)). In general, Abel equations are classified as non-integrable ODEs. The most comprehensive list of solvable Abel equations, whose solutions are expressed in quadratures (or through special functions) in parametric form, is provided in the handbooks [24, 25]. For the integration of many of these equations, the discrete-group method developed by Zaitsev [26,27] was used (see also [25]), which is based on cyclic chains of Bäcklund transformations combined with several point transformations.

It is important to note that five of the six equations that DeepSeek failed to solve are not found in the handbook by Kamke [23] (this is a basic reference on ODEs for testing AI neural networks and specialized computer algebra systems Mathematica and Maple) or in the book by Murphi [28].

A complete report (including the necessary intermediate calculations and our additional questions) on the construction of solutions to the test equations (27)–(40) by the DeepSeek neural network can be viewed online at:
https://chat.deepseek.com/share/ypl8c94nwp6tq361fk,
https://chat.deepseek.com/share/yvhpawyjnp4wm7amml,
https://chat.deepseek.com/share/59yufjp695rp8xo4m9.

The analysis of the results from applying DeepSeek to solve ODEs reveals a lack of a reference and information based on solvable Abel equations and generalized Emden–Fowler equations of the form $y''_{xx} = ax^k y^m (y'_x)^n$ (a four-parameter family which includes ODE (39)), as well as a lack of knowledge regarding methods for integrating these and related equations. The complete information on these topics can be found in the books [24, 25, 27].

Remark 9. Of the six equations (29), (31), (33), (38)–(40), which DeepSeek failed to solve, the Gemini was able to solve four equations (29), (31), (38), (40) and failed to solve two equations (33), (39).

Remark 10. Of the six aforementioned ODEs that DeepSeek failed to solve, Mathematica managed to solve only one equation (31).

6 Equations of mathematical physics and other partial differential equations

Next, the authors provided the following definition, which DeepSeek was instructed to use when solving the test problems.

Exact solutions to nonlinear equations of mathematical physics are understood as the following types of solutions:

1. Solutions expressed in terms of elementary functions, functions included in the equation (this is necessary when the equation depends on arbitrary functions), and indefinite integrals. Solutions may be presented in explicit, implicit, or parametric form.

2. Solutions expressed in terms of solutions to ODEs or systems of ODEs. Overdetermined systems of ODEs are not included in this definition.

Several simple examples showed that DeepSeek is proficient at finding exact solutions of the traveling wave type, self-similar solutions, and solutions with simple (multiplicative and additive) separation of variables. Consequently, it was assigned multivariate test problems of research grade, where the type of sought solution was not prespecified. Such test problems are valuable to use since AI can often discover solutions whose existence a researcher may be unaware of or might not suspect.

Below are the statements of the test problems, along with DeepSeek's answers and our comments (all equations are taken from the handbooks [16, 21]). In this section, the testing results and the obtained and the missing solutions are described in greater detail than was done previously.

Test problem 1. *Find as many exact solutions as you can to the nonlinear equation*

$$u_t = (uu_x)_x. \tag{42}$$

DeepSeek found five solutions to equation (42): a traveling wave solution, a self-similar solution, a solution with additive separation of variables, a solution with multiplicative separation of variables, and a solution with generalized separation of variables that is quadratic in x (the process took 238 seconds in total). It is possible that DeepSeek could have found several other solutions had we asked it to continue (the handbooks [16, 21] describe 9 solutions for this equation). Instead, we gave it an additional hint: *Try to find another solution using generalized separation of variables in the form* $u = a(t)x^2 + b(t)x^k$*, where* $a(t)$ *and* $b(t)$ *are unknown functions, and* k *is an unknown constant.* Following this, DeepSeek found a solution with $k = 1/2$.

The complete report on the construction of solutions for this equation can be found at: `https://chat.deepseek.com/share/l05lqt6yqel77tyw4f`.

Test problem 2. *Find as many exact solutions as you can to the nonlinear equation*

$$u_{xx} = u_y u_{yy}. \tag{43}$$

DeepSeek found two solutions (with additive and multiplicative separation of variables) for equation (43) (the entire process took 321 seconds).

1°. We asked an additional question: *Does the considered equation admit a self-similar solution?* After this, DeepSeek found a self-similar solution.

2°. We further requested: *Show that the considered equation has an exact solution in the form of a third-degree polynomial in y with functional coefficients depending on x.* After this, DeepSeek found such a solution.

