ISSUE 2007

PROGRESS IN PHYSICS

VOLUME 4

ISSN 1555-5534

The Journal on Advanced Studies in Theoretical and Experimental Physics, including Related Themes from Mathematics

PROGRESS IN PHYSICS

A quarterly issue scientific journal, registered with the Library of Congress (DC, USA). This journal is peer reviewed and included in the abstracting and indexing coverage of: Mathematical Reviews and MathSciNet (AMS, USA), DOAJ of Lund University (Sweden), Zentralblatt MATH (Germany), Referativnyi Zhurnal VINITI (Russia), etc.

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Postal address for correspondence:
Chair of the Department
of Mathematics and Science,
University of New Mexico,
200 College Road,
Gallup, NM 87301, USA

ISSN: 1555-5534 (print)
ISSN: 1555-5615 (online)

Standard Address Number: 297-5092
Printed in the United States of America

OCTOBER 2007 **VOLUME 4**

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On the Gravitational Field of a Pulsating Source

Nikias Stavroulakis

Solomou 35, 15233 Chalandri, Greece

E-mail: nikias.stavroulakis@yahoo.fr

Because of the pseudo-theorem of Birkhoff, the important problem related to the dynamical gravitational field of a non-stationary spherical mass is ignored by the relativists. A clear formulation of this problem appears in the paper [5], which deals also with the establishment of the appropriate form of the spacetime metric. In the present paper we establish the corresponding equations of gravitation and bring out their solutions.

1 Introduction

As is shown in the paper [5], the propagation of gravitation from a spherical pulsating source is governed by a function $\pi(t,\rho)$, termed *propagation function*, satisfying the following conditions

$$\frac{\partial \pi(t,\rho)}{\partial t} > 0, \quad \frac{\partial \pi(t,\rho)}{\partial \rho} \leqslant 0, \quad \pi(t,\sigma(t)) = t,$$

where $\sigma(t)$ denotes the time-dependent radius of the sphere bounding the matter. The propagation function is not uniquely defined. Any function fulfilling the above conditions characterizes the propagation of gravitation according to the following rule: If the gravitational disturbance reaches the sphere $\|x\| = \rho$ at the instant t, then $\tau = \pi(t,\rho)$ is the instant of its radial emission from the entirety of the sphere bounding the matter. Among the infinity of possible choices of $\pi(t,\rho)$, we distinguish principally the one identified with the time coordinate, namely the propagation function giving rise to the *canonical* $\Theta(4)$*-invariant metric*

$$ds^2 = \left(f(\tau,\rho)d\tau + \ell(\tau,\rho)\,\frac{x\,dx}{\rho}\right)^2 - \\ -\left[\left(\frac{g(\tau,\rho)}{\rho}\right)^2 dx^2 + \left(\big(\ell(\tau,\rho)\big)^2 - \left(\frac{g(\tau,\rho)}{\rho}\right)^2\right)\frac{(x\,dx)^2}{\rho^2}\right] \tag{1.1}$$

(here τ denotes the time coordinate instead of the notation u used in the paper [5]).

Any other $\Theta(4)$-invariant metric results from (1.1) if we replace τ by a conveniently chosen propagation function $\pi(t,\rho)$. Consequently the general form of a $\Theta(4)$-invariant metric outside the matter can be written as

$$ds^2 = \left[\left(f\big(\pi(t,\rho),\rho\big)\frac{\partial \pi(t,\rho)}{\partial t}\right)dt + \\ + \left(f\big(\pi(t,\rho),\rho\big)\frac{\partial \pi(t,\rho)}{\partial \rho} + \ell\big(\pi(t,\rho),\rho\big)\right)\frac{x\,dx}{\rho}\right]^2 - \\ -\left[\left(\frac{g\big(\pi(t,\rho),\rho\big)}{\rho}\right)^2 dx^2 + \left(\Big(\ell\big(\pi(t,\rho),\rho\big)\Big)^2 - \\ -\left(\frac{g\big(\pi(t,\rho),\rho\big)}{\rho}\right)^2\right)\frac{(x\,dx)^2}{\rho^2}\right]. \tag{1.2}$$

The equations of gravitation related to (1.2) are very complicated, but we do not need to write them explicitly, because the propagation function occurs in them as an arbitrary function. So their solution results from that of the equations related to (1.1) if we replace τ by a general propagation function $\pi(t,\rho)$. It follows that the investigation of the $\Theta(4)$-invariant gravitational field must by based on the canonical metric (1.1). The metric (1.2) indicates the dependence of the gravitational field upon the general propagation function $\pi(t,\rho)$, but it is of no interest in dealing with specific problems of gravitation for the following reason. Each allowable propagation function is connected with a certain conception of time, so that the infinity of allowable propagation functions introduces an infinity of definitions of time with respect to the general $\Theta(4)$-invariant metric. This is why the notion of time involved in (1.2) is not clear.

On the other hand, the notion of time related to the canonical metric, although unusual, is uniquely defined and conceptually easily understandable.

This being said, from now on we will confine ourselves to the explicit form of the canonical metric, namely

$$ds^2 = \big(f(\tau,\rho)\big)^2 d\tau^2 + 2f(\tau,\rho)\,\ell(\tau,\rho)\frac{(x\,dx)}{\rho}\,d\tau - \\ -\left(\frac{g(\tau,\rho)}{\rho}\right)^2 dx^2 + \left(\frac{g(\tau,\rho)}{\rho}\right)^2\frac{(x\,dx)^2}{\rho^2} \tag{1.3}$$

which brings out its components:

$$g_{00} = \big(f(\tau,\rho)\big)^2, \qquad g_{0i} = f(\tau,\rho)\,\ell(\tau,\rho)\frac{x_i}{\rho},$$

$$g_{ii} = -\left(\frac{g(\tau,\rho)}{\rho}\right)^2 + \left(\frac{g(\tau,\rho)}{\rho}\right)^2\frac{x_i^2}{\rho^2},$$

$$g_{ij} = \left(\frac{g(\tau,\rho)}{\rho}\right)^2\frac{x_i x_j}{\rho^2}, \qquad (i,j = 1,2,3;\ i \neq j).$$

Note that, since the canonical metric, on account of its own definition, is conceived outside the matter, we have not to bother ourselves about questions of differentiability on the subspace $\mathbb{R}\times\{(0,0,0)\}$ of $\mathbb{R}\times\mathbb{R}^3$. It will be always understood that the spacetime metric is defined for $(\tau,\rho)\in\overline{U}$, $\rho = \|x\|$, $\overline{U}$ being the closed set $\{(\tau,\rho)\in\mathbb{R}^2 \,|\, \rho \geq \sigma(\tau)\}$.

