Fourier Cosine Transforms: Expressions with Trigonometric Functions

No	*Original function*, $f(x)$	*Cosine transform*, $\check{f}_c(u) = \int_0^\infty f(x)\cos(ux)\,dx$
1	$x^{\nu-1}\sin(ax), \quad a>0,\ \lvert\nu\rvert<1$	$\pi\dfrac{(u+a)^{-\nu}-\lvert u+a\rvert^{-\nu}\operatorname{sign}(u-a)}{4\Gamma(1-\nu)\cos\left(\frac{1}{2}\pi\nu\right)}$
2	$e^{-bx}\sin(ax), \quad a,b>0$	$\dfrac{1}{2}\left[\dfrac{a+u}{(a+u)^2+b^2}+\dfrac{a-u}{(a-u)^2+b^2}\right]$
3	$\dfrac{1}{x}\sin^2(ax), \quad a>0$	$\dfrac{1}{4}\ln\left\lvert 1-4\dfrac{a^2}{u^2}\right\rvert$
4	$\dfrac{1}{x^2}\sin^2(ax), \quad a>0$	$\begin{cases}\frac{1}{4}\pi(2a-u) & \text{if } u<2a,\\ 0 & \text{if } u>2a\end{cases}$
5	$\dfrac{1}{x}\sin\left(\dfrac{a}{x}\right), \quad a>0$	$\dfrac{\pi}{2}J_0\left(2\sqrt{au}\right)$
6	$\dfrac{1}{\sqrt{x}}\sin\left(a\sqrt{x}\right)\sin\left(b\sqrt{x}\right), \quad a,b>0$	$\sqrt{\dfrac{\pi}{u}}\sin\left(\dfrac{ab}{2u}\right)\sin\left(\dfrac{a^2+b^2}{4u}-\dfrac{\pi}{4}\right)$
7	$\sin\left(ax^2\right), \quad a>0$	$\sqrt{\dfrac{\pi}{8a}}\left[\cos\left(\dfrac{u^2}{4a}\right)-\sin\left(\dfrac{u^2}{4a}\right)\right]$
8	$\dfrac{1-\cos(ax)}{x}, \quad a>0$	$\dfrac{1}{2}\ln\left\lvert 1-\dfrac{a^2}{u^2}\right\rvert$
9	$x^{\nu-1}\cos(ax), \quad a>0,\ 0<\nu<1$	$\frac{1}{2}\Gamma(\nu)\cos\left(\frac{1}{2}\pi\nu\right)\left[\lvert u-a\rvert^{-\nu}+(u+a)^{-\nu}\right]$
10	$e^{-bx}\cos(ax), \quad a,b>0$	$\dfrac{b}{2}\left[\dfrac{1}{(a+u)^2+b^2}+\dfrac{1}{(a-u)^2+b^2}\right]$
11	$\dfrac{1}{\sqrt{x}}\cos\left(a\sqrt{x}\right)$	$\sqrt{\dfrac{\pi}{u}}\sin\left(\dfrac{a^2}{4u}+\dfrac{\pi}{4}\right)$
12	$\dfrac{1}{\sqrt{x}}\cos\left(a\sqrt{x}\right)\cos\left(b\sqrt{x}\right)$	$\sqrt{\dfrac{\pi}{u}}\cos\left(\dfrac{ab}{2u}\right)\sin\left(\dfrac{a^2+b^2}{4u}+\dfrac{\pi}{4}\right)$
13	$\cos\left(ax^2\right), \quad a>0$	$\sqrt{\dfrac{\pi}{8a}}\left[\cos\left(\frac{1}{4}a^{-1}u^2\right)+\sin\left(\frac{1}{4}a^{-1}u^2\right)\right]$

Notation: $J_0(z)$ is the Bessel function of the first kind, $\Gamma(z)$ is the gamma function.

References

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