3°. We additionally asked: *Show that the considered equation has an exact solution of the form $u = a(x) + b(x)y^{3/2} + c(x)y^3$.* After this, DeepSeek found such a solution.

The complete report on the construction of solutions for equation (43) can be found at: https://chat.deepseek.com/share/acmdcywq4p03ucoihv.

It should be noted that the Guderley equation (43) arises in the theory of transonic gas flows. The solutions described above, as well as some others for this equation, can be found in [21] (see also [16,29,30]). This equation can be linearized via the Legendre transformation [21].

Test problem 3. *Find as many exact solutions as you can to the nonlinear equation*

$$u_t = u_{xx} + u_x^2 + ku^2, \tag{44}$$

where k is a free parameter.

DeepSeek found only a traveling wave solution and solutions for $k = 0$ (the entire process took 209 seconds).

We asked an additional question: *Does the considered equation admit a solution of the form $u = a(t) + b(t)\cos(cx)$, where a and b are unknown functions and c is an unknown constant?* Following this, DeepSeek proved that a solution exists for $c = \sqrt{k}$ and, in doing so, also showed that for $k < 0$ there exists a solution of the form $u = a(t) + b(t)\cosh(cx)$ with $c = \sqrt{-k}$.

The complete report on the construction of solutions for equation (44) can be found via the link provided at the end of Test problem 2.

Test problem 4. *Find as many exact solutions as you can to the nonlinear equation*

$$u_y u_{xy} - u_x u_{yy} = u_{yyy}. \tag{45}$$

DeepSeek found three nondegenerate solutions: solutions with additive and generalized separation of variables (both linear in x), a solution with multiplicative separation of variables, as well as a degenerate solution independent of x (the process took 168 seconds).

We asked an additional question: *Does the considered equation admit a solution of the form $u = a(x) + b(x)\exp(cy)$, where a and b are unknown* *functions, and c is an unknown constant?* Following this, DeepSeek found such a solution, which depends on an arbitrary function $b = b(x)$.

The complete report on the construction of solutions for equation (45) can be found at: https://chat.deepseek.com/share/acmdcywq4p03ucoihv.

It should be noted that equation (45) describes the boundary layer arising from the longitudinal flow of a viscous incompressible fluid near a flat plate (u is the stream function). The solutions described above, along with a number of other exact solutions to this equation, can be found in [16, 21]. The order of equation (45) can be reduced using the von Mises transformation [21].

Test problem 5. *Find as many exact solutions as you can to the nonlinear equation*

$$u_t = (ax^n u_x)_x + bu \ln u, \tag{46}$$

where a and b are free parameters.

DeepSeek found a solution with multiplicative separation of variables, and also, unexpectedly for us, found a new solution for $n = 1$ with functional separation of variables of the form $u = \exp[\beta(t)x + \gamma(t)]$. Note that equation (46) is a special case of the equation considered in [16, 21].

The complete report on the construction of solutions for equation (46) can be found at: https://chat.deepseek.com/share/acmdcywq4p03ucoihv

Test problem 6. *Find exact solutions to the nonlinear equation*

$$u_t = u_{xx} + x^2 f(u), \tag{47}$$

where $f(u)$ is an arbitrary function.

DeepSeek made an error: it found a self-similar solution of the form $u = U(z)$, where $z = xt^{-1/2}$, which does not exist. We wrote: *Check the obtained self-similar solution by direct substitution.* DeepSeek then confirmed that no self-similar solutions exist.

We then provided an additional request: *Try seeking a solution of the generalized traveling wave type in the form $u = U(z)$, where $z = t + \varphi(x)$. Here $\varphi(x)$ and $U(z)$ are the unknown functions.* After this, DeepSeek found this solution.

The complete report on the construction of solutions for equation (47) can be found at: https://chat.deepseek.com/share/acmdcywq4p03ucoihv

Test problem 7. *Find as many exact solutions as you can to the nonlinear equation*

$$u_t = (e^u u_x)_x. \tag{48}$$

DeepSeek found three nondegenerate solutions: a solution with additive separation of variables, a traveling wave solution, an invariant solution (which generalizes the self-similar solution given in [21]), and a stationary solution of little interest (the process took 220 minutes). It should be noted that [21] describes two additional exact solutions to equation (48).

The complete report on the construction of solutions for equation (48) can be found at: https://chat.deepseek.com/share/acmdcywq4p03ucoihv

Test problem 8. *Find exact solutions (as many as possible) to the nonlinear Schrödinger equation*

$$iu_t + u_{xx} + f(|u|)u = 0, \tag{49}$$

where u is a complex-valued function of real variables and $i^2 = -1$.