2 Summary of auxiliary results

We recall that the Christoffel symbols of second kind related to a given $\Theta(4)$-invariant spacetime metric [3] are the components of a $\Theta(4)$-invariant tensor field and depend on ten functions $B_\alpha = B_\alpha(t,\rho)$, $(\alpha = 0,1,\ldots,9)$, according to the following formulae

$$\Gamma^0_{00} = B_0,\ \ \Gamma^0_{0i} = \Gamma^0_{i0} = B_1 x_i\,,\quad \Gamma^i_{00} = B_2 x_i\,,$$

$$\Gamma^0_{ii} = B_3 + B_4 x_i^2\,,\ \ \Gamma^0_{ij} = \Gamma^0_{ji} = B_4 x_i x_j\,,$$

$$\Gamma^i_{i0} = \Gamma^i_{0i} = B_5 + B_6 x_i^2\,,\ \ \Gamma^i_{j0} = \Gamma^i_{0j} = B_6 x_i x_j\,,$$

$$\Gamma^i_{ii} = B_7 x_i^3 + (B_8 + 2B_9)x_i\,,$$

$$\Gamma^i_{jj} = B_7 x_i x_j^2 + B_8 x_i\,,\quad \Gamma^j_{ij} = \Gamma^j_{ji} = B_7 x_i x_j^2 + B_9 x_i\,,$$

$$\Gamma^i_{jk} = B_7 x_i x_j x_k\,,\quad (i,j,k = 1,2,3;\ i \neq j \neq k \neq i).$$

We recall also that the corresponding Ricci tensor is a symmetric $\Theta(4)$-invariant tensor defined by four functions Q_{00}, Q_{01}, Q_{11}, Q_{22}, the computation of which is carried out by means of the functions B_α occurring in the Christoffel symbols:

$$Q_{00} = \frac{\partial}{\partial t}(3B_5 + \rho^2 B_6) - \rho\frac{\partial B_2}{\partial \rho} - B_2(3 + 4\rho^2 B_9 - \rho^2 B_1 + \rho^2 B_8 + \rho^2 B_7) - 3B_0B_5 + 3B_5^2 + \rho^2 B_6(-B_0 + 2B_5 + \rho^2 B_6)\,,$$

$$Q_{01} = \frac{\partial}{\partial t}(\rho^2 B_7 + B_8 + 4B_9) - \frac{1}{\rho}\frac{\partial B_5}{\partial \rho} - \rho\frac{\partial B_6}{\partial \rho} + B_2(B_3 + \rho^2 B_4) - 2B_6(2 + \rho^2 B_9) - B_1(3B_5 + \rho^2 B_6)\,,$$

$$Q_{11} = -\frac{\partial B_3}{\partial t} - \rho\frac{\partial B_8}{\partial \rho} - (B_0 + B_5 + \rho^2 B_6)B_3 + (1 - \rho^2 B_8)(B_1 + \rho^2 B_7 + B_8 + 2B_9) - 3B_8\,,$$

$$Q_{22} = -\frac{\partial B_4}{\partial t} + \frac{1}{\rho}\frac{\partial}{\partial \rho}(B_1 + B_8 + 2B_9) + B_1^2 + B_8^2 - 2B_9^2 - 2B_1B_9 + 2B_3B_6 + (-B_0 - B_5 + \rho^2 B_6)B_4 + \big(-3 + \rho^2(-B_1 + B_8 - 2B_9)\big)B_7\,.$$

3 The Ricci tensor related to the canonical metric (1.3)

In order to find out the functions B_α, $(\alpha = 0,1,\ldots,9)$, resulting from the metric (1.3), we have simply to write down the explicit expressions of the Christoffel symbols Γ^0_{00}, Γ^0_{01}, Γ^1_{00}, Γ^0_{11}, Γ^1_{01}, Γ^1_{12}, Γ^1_{22}, thus obtaining

$$B_0 = \frac{1}{f}\frac{\partial f}{\partial \tau} + \frac{1}{\ell}\frac{\partial \ell}{\partial \tau} - \frac{1}{\ell}\frac{\partial f}{\partial \rho}\,,\qquad B_1 = 0\,,$$

$$B_2 = -\frac{f}{\rho\ell^2}\frac{\partial \ell}{\partial \tau} + \frac{f}{\rho\ell^2}\frac{\partial f}{\partial \rho}\,,$$

$$B_3 = \frac{g}{\rho^2 f\ell}\frac{\partial g}{\partial \rho}\,,\qquad B_4 = -\frac{g}{\rho^4 f\ell}\frac{\partial g}{\partial \rho}\,,$$

$$B_5 = \frac{1}{g}\frac{\partial g}{\partial \tau}\,,\qquad B_6 = \frac{1}{\rho^2\ell}\frac{\partial f}{\partial \rho} - \frac{1}{\rho^2 g}\frac{\partial g}{\partial \tau}\,,$$

$$B_7 = -\frac{g}{\rho^5 f\ell}\frac{\partial g}{\partial \tau} + \frac{1}{\rho^3 f}\frac{\partial f}{\partial \rho} + \frac{1}{\rho^4} + \frac{g}{\rho^5\ell^2}\frac{\partial g}{\partial \rho} + \frac{1}{\rho^3\ell}\frac{\partial \ell}{\partial \rho} - \frac{2}{\rho^3 g}\frac{\partial g}{\partial \rho}\,,$$

$$B_8 = \frac{g}{\rho^3 f\ell}\frac{\partial g}{\partial \tau} + \frac{1}{\rho^2} - \frac{g}{\rho^3\ell^2}\frac{\partial g}{\partial \rho}\,,$$

$$B_9 = -\frac{1}{\rho^2} + \frac{1}{\rho g}\frac{\partial g}{\partial \rho}\,.$$

The conditions $B_1 = 0$, $B_3 + \rho^2 B_4 = 0$ imply several simplifications. Moreover an easy computation gives

$$Q_{11} + \rho^2 Q_{22} = 2\rho\frac{\partial B_9}{\partial \rho} - 2(1 + \rho^2 B_9)(B_8 + B_9 + \rho^2 B_7) + 4B_9\,.$$

Replacing now everywhere the functions B_α, $(\alpha = 0,1,\ldots,9)$, by their expressions, we obtain the four functions defining the Ricci tensor.

Proposition 3.1 *The functions Q_{00}, Q_{01}, Q_{11}, Q_{22} related to (1.3) are defined by the following formulae.*

$$\begin{aligned} Q_{00} = {} & \frac{1}{\ell}\frac{\partial^2 f}{\partial\tau\partial\rho} - \frac{f}{\ell^2}\frac{\partial^2 f}{\partial\rho^2} + \frac{f}{\ell^2}\frac{\partial^2 \ell}{\partial\tau\partial\rho} + \frac{2}{g}\frac{\partial^2 g}{\partial\tau^2} - \\ & - \frac{f}{\ell^3}\frac{\partial \ell}{\partial \tau}\frac{\partial \ell}{\partial \rho} + \frac{f}{\ell^3}\frac{\partial f}{\partial \rho}\frac{\partial \ell}{\partial \rho} + \frac{2f}{\ell^2 g}\frac{\partial \ell}{\partial \tau}\frac{\partial g}{\partial \rho} - \\ & - \frac{2f}{\ell^2 g}\frac{\partial f}{\partial \rho}\frac{\partial g}{\partial \rho} - \frac{2}{fg}\frac{\partial f}{\partial \tau}\frac{\partial g}{\partial \tau} - \frac{2}{\ell g}\frac{\partial \ell}{\partial \tau}\frac{\partial g}{\partial \tau} + \\ & + \frac{2}{\ell g}\frac{\partial f}{\partial \rho}\frac{\partial g}{\partial \tau} - \frac{1}{f\ell}\frac{\partial f}{\partial \tau}\frac{\partial f}{\partial \rho}\,, \end{aligned} \tag{3.1}$$

