Fourier Sine Transforms: Expressions with Logarithmic Functions

No	*Original function*, $f(x)$	*Sine transform*, $\check{f}_s(u) = \int_0^\infty f(x)\sin(ux)\,dx$
1	$\begin{cases} \ln x & \text{if } 0 < x < 1, \\ 0 & \text{if } 1 < x \end{cases}$	$\frac{1}{u}\left[\mathrm{Ci}(u) - \ln u - \mathcal{C}\right]$
2	$\frac{\ln x}{x}$	$-\frac{1}{2}\pi(\ln u + \mathcal{C})$
3	$\frac{\ln x}{\sqrt{x}}$	$-\sqrt{\frac{\pi}{2u}}\left[\ln(4u) + \mathcal{C} - \frac{\pi}{2}\right]$
4	$x^{\nu-1}\ln x, \quad \lvert\nu\rvert < 1$	$\frac{\pi u^{-\nu}\left[\psi(\nu) + \frac{\pi}{2}\cot\left(\frac{\pi\nu}{2}\right) - \ln u\right]}{2\Gamma(1-\nu)\cos\left(\frac{\pi\nu}{2}\right)}$
5	$\ln\left\lvert\frac{a+x}{a-x}\right\rvert, \quad a > 0$	$\frac{\pi}{u}\sin(au)$
6	$\ln\frac{(x+b)^2+a^2}{(x-b)^2+a^2}, \quad a, b > 0$	$\frac{2\pi}{u}e^{-au}\sin(bu)$
7	$e^{-ax}\ln x, \quad a > 0$	$\frac{a\arctan(u/a) - \frac{1}{2}u\ln(u^2+a^2) - e^C u}{u^2+a^2}$
8	$\frac{1}{x}\ln\left(1+a^2x^2\right), \quad a > 0$	$-\pi\,\mathrm{Ei}\left(-\frac{u}{a}\right)$

Notation: $\mathcal{C} = 0.5772\ldots$ is the Euler constant, $\mathrm{Ci}(z)$ is the integral cosine, $\mathrm{Ei}(z)$ is the integral exponent, $\Gamma(z)$ is the gamma function, $\psi(z)$ is the logarithmic derivative of the gamma function.

References

Bateman, H. and Erdélyi, A., *Tables of Integral Transforms. Vols. 1 and 2*, McGraw-Hill Book Co., New York, 1954.

Ditkin, V. A. and Prudnikov, A. P., *Integral Transforms and Operational Calculus*, Pergamon Press, New York, 1965.

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations* , CRC Press, Boca Raton, 1998.