DeepSeek described two solutions (indicating the information was taken from literature): a periodic solution with a stationary amplitude and a solution with an amplitude depending on a traveling wave variable. It should be noted that four other exact solutions of equation (49) are described in [21].

Test problem 9. *Find a solution in closed form to the equation*

$$u_t = u_{xx} + x^2 \cos u. \tag{50}$$

This problem is a special case of Test problem 6 and was posed in a new chat window (to check whether DeepSeek retains memory of previous problems). Just as before, DeepSeek failed to solve this problem.

Remark 11. The Gemini also failed to find a solution to equation (50).

Test problem 10. *Find exact solutions to the equation*

$$u_{xy}^2 - u_{xx}u_{yy} = f(x)y^k, \tag{51}$$

where $f(x)$ is an arbitrary function, and k is a free parameter, which are expressible in quadratures or through solutions of an ordinary differential equation.

DeepSeek found one solution and missed two (these solutions are given in [21]). We then provided an additional request: *Try seeking solutions in the form $u = \varphi(x)y^{k+2} + \psi(x)$*. After this, DeepSeek found both missing solutions and presented them in a unified formula.

The complete report on the construction of solutions for equation (51) can be found at: https://chat.deepseek.com/share/ml1z4zykmzdv1gk8xy

Test problem 11. *Find exact solutions to the equation*

$$u_{xy}^2 - u_{xx}u_{yy} = f(x)u^k, \tag{52}$$

where $f(x)$ is an arbitrary function, and k is a free parameter, which are expressible in quadratures or through solutions of an ordinary differential equation.

DeepSeek found two solutions for $k \neq 2$: one of them is in [21], and the other is a new solution of the form

$$u = A(x+x_0)^{-\frac{1+k}{2-k}}[y+g(x)]^{\frac{3}{2-k}},$$
$$g''_{xx} = -\frac{(2-k)^3}{9(1+k)}A^{k-2}(x+x_0)^{1+k}f(x).$$

The complete report on the construction of solutions for equation (52) can be found at: https://chat.deepseek.com/share/ml1z4zykmzdv1gk8xy

Test problem 12. *Consider the equation*

$$u_t = u_{xx} + F(u, u_x). \tag{53}$$

It is required to find the most general function $F(u, w)$ for which this equation admits a self-similar solution.

DeepSeek correctly solved this problem (its solution is given in [19]).

A detailed account of the solution procedure for Test problem 12 is available at: https://chat.deepseek.com/share/w8ygqza37sqo1d0b66

Test problem 13. *Determine the type of the strongly nonlinear equation (elliptic, hyperbolic, parabolic):*

$$u_{xx}u_{yy} - u_{xy}^2 = f(x, y). \tag{54}$$

DeepSeek identified it as the Monge–Ampére equation and correctly described the type of this nonlinear equation (with detailed explanations), which is determined by the sign of the function f (for $f > 0$ it is an equation of elliptic type, and for $f < 0$ it is of hyperbolic type). It should be noted that the classification of equation (54) is described in [21].

A complete report on the solution of Test problem 13 can be found at: https://chat.deepseek.com/share/bvbdhiggpnuwz4nhwi

Test problem 14. *Consider the equation*

$$u_{xxxx} + au_{xxyy} + bu_{yyyy} = 0, \tag{55}$$

where a and b are free parameters. Question: Find the conditions on the parameters under which the considered equation will be elliptic and hyperbolic.

DeepSeek correctly wrote the sought conditions under which the considered equation will be elliptic and hyperbolic.

We asked an additional question: *Can you find the fundamental solution of the considered linear equation in the elliptic case?* DeepSeek obtained the solution by correctly deducing the general structure of the solution in polar coordinates with two unknown functions (this is a new result).

We gave an additional assignment: *Try to verify the obtained fundamental solution by direct substitution into the original equation.* It verified and confirmed that the solution is correct.

A complete report on the solution of Test problem 14 can be found at: https://chat.deepseek.com/share/bvbdhiggpnuwz4nhwi

It should be noted that equation (55) describes the displacements of an anisotropic linear-elastic medium. Some of its exact solutions can be found in [31]. A related three-dimensional equation was considered in [32] (see also [31, 33]).

Test problem 15. *Find as many exact solutions as you can to the nonlinear equation*

$$u_t = au_{xx} + uf(u_x^2 - au^2), \tag{56}$$

where $f(z)$ is an arbitrary functions, and a is a free parameter.