$$\begin{aligned} \rho Q_{01} = {} & \frac{\partial}{\partial \tau}\left(\frac{1}{f\ell}\frac{\partial (f\ell)}{\partial \rho}\right) - \frac{\partial}{\partial \rho}\left(\frac{1}{\ell}\frac{\partial f}{\partial \rho}\right) + \\ & + \frac{2}{g}\frac{\partial^2 g}{\partial\tau\partial\rho} - \frac{2}{\ell g}\frac{\partial f}{\partial \rho}\frac{\partial g}{\partial \rho}\,, \end{aligned} \tag{3.2}$$

$$\begin{aligned} \rho^2 Q_{11} = {} & -1 - \frac{2g}{f\ell}\frac{\partial^2 g}{\partial\tau\partial\rho} + \frac{g}{\ell^2}\frac{\partial^2 g}{\partial\rho^2} - \frac{2}{f\ell}\frac{\partial g}{\partial \tau}\frac{\partial g}{\partial \rho} - \\ & - \frac{g}{\ell^3}\frac{\partial \ell}{\partial \rho}\frac{\partial g}{\partial \rho} + \frac{1}{\ell^2}\left(\frac{\partial g}{\partial \rho}\right)^2 + \frac{g}{f\ell^2}\frac{\partial f}{\partial \rho}\frac{\partial g}{\partial \rho}\,, \end{aligned} \tag{3.3}$$

$$Q_{11} + \rho^2 Q_{22} = \frac{2}{g}\left(\frac{\partial^2 g}{\partial \rho^2} - \frac{\partial g}{\partial \rho}\frac{1}{f\ell}\frac{\partial(f\ell)}{\partial \rho}\right). \tag{3.4}$$

Note that from (3.1) and (3.2) we deduce the following useful relation

$$\ell Q_{00} - f\rho\, Q_{01} = \frac{2\ell}{g}\frac{\partial^2 g}{\partial \tau^2} + \frac{2f}{\ell g}\frac{\partial \ell}{\partial \tau}\frac{\partial g}{\partial \rho} - \\ - \frac{2\ell}{fg}\frac{\partial f}{\partial \tau}\frac{\partial g}{\partial \tau} - \frac{2}{g}\frac{\partial \ell}{\partial \tau}\frac{\partial g}{\partial \tau} + \frac{2}{g}\frac{\partial f}{\partial \rho}\frac{\partial g}{\partial \tau} - \frac{2f}{g}\frac{\partial^2 g}{\partial \tau \partial \rho}. \tag{3.5}$$

4 Reducing the system of the equations of gravitation

In order to clarify the fundamental problems with a minimum of computations, we will assume that the spherical source is not charged and neglect the cosmological constant. The charge of the source and the cosmological constant do not add difficulties in the discussion of the main problems, so that they may be considered afterwards.

Of course, the equations of gravitation outside the pulsating source are obtained by writing simply that the Ricci tensor vanishes, namely

$$Q_{00} = 0, \quad Q_{01} = 0, \quad Q_{11} = 0, \quad Q_{11} + \rho^2 Q_{22} = 0.$$

The first equation $Q_{00} = 0$ is to be replaced by the equation

$$\ell Q_{00} - f\rho\, Q_{01} = 0$$

which, on account of (3.5), is easier to deal with.

This being said, in order to investigate the equations of gravitation, we assume that the dynamical states of the gravitational field alternate with the stationary ones without diffusion of gravitational waves.

We begin with the equation $Q_{11} + \rho^2 Q_{22} = 0$, which, on account of (3.4), can be written as

$$\frac{\partial}{\partial \rho}\left(\frac{1}{f\ell}\frac{\partial g}{\partial \rho}\right) = 0$$

so that

$$\frac{\partial g}{\partial \rho} = \beta f \ell$$

where β is a function depending uniquely on the time τ.

Let us consider a succession of three intervals of time,

$$[\tau_1, \tau_2] \quad]\tau_2, \tau_3[\quad [\tau_3, \tau_4],$$

such that the gravitational field is stationary during $[\tau_1, \tau_2]$ and $[\tau_3, \tau_4]$ and dynamical during $]\tau_2, \tau_3[$.

When τ describes $[\tau_1, \tau_2]$ and $[\tau_3, \tau_4]$, the functions f, ℓ, g depend uniquely on ρ, so that β reduces then necessarily to a constant, which, according to the known theory of the stationary vacuum solutions, equals $\frac{1}{c}$, c being the classical constant (which, in the present situation, does not represent the velocity of propagation of light in vacuum). It follows that, if β depends effectively on τ during $]\tau_2, \tau_3[$, then it appears as a boundary condition at finite distance, like the radius and the curvature radius of the sphere bounding the matter. However, we cannot conceive a physical situation related to such a boundary condition. So we are led to assume that β is a universal constant, namely $\frac{1}{c}$, keeping this value even during the dynamical states of the gravitational field. However, before accepting finally the universal constancy of β, it is convenient to investigate the equations of gravitation under the assumption that β depends effectively on time during the interval $]\tau_2, \tau_3[$.

We first prove that $\beta = \beta(\tau)$ does not vanish in $]\tau_2, \tau_3[$. We argue by contradiction, assuming that $\beta(\tau_0) = 0$ for some value $\tau_0 \in]\tau_2, \tau_3[$. Then $\frac{\partial g}{\partial \rho}$ and $\frac{\partial^2 g}{\partial \rho^2} = \beta \frac{\partial(f\ell)}{\partial \rho}$ vanish for $\tau = \tau_0$, whereas $\frac{\partial^2 g}{\partial \tau \partial \rho} = (f\ell)\beta' + \beta\frac{\partial(f\ell)}{\partial \tau}$ reduces to $(f\ell)\beta'(\tau_0)$ for $\tau = \tau_0$. Consequently the equation $\rho^2 Q_{11} = 0$ reduces to the condition $1 + 2g\beta'(\tau_0) = 0$ whence $\beta'(\tau_0) < 0$ (since $g > 0$). It follows that $\beta(\tau)$ is strictly decreasing on a certain interval $[\tau_0 - \varepsilon, \tau_0 + \varepsilon] \subset]\tau_2, \tau_3[$, $\varepsilon > 0$, so that $\beta(\tau) < 0$ for every $\tau \in]\tau_0, \tau_0 + \varepsilon]$. Let τ_{00} be the least upper bound of the set of values $\tau \in]\tau_0 + \varepsilon, \tau_3[$ for which $\beta(\tau) = 0$ (This value exists because $\beta(\tau) = \frac{1}{c} > 0$ on $[\tau_3, \tau_4]$). Then $\beta(\tau_{00}) = 0$ and $\beta(\tau) > \tau_{00}$ for $\tau > \tau_{00}$. But, according to what has just been proved, the condition $\beta(\tau_{00}) = 0$ implies that $\beta(\tau) < 0$ on a certain interval $]\tau_{00}, \tau_{00} + \eta]$, $\eta > 0$, giving a contradiction. It follows that the function $\beta(\tau)$ is strictly positive on $]\tau_2, \tau_3[$, hence also on any interval of non-stationarity, and since $\beta(\tau) = \frac{1}{c}$ on the intervals of stationarity, it is strictly positive everywhere. Consequently we are allowed to introduce the inverse function $\alpha = \alpha(\tau) = \frac{1}{\beta(\tau)}$ and write

