



Inverse Laplace Transforms: Expressions with Error Functions

No	<i>Laplace transform</i> , $\tilde{f}(p)$	<i>Inverse transform</i> , $f(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{px} \tilde{f}(p) dp$
1	$\exp(ap^2) \operatorname{erfc}(p\sqrt{a})$	$\frac{1}{\sqrt{\pi a}} \exp\left(-\frac{x^2}{4a}\right)$
2	$\frac{1}{p} \exp(ap^2) \operatorname{erfc}(p\sqrt{a})$	$\operatorname{erf}\left(\frac{x}{2\sqrt{a}}\right)$
3	$\operatorname{erfc}(\sqrt{ap}), \quad a > 0$	$\begin{cases} 0 & \text{if } 0 < x < a, \\ \frac{\sqrt{a}}{\pi x \sqrt{x-a}} & \text{if } a < x. \end{cases}$
4	$\frac{1}{\sqrt{p}} \operatorname{erfc}(\sqrt{ap}), \quad a > 0$	$\begin{cases} 0 & \text{if } 0 < x < a, \\ \frac{1}{\sqrt{\pi x}} & \text{if } x > a. \end{cases}$
5	$e^{ap} \operatorname{erfc}(\sqrt{ap})$	$\frac{\sqrt{a}}{\pi \sqrt{x} (x+a)}$
6	$\frac{1}{\sqrt{p}} e^{ap} \operatorname{erfc}(\sqrt{ap})$	$\frac{1}{\sqrt{\pi(x+a)}}$
7	$\frac{1}{\sqrt{p}} \operatorname{erf}(\sqrt{ap}), \quad a > 0$	$\begin{cases} \frac{1}{\sqrt{\pi x}} & \text{if } 0 < x < a, \\ 0 & \text{if } x > a. \end{cases}$
8	$\operatorname{erf}(\sqrt{a/p})$	$\frac{1}{\pi x} \sin(2\sqrt{ax})$
9	$\frac{1}{\sqrt{p}} \exp(a/p) \operatorname{erf}(\sqrt{a/p})$	$\frac{1}{\sqrt{\pi x}} \sinh(2\sqrt{ax})$
10	$\frac{1}{\sqrt{p}} \exp(a/p) \operatorname{erfc}(\sqrt{a/p})$	$\frac{1}{\sqrt{\pi x}} \exp(-2\sqrt{ax})$

Notation: $\operatorname{erf} z$ is the error function and $\operatorname{erfc} z$ is the complementary error function.

References

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