DeepSeek has solved this problem correctly.

A complete report on the solution of Test problem 15 can be found at: https://chat.deepseek.com/share/wiqnwg6f0occkqnkn9

Test problem 16. *Find as many exact solutions as you can to the nonlinear equation*

$$u_{tt} = (x^k u_x)_x + f(u), \tag{57}$$

where $f(u)$ is an arbitrary functions, and k is a free parameter.

DeepSeek found some exact solutions only for the power function $f(u)$, see https://chat.deepseek.com/share/v8cs4ml6nllbag6dim

1. An additional assignment was given: *To clarify: solutions to the equation $u_{tt} = (x^k u_x)_x + f(u)$ are sought in the form $u = U(z)$, where $z = \varphi(t) + \psi(x)$.* DeepSeek did not find a solution.

2. An additional assignment was given: *To simplify: solutions to the equation $u_{tt} = (x^k u_x)_x + f(u)$ are sought in the form $u = U(z)$, where $z = bt^2 + cx^m$, with b, c, and m being undetermined coefficients.* DeepSeek found this solution. See: https://chat.deepseek.com/share/jypo8yl9bnf3agolf9

Test problem 17. *Find exact solutions to the nonlinear equation*

$$u_{tt} = [f(u)u_x]_x, \tag{58}$$

where $f(u)$ is an arbitrary function.

DeepSeek found a traveling wave solution and a self-similar solution. See: https://chat.deepseek.com/share/1e4q9l29ludd8fnwoo

An additional assignment was given: *Show that the latter equation also admits a solution that can be presented in implicit form:*

$$x - t\sqrt{f(u)} = \varphi(u),$$

where $\varphi(u)$ *is an arbitrary function.* DeepSeek demonstrated that such a solution exists (the solution is given, for example, in [16]).

Test problem 18. *Find exact solutions to the nonlinear equation*

$$u_t = [f(u)u_x]_x + \frac{a}{f(u)} + b, \tag{59}$$

where $f(u)$ *is an arbitrary function, and* a *and* b *are free parameters.*

DeepSeek found a traveling wave solution and several degenerate solutions. See: https://chat.deepseek.com/share/mjvj2r9rpb2rqmlpj8.

An additional assignment was given: *Show that the latter equation also admits a solution that can be presented in implicit form:*

$$\int f(u)\,du = At + Bx^2 + Cx + D,$$

where A*,* B*,* C*, and* D *are the unknown constants.* DeepSeek demonstrated that such a solution exists (this solution is given, for example, in [16, 21]).

The conducted testing allows us to draw the following conclusions:

1. For nonlinear partial differential equations, DeepSeek successfully finds invariant solutions of the traveling wave type and self-similar solutions. However, it often misses solutions invariant under a combination of translation and scaling transformations.

2. DeepSeek effectively constructs solutions with additive and multiplicative separation of variables, as well as solutions with generalized separation of variables in the form of a quadratic polynomial in one of the independent variables. However, it cannot find solutions with more complex forms of generalized separation of variables, as well as solutions with functional separation of variables.

Importantly, during the testing process, DeepSeek discovered several new solutions to partial differential equations.

7 Brief conclusions

While solving the test mathematical equations considered in Sections 2–6, DeepSeek performed numerous pointwise transformations, differentiated expressions, computed definite and indefinite integrals, utilized various integral transforms and their inverses, successfully handled special functions and infinite series, etc. Thus, it can be stated that DeepSeek has a good command of the fundamental concepts of mathematical analysis and can apply them successfully in practice.

It is also noted that, in addition to the test problems described in Sections 2–6, DeepSeek solved a number of Diophantine equations, which are not presented here, demonstrating very good results. A complete description of the formulations of test problems and solutions of Diophantine equations can be found at: https://chat.deepseek.com/share/aa42ggcizytj0oxck7

When solving mathematical equations, DeepSeek occasionally made mistakes, but with the help of guiding questions or hints, it ultimately found the solution.

The results of the tests conducted in Sections 2–4 show that DeepSeek appears to be better at finding solutions to algebraic, functional, and integral equations than the specialized computer algebra systems Mathematica and Maple.

Overall, the testing results demonstrate that DeepSeek is a user-friendly and powerful analytical tool useful for researchers, professors and lecturers, graduate students, and students specializing in applied mathematics, physics, mechanics, and engineering sciences.

Acknowledgments

The authors thank A.V. Aksenov and D.A. Polyanin for discussions and useful comments.

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