$$f\ell = \alpha \frac{\partial g}{\partial \rho} \tag{4.1}$$

and

$$f = \frac{\alpha}{\ell}\frac{\partial g}{\partial \rho}. \tag{4.2}$$

Inserting this expression of f into the equation $\rho^2 Q_{11} = 0$ and then multiplying throughout by $\frac{\partial g}{\partial \rho}$, we obtain an equation which can be written as

$$\frac{\partial}{\partial \rho}\left(-\frac{2g}{\alpha}\frac{\partial g}{\partial \tau} + \frac{g}{\ell^2}\left(\frac{\partial g}{\partial \rho}\right)^2 - g\right) = 0$$

whence

$$-\frac{2g}{\alpha}\frac{\partial g}{\partial \tau} + \frac{g}{\ell^2}\left(\frac{\partial g}{\partial \rho}\right)^2 - g = -2\mu = \text{function of } \tau,$$

and

$$\frac{\partial g}{\partial \tau} = \frac{\alpha}{2}\left(-1 + \frac{2\mu}{g} + \frac{1}{\ell^2}\left(\frac{\partial g}{\partial \rho}\right)^2\right). \tag{4.3}$$

It follows that

$$\frac{\partial^2 g}{\partial \tau \partial \rho} = \alpha\left(-\frac{\mu}{g^2}\frac{\partial g}{\partial \rho} - \frac{1}{\ell^3}\frac{\partial \ell}{\partial \rho}\left(\frac{\partial g}{\partial \rho}\right)^2 + \frac{1}{\ell^2}\frac{\partial g}{\partial \rho}\frac{\partial^2 g}{\partial \rho^2}\right) \tag{4.4}$$

and

$$\frac{\partial^3 g}{\partial\tau\partial\rho^2} = \alpha\left(\frac{2\mu}{g^3}\left(\frac{\partial g}{\partial\rho}\right)^2 - \frac{\mu}{g^2}\frac{\partial^2 g}{\partial\rho^2} + \right.$$
$$+ \frac{3}{\ell^4}\left(\frac{\partial\ell}{\partial\rho}\right)^2\left(\frac{\partial g}{\partial\rho}\right)^2 - \frac{1}{\ell^3}\frac{\partial^2\ell}{\partial\rho^2}\left(\frac{\partial g}{\partial\rho}\right)^2 - \tag{4.5}$$
$$\left. - \frac{4}{\ell^3}\frac{\partial\ell}{\partial\rho}\frac{\partial g}{\partial\rho}\frac{\partial^2 g}{\partial\rho^2} + \frac{1}{\ell^2}\left(\frac{\partial^2 g}{\partial\rho^2}\right)^2 + \frac{1}{\ell^2}\frac{\partial g}{\partial\rho}\frac{\partial^3 g}{\partial\rho^3}\right).$$

On the other hand, since $f\ell = \alpha\frac{\partial g}{\partial\rho}$, the expression (3.2) is transformed as follows

$$\rho\, Q_{01} = \frac{1}{\left(\frac{\partial g}{\partial\rho}\right)^2}\left(\frac{\partial g}{\partial\rho}\frac{\partial^3 g}{\partial\tau\partial\rho^2} - \frac{\partial^2 g}{\partial\rho^2}\frac{\partial^2 g}{\partial\tau\partial\rho}\right) +$$
$$+ \alpha\left(- \frac{3}{\ell^4}\left(\frac{\partial\ell}{\partial\rho}\right)^2\frac{\partial g}{\partial\rho} + \frac{1}{\ell^3}\frac{\partial^2\ell}{\partial\rho^2}\frac{\partial g}{\partial\rho} + \right.$$
$$\left. + \frac{3}{\ell^3}\frac{\partial\ell}{\partial\rho}\frac{\partial^2 g}{\partial\rho^2} - \frac{1}{\ell^2}\frac{\partial^3 g}{\partial\rho^3} - \frac{2\mu}{g^3}\frac{\partial g}{\partial\rho}\right)$$

and replacing in it $\frac{\partial^2 g}{\partial\tau\partial\rho}$ and $\frac{\partial^3 g}{\partial\tau\partial\rho^2}$ by their expressions (4.4) and (4.5), we find $\rho\, Q_{01} = 0$. Consequently the equation of gravitation $\rho\, Q_{01} = 0$ is verified. It remains to examine the equation $\ell Q_{00} - f\rho\, Q_{01} = 0$. We need some preliminary computations. First we consider the expression of $\frac{\partial^2 g}{\partial\tau^2}$ resulting from the derivation of (4.3) with respect to τ, and then replacing in it $\frac{\partial g}{\partial\tau}$ and $\frac{\partial^2 g}{\partial\tau\partial\rho}$ by their expressions (4.3) and (4.4), we obtain

$$2\frac{\partial^2 g}{\partial\tau^2} = -\frac{d\alpha}{d\tau} + 2\frac{d\alpha}{d\tau}\frac{\mu}{g} + \frac{1}{\ell^2}\frac{d\alpha}{d\tau}\left(\frac{\partial g}{\partial\rho}\right)^2 +$$
$$+ \frac{2\alpha}{g}\frac{d\mu}{d\tau} - \frac{2\alpha^2\mu^2}{g^3} + \frac{\alpha^2\mu}{g^2} - \frac{2\alpha}{\ell^3}\frac{\partial\ell}{\partial\tau}\left(\frac{\partial g}{\partial\rho}\right)^2 - \tag{4.6}$$
$$- 3\frac{\alpha^2\mu}{\ell^2 g^2}\left(\frac{\partial g}{\partial\rho}\right)^2 - \frac{2\alpha^2}{\ell^5}\frac{\partial\ell}{\partial\rho}\left(\frac{\partial g}{\partial\rho}\right)^3 + \frac{2\alpha^2}{\ell^4}\left(\frac{\partial g}{\partial\rho}\right)^2\frac{\partial^2 g}{\partial\rho^2}.$$

Next, because of (4.2), we have

$$\frac{\partial f}{\partial\rho} = -\frac{\alpha}{\ell^2}\frac{\partial\ell}{\partial\rho}\frac{\partial g}{\partial\rho} + \frac{\alpha}{\ell}\frac{\partial^2 g}{\partial\rho^2} \tag{4.7}$$

and

$$\frac{\partial f}{\partial\tau} = \frac{1}{\ell}\frac{d\alpha}{d\tau}\frac{\partial g}{\partial\rho} - \frac{\alpha}{\ell^2}\frac{\partial\ell}{\partial\tau}\frac{\partial g}{\partial\rho} + \frac{\alpha}{\ell}\frac{\partial^2 g}{\partial\tau\partial\rho}.$$

Lastly taking into account (4.4), we obtain

$$\frac{\partial f}{\partial\tau} = \frac{1}{\ell}\frac{d\alpha}{d\tau}\frac{\partial g}{\partial\rho} - \frac{\alpha}{\ell^2}\frac{\partial\ell}{\partial\tau}\frac{\partial g}{\partial\rho} - \frac{\alpha^2\mu}{\ell g^2}\frac{\partial g}{\partial\rho} -$$
$$- \frac{\alpha^2}{\ell^4}\frac{\partial\ell}{\partial\rho}\left(\frac{\partial g}{\partial\rho}\right)^2 + \frac{\alpha^2}{\ell^3}\frac{\partial g}{\partial\rho}\frac{\partial^2 g}{\partial\rho^2}. \tag{4.8}$$

Now inserting (4.2), (4.3), (4.4), (4.6), (4.7), (4.8) into (3.5), we obtain, after cancelations, the very simple expression

$$\ell Q_{00} - f\rho\, Q_{01} = \frac{2\alpha\ell}{g^2}\frac{d\mu}{d\tau}.$$

Consequently the last equation of gravitation, namely $\ell Q_{00} - f\rho\, Q_{01} = 0$, implies that $\frac{d\mu}{d\tau} = 0$, namely that μ reduces to a constant.

Finally the system of the equations of gravitation is reduced to a system of two equations, namely (4.1) and (4.3), where μ is a constant valid whatever is the state of the field, and α is a strictly positive function of time reducing to the constant c during the stationary states of the field. As already remarked, if α depends effectively on τ during the dynamical states, then it plays the part of a boundary condition the origin of which is indefinable. The following reasoning, which is allowed according to the principles of General Relativity, corroborates the idea that α must be taken everywhere equal to c.

Since $\alpha(\tau) > 0$ everywhere, we can introduce the new time coordinate

$$u = \frac{1}{c}\int_{\tau_0}^{\tau}\alpha(v)dv$$

which amounts to a change of coordinate in the sphere bounding the matter. The function

$$\psi(\tau) = \frac{1}{c}\int_{\tau_0}^{\tau}\alpha(v)dv$$

being strictly increasing, its inverse $\tau = \varphi(u)$ is well defined and $\varphi' = \frac{1}{\psi'} = \frac{c}{\alpha}$. Instead of $\ell(\tau,\rho)$ and $g(\tau,\rho)$ we have now the functions $L(u,\rho) = \ell(\varphi(u),\rho)$ and $G(u,\rho) = g(\varphi(u),\rho)$,

Moreover, since $f d\tau = f\varphi' du$, $f(\tau,\rho)$ is replaced by the function $F(u,\rho) = \varphi'(u)f(\varphi(u),\rho) = \frac{c}{\alpha}f(\varphi(u),\rho)$.

It follows that

$$FL = \varphi' f\ell = \frac{c}{\alpha}\,\alpha\frac{\partial g}{\partial\rho} = c\frac{\partial G}{\partial\rho} \tag{4.9}$$

and

$$\frac{\partial G}{\partial u} = \frac{\partial g}{\partial\tau}\frac{d\tau}{du} = \frac{\alpha}{2}\left(- 1 + \frac{2\mu}{g} + \frac{1}{\ell^2}\left(\frac{\partial g}{\partial\rho}\right)^2\right)\frac{c}{\alpha} =$$
$$= \frac{c}{2}\left(- 1 + \frac{2\mu}{G} + \frac{1}{L^2}\left(\frac{\partial G}{\partial\rho}\right)^2\right). \tag{4.10}$$

Writing again $f(\tau,\rho)$, $\ell(\tau,\rho)$, $g(\tau,\rho)$ respectively instead of $F(u,\rho)$, $L(u,\rho)$, $G(u,\rho)$, we see that the equations (4.9) and (4.10) are rewritten as

$$f\ell = c\frac{\partial g}{\partial\rho} \tag{4.11}$$

$$\frac{\partial g}{\partial\tau} = \frac{c}{2}\left(- 1 + \frac{2\mu}{g} + \frac{1}{\ell^2}\left(\frac{\partial g}{\partial\rho}\right)^2\right). \tag{4.12}$$

So (4.1) and (4.3) preserve their form, but the function α is now replaced by the constant c. Finally we are allowed to dispense with the function α and deal subsequently with the equations (4.11) and (4.12).

5 Stationary and non-stationary solutions

If the field is stationary during a certain interval of time, then the derivative $\frac{\partial g}{\partial \tau}$ vanishes on this interval. The converse is also true. In order to clarify the situation, consider the succession of three intervals of time $]\tau_1, \tau_2[$, $[\tau_2, \tau_3]$, $]\tau_3, \tau_4[$ such that $]\tau_1, \tau_2[$ and $]\tau_3, \tau_4[$ be maximal intervals of non-stationarity, and $\frac{\partial g}{\partial \tau} = 0$ on $[\tau_2, \tau_3]$. Then we have on $[\tau_2, \tau_3]$ the equation

$$-1 + \frac{2\mu}{g} + \frac{1}{\ell^2}\left(\frac{\partial g}{\partial \rho}\right)^2 = 0$$

from which it follows that ℓ does not depend either on τ. On account of (4.11), this property is also valid for f. Consequently the vanishing of $\frac{\partial g}{\partial \tau}$ on $[\tau_2, \tau_3]$ implies the establishment of a stationary state.

During the stationary state we are allowed to introduce the radial geodesic distance

$$\delta = \int_0^\rho \ell(v)\,dv$$

and investigate subsequently the stationary equations in accordance with the exposition appearing in the paper [4]. Since

$$\delta = \beta(\rho)$$

is a strictly increasing function of ρ, the inverse function $\rho = \gamma(\delta)$ is well defined and allows to consider as function of δ every function of ρ. In particular the curvature radius $G(\delta) = g(\gamma(\delta))$ appears as a function of the geodesic distance δ and gives rise to a complete study of the stationary field. From this study it follows that the constant μ equals $\frac{km}{c^2}$ and that the solution $G(\delta)$ possesses the greatest lower bound 2μ. Moreover $G(\delta)$ is defined by the equation

$$\int_{2\mu}^{G} \frac{du}{\sqrt{1 - \frac{2\mu}{u}}} = \delta - \delta_0 \tag{5.1}$$

where δ_0 is a new constant unknown in the classical theory of gravitation. This constant is defined by means of the radius δ_1 and the curvature radius $\zeta_1 = G(\delta_1)$ of the sphere bounding the matter:

$$\delta_0 = \delta_1 - \sqrt{G(\delta_1)(G(\delta_1) - 2\mu)} - $$
$$- 2\mu \ln\left(\sqrt{\frac{G(\delta_1)}{2\mu}} + \sqrt{\frac{G(\delta_1)}{2\mu} - 1}\right).$$

So the values δ_1 and $\zeta_1 = G(\delta_1)$ constitute the boundary conditions at finite distance. Regarding $F = F(\delta) = f(\gamma(\delta))$, it is defined by means of G:

$$F = cG' = c\sqrt{1 - \frac{2\mu}{G}}\,, \quad (G \geqslant 2\mu)\,.$$

The so obtained solution does not extend beyond the interval $[\tau_2, \tau_3]$ and even its validity for $\tau = \tau_2$ and $\tau = \tau_3$ is questionable. The notion of radial geodesic distance does not make sense in the intervals of non-stationarity such as $]\tau_1, \tau_2[$ and $]\tau_3, \tau_4[$. Then the integral

$$\int_0^\rho \ell(\tau, v)\,dv$$

depends on the time τ and does not define an invariant length. As a way out of the difficulty we confine ourselves to the consideration of the radical coordinate related to the manifold itself, namely $\rho = \|x\|$.

Regarding the curvature radius $\zeta(\tau)$, it is needed in order to conceive the solution of the equations of gravitation. The function $g(\tau, \rho)$ must be so defined that $g(\tau, \sigma(\tau)) = \zeta(\tau)$. The functions $\sigma(\tau)$ and $\zeta(\tau)$ are the boundary conditions at finite distance for the non-stationary field. They are not directly connected with the boundary conditions of the stationary field defined by means of the radial geodesic distance.

6 On the non-stationary solutions

According to very strong arguments summarized in the paper [2], the relation $g \geqslant 2\mu$ is always valid outside the matter whatever is the state of the field. This is why the first attempt to obtain dynamical solutions was based on an equation analogous to (5.1), namely

$$\int_{2\mu}^{g} \frac{du}{\sqrt{1 - \frac{2\mu}{u}}} = \gamma(\tau, \rho)$$

where $\gamma(\tau, \rho)$ is a new function satisfying certain conditions. This idea underlies the results presented briefly in the paper [1]. However the usefulness of introduction of a new function is questionable. It is more natural to deal directly with the functions f, ℓ, g involved in the metric. In any case we have to do with two equations, namely (4.11) and (4.12), so that we cannot expect to define completely the three unknown functions. Note also that, even in the considered stationary solution, the equation (5.1) does not define completely the function G on account of the new unknown constant δ_0. In the general case there is no way to define the function $g(\tau, \rho)$ by means of parameters and simpler functions. The only available equation, namely (4.12), a partial differential equation including the unknown function $\ell(\tau, \rho)$, is, in fact, intractable. As a way out of the difficulties, we propose to consider the function $g(\tau, \rho)$ as a new entity required by the non-Euclidean structure involved in the dynamical gravitational field. In the present state of our knowledge, we confine ourselves to put forward the main features of $g(\tau, \rho)$ in the closed set

$$\overline{U} = \{(\tau, \rho) \in \mathbb{R}^2 | \rho \geqslant \sigma(\tau)\}.$$

Since the vanishing of f or ℓ would imply the degeneracy of the spacetime metric, these two functions are necessarily strictly positive on $\overline{U}$. Then from the equation (4.11) it follows that

$$\frac{\partial g(\tau,\rho)}{\partial\rho} > 0 \tag{6.1}$$

on the closed set $\overline{U}$. On the other hand, since (4.12) can be rewritten as

$$\frac{2}{c}\frac{\partial g}{\partial\tau} + 1 - \frac{2\mu}{g} = \frac{1}{\ell^2}\left(\frac{\partial g}{\partial\rho}\right)^2$$

we have also

$$\frac{2}{c}\frac{\partial g}{\partial\tau} + 1 - \frac{2\mu}{g} > 0 \tag{6.2}$$

on the closed set $\overline{U}$. Now, on account of (6.1) and (6.2), the equations (4.11) and (4.12) define uniquely the functions f and ℓ by means of g:

$$f = c\sqrt{\frac{2}{c}\frac{\partial g}{\partial\tau} + 1 - \frac{2\mu}{g}} \tag{6.3}$$

$$\ell = \frac{\partial g/\partial\rho}{\sqrt{\frac{2}{c}\frac{\partial g}{\partial\tau} + 1 - \frac{2\mu}{g}}}\,. \tag{6.4}$$

It is now obvious that the curvature radius $g(\tau,\rho)$ plays the main part in the conception of the gravitational field. Although it has nothing to do with coordinates, the relativists have reduced it to a so-called radial coordinate from the beginnings of General Relativity. This glaring mistake has given rise to intolerable misunderstandings and distorted completely the theory of the gravitational field.

Let $]\tau_1,\tau_2[$ be a maximal bounded open interval of non-stationarity. Then $\frac{\partial g}{\partial\tau} = 0$ for $\tau = \tau_1$ and $\tau = \tau_2$, but $\frac{\partial g}{\partial\tau} \neq 0$ on an open dense subset of $]\tau_1,\tau_2[$. So $\frac{\partial g}{\partial\tau}$ appears as a gravitational wave travelling to infinity, and it is natural to assume that $\frac{\partial g}{\partial\tau}$ tends uniformly to zero on $[\tau_1,\tau_2]$ as $\rho\to+\infty$. Of course the behaviour of $\frac{\partial g}{\partial\tau}$ depends on the boundary conditions which do not appear in the obtained general solution. They are to be introduced in accordance with the envisaged problem. In any case the gravitational disturbance plays the fundamental part in the conception of the dynamical gravitation, but the state of the field does not follow always a simple rule.

In particular, if the gravitational disturbance vanishes during a certain interval of time $[\tau_1,\tau_2]$, the function $g(\tau,\rho)$ does not depend necessarily only on ρ during $[\tau_1,\tau_2]$. In other words, the gravitational field does not follow necessarily the Huyghens principle contrary to the solutions of the classical wave equation in $\mathbb{R}^3$.

We deal briefly with the case of *a Huyghens type field*, namely a $\Theta(4)$-invariant gravitational field such that the vanishing of the gravitational disturbance on a time interval implies the establishment of a universal stationary state. Then the time is involved in the curvature radius by means of the boundary conditions $\sigma(\tau)$, $\zeta(\tau)$, so that $g(\tau,\rho)$ is in fact a function of $(\sigma(\tau),\zeta(\tau),\rho)$: $g(\sigma(\tau),\zeta(\tau),\rho)$. The corresponding expressions for f and ℓ result from (6.3) and (6.4):

$$f = c\sqrt{\frac{2}{c}\left(\frac{\partial g}{\partial\sigma}\,\sigma'(\tau) + \frac{\partial g}{\partial\zeta}\,\zeta'(\tau)\right) + 1 - \frac{2\mu}{g}}$$

$$\ell = \frac{\frac{\partial g}{\partial\rho}}{\sqrt{\frac{2}{c}\left(\frac{\partial g}{\partial\sigma}\,\sigma'(\tau) + \frac{\partial g}{\partial\zeta}\,\zeta'(\tau)\right) + 1 - \frac{2\mu}{g}}}$$

where g denotes $g(\sigma(\tau),\zeta(\tau),\rho)$.

If $\sigma'(\tau) = \zeta'(\tau) = 0$ during an interval of time, the boundary conditions $\sigma(\tau)$, $\zeta(\tau)$ reduce to positive constants σ_0, ζ_0 on this interval, so that the curvature radius defining the stationary states depends on the constants $\sigma_0, \zeta_0 : g(\sigma_0,\zeta_0,\rho)$. It is easy to write down the conditions satisfied by $g(\sigma_0,\zeta_0,\rho)$, considered as function of three variables.

Submitted on June 12, 2007
Accepted on June 13, 2007

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Dynamical 3-Space: Supernovae and the Hubble Expansion — the Older Universe without Dark Energy

Reginald T. Cahill

School of Chemistry, Physics and Earth Sciences, Flinders University, Adelaide 5001, Australia

E-mail: Reg.Cahill@flinders.edu.au

We apply the new dynamics of 3-space to cosmology by deriving a Hubble expansion solution. This dynamics involves two constants; G and α — the fine structure constant. This solution gives an excellent parameter-free fit to the recent supernova and gamma-ray burst redshift data without the need for "dark energy" or "dark matter". The data and theory together imply an older age for the universe of some 14.7Gyrs. The 3-space dynamics has explained the bore hole anomaly, spiral galaxy flat rotation speeds, the masses of black holes in spherical galaxies, gravitational light bending and lensing, all without invoking "dark matter" or "dark energy". These developments imply that a new understanding of the universe is now available.

1 Introduction

There are theoretical claims based on observations of Type Ia supernova (SNe Ia) redshifts [1, 2] that the universe expansion is accelerating. The cause of this acceleration has been attributed to an undetected "dark energy". Here the dynamical theory of 3-space is applied to Hubble expansion dynamics, with the result that the supernova and gamma-ray burst redshift data is well fitted without an acceleration effect and without the need to introduce any notion of "dark energy". So, like "dark matter", "dark energy" is an unnecessary and spurious notion. These developments imply that a new understanding of the universe is now available.

1.1 Dynamical 3-Space

At a deeper level an information-theoretic approach to modelling reality, *Process Physics* [3, 4], leads to an emergent structured "space" which is 3-dimensional and dynamic, but where the 3-dimensionality is only approximate, in that if we ignore non-trivial topological aspects of the space, then it may be embedded in a 3-dimensional geometrical manifold. Here the space is a real existent discrete fractal network of relationships or connectivities, but the embedding space is purely a mathematical way of characterising the 3-dimensionality of the network. Embedding the network in the embedding space is very arbitrary; we could equally well rotate the embedding or use an embedding that has the network translated or translating. These general requirements then dictate the minimal dynamics for the actual network, at a phenomenological level. To see this we assume at a coarse grained level that the dynamical patterns within the network may be described by a velocity field $\mathbf{v}(\mathbf{r}, t)$, where $\mathbf{r}$ is the location of a small region in the network according to some arbitrary embedding. The 3-space velocity field has been observed in at least 8 experiments [3, 4]. For simplicity we assume here that the global topology of the network is not significant for the local dynamics, and so we embed in an E^3, although a generalisation to an embedding in S^3 is straightforward and might be relevant to cosmology. The minimal dynamics is then obtained by writing down the lowest-order zero-rank tensors, of dimension $1/t^2$, that are invariant under translation and rotation, giving

$$\nabla\cdot\left(\frac{\partial\mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{v}\right)+\frac{\alpha}{8}\left((\mathrm{tr}D)^2-\mathrm{tr}(D^2)\right)=-4\pi G\rho\,, \quad (1)$$

$$D_{ij}=\frac{1}{2}\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right), \quad (2)$$

where $\rho(\mathbf{r}, t)$ is the effective matter density. The embedding space coordinates provide a coordinate system or frame of reference that is convenient to describing the velocity field, but which is not real. In *Process Physics* quantum matter are topological defects in the network, but here it is sufficient to give a simple description in terms of an effective density. G is Newton's gravitational constant, and describes the rate of non-conservative flow of space into matter, and data from the bore hole g anomaly and the mass spectrum of black holes reveals that α is the fine structure constant $\approx$1/137, to within experimental error [5, 6, 7].

Now the acceleration $\mathbf{a}$ of the dynamical patterns in the 3-space is given by the Euler or convective expression

$$\mathbf{a}(\mathbf{r},t)=\lim_{\Delta t\to 0}\frac{\mathbf{v}\big(\mathbf{r}+\mathbf{v}(\mathbf{r},t)\Delta t, t+\Delta t\big)-\mathbf{v}(\mathbf{r},t)}{\Delta t}= \\ =\frac{\partial\mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{v}\,. \quad (3)$$

As shown in [8] the acceleration $\mathbf{g}$ of quantum matter is identical to the acceleration of the 3-space itself, apart from vorticity and relativistic effects, and so the gravitational acceleration of matter is also given by (3). Eqn. (1) has black hole solutions for which the effective masses agree with observational data for spherical star systems [5, 6, 7]. Theses black holes also explain the flat rotation curves in spiral galaxies [9].

2 Supernova and gamma-ray burst data

The supernovae and gamma-ray bursts provide standard candles that enable observation of the expansion of the universe. The supernova data set used herein and shown in Figs. 2 and 3 is available at [10]. Quoting from [10] we note that Davis *et al.* [11] combined several data sets by taking the ESSENCE data set from Table 9 of Wood–Vassey *et al.* (2007) [13], using only the supernova that passed the light-curve-fit quality criteria. They took the HST data from Table 6 of Riess *et al.* (2007) [12], using only the supernovae classified as gold. To put these data sets on the same Hubble diagram Davis *et al.* used 36 local supernovae that are in common between these two data sets. When discarding supernovae with $z < 0.0233$ (due to larger uncertainties in the peculiar velocities) they found an offset of 0.037 ± 0.021 magnitude between the data sets, which they then corrected for by subtracting this constant from the HST data set. The dispersion in this offset was also accounted for in the uncertainties. The HST data set had an additional 0.08 magnitude added to the distance modulus errors to allow for the intrinsic dispersion of the supernova luminosities. The value used by Wood–Vassey *et al.* (2007) [13] was instead 0.10 mag. Davis *et al.* adjusted for this difference by putting the Gold supernovae on the same scale as the ESSENCE supernovae. Finally, they also added the dispersion of 0.021 magnitude introduced by the simple offset described above to the errors of the 30 supernovae in the HST data set. The final supernova data base for the distance modulus $\mu_{obs}(z)$ is shown in Figs. 2 and 3. The gamma-ray burst (GRB) data is from Schaefer [14].

3 Expanding 3-space — the Hubble solution

Suppose that we have a radially symmetric density $\rho(r,t)$ and that we look for a radially symmetric time-dependent flow $\mathbf{v}(\mathbf{r},t) = v(r,t)\hat{\mathbf{r}}$ from (1). Then $v(r,t)$ satisfies the equation, with $v' = \frac{\partial v(r,t)}{\partial r}$,

$$\frac{\partial}{\partial t}\left(\frac{2v}{r}+v'\right)+vv''+2\frac{vv'}{r}+(v')^2+ \\ +\frac{\alpha}{4}\left(\frac{v^2}{r^2}+\frac{2vv'}{r}\right)=-4\pi G\rho(r,t). \tag{4}$$

Consider first the zero energy case $\rho = 0$. Then we have a Hubble solution $v(r,t) = H(t)r$, a centreless flow, determined by

$$\dot{H}+\left(1+\frac{\alpha}{4}\right)H^2=0 \tag{5}$$

with $\dot{H} = \frac{dH}{dt}$. We also introduce in the usual manner the scale factor $R(t)$ according to $H(t) = \frac{1}{R}\frac{dR}{dt}$. We then obtain the solution

$$H(t)=\frac{1}{(1+\frac{\alpha}{4})t}=H_0\frac{t_0}{t};\quad R(t)=R_0\left(\frac{t}{t_0}\right)^{4/(4+\alpha)} \tag{6}$$

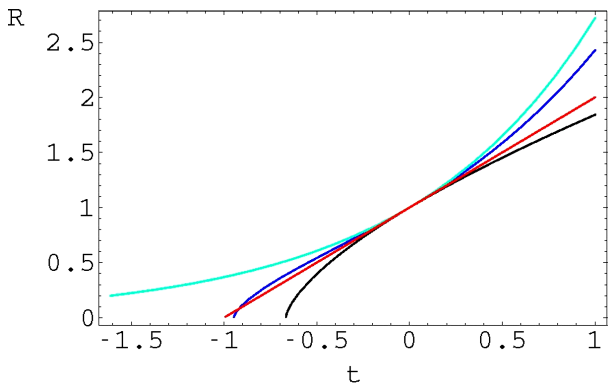

Fig. 1: Plot of the scale factor $R(t)$ vs t, with $t = 0$ being "now" with $R(0) = 1$, for the four cases discussed in the text, and corresponding to the plots in Figs. 2 and 3: (i) the upper curve (green) is the "dark energy" only case, resulting in an exponential acceleration at all times, (ii) the bottom curve (black) is the matter only prediction, (iii) the 2nd highest curve (to the right of $t = 0$) is the best-fit "dark energy" plus matter case (blue) showing a past deceleration and future exponential acceleration effect. The straight line plot (red) is the dynamical 3-space prediction showing a slightly older universe compared to case (iii). We see that the best-fit "dark energy"-matter curve essentially converges on the dynamical 3-space result. All plots have the same slope at $t = 0$, i.e. the same value of H_0. If the age of the universe is inferred to be some 14Gyrs for case (iii) then the age of the universe is changed to some 14.7Gyr for case (iv).

where $H_0 = H(t_0)$ and $R_0 = R(t_0)$. We can write the Hubble function $H(t)$ in terms of $R(t)$ via the inverse function $t(R)$, i.e. $H(t(R))$ and finally as $H(z)$, where the redshift observed now, t_0, relative to the wavelengths at time t, is $z = R_0/R - 1$. Then we obtain

$$H(z)=H_0(1+z)^{1+\alpha/4}. \tag{7}$$

We need to determine the distance travelled by the light from a supernova before detection. Using a choice of coordinate system with $r = 0$ at the location of a supernova the speed of light relative to this embedding space frame is $c + v(r(t),t)$, i.e. c wrt the space itself, where $r(t)$ is the distance from the source. Then the distance travelled by the light at time t after emission at time t_1 is determined implicitly by

$$r(t)=\int_{t_1}^{t}dt'\left(c+v\left(r(t'),t'\right)\right), \tag{8}$$

which has the solution on using $v(r,t) = H(t)r$

$$r(t)=cR(t)\int_{t_1}^{t}\frac{dt'}{R(t')}. \tag{9}$$

Expressed in terms of the observable redshift z this gives

$$r(z)=c(1+z)\int_0^z\frac{dz'}{H(z')}. \tag{10}$$

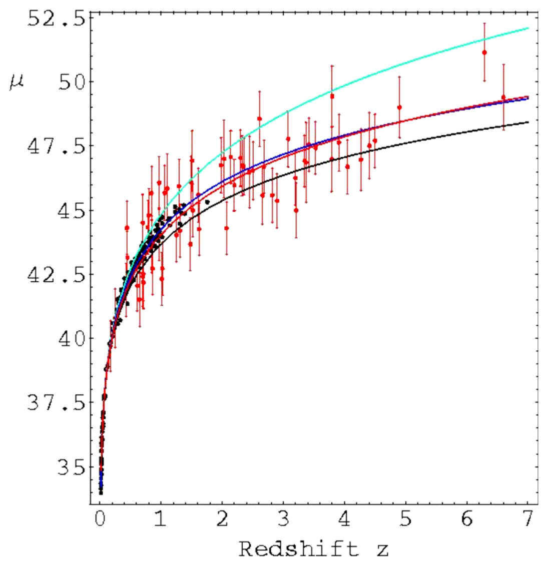

Fig. 2: Hubble diagram showing the combined supernovae data from Davis *et al.* [11] using several data sets from Riess *et al.* (2007) [12] and Wood-Vassey *et al.* (2007) [13] (dots without error bars for clarity — see Fig. 3 for error bars) and the Gamma-Ray Bursts data (with error bars) from Schaefer [14]. Upper curve (green) is "dark energy" only $\Omega_\Lambda = 1$, lower curve (black) is matter only $\Omega_m = 1$. Two middle curves show best-fit of "dark energy"-matter (blue) and dynamical 3-space prediction (red), and are essentially indistinguishable. However the theories make very different predictions for the future and for the age of the universe. We see that the best-fit 'dark energy'-matter curve essentially converges on the dynamical 3-space prediction.

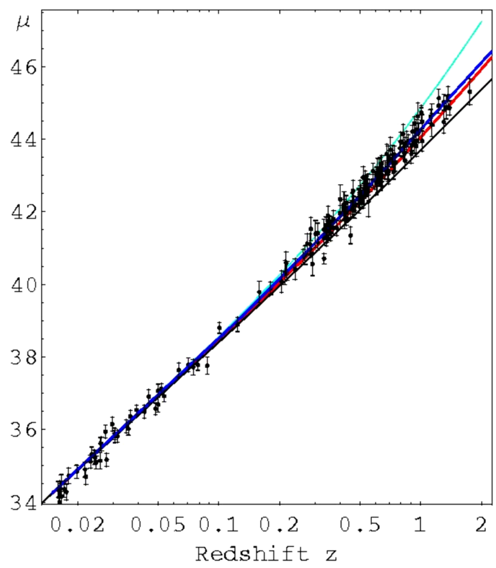

Fig. 3: Hubble diagram as in Fig. 2 but plotted logarithmically to reveal details for $z < 2$, and without GRB data. Upper curve (green) is "dark-energy" only, next curve down (blue) is best fit of "dark energy"-matter. Lower curve (black) is matter only $\Omega_m = 1$. Lower of two middle curves (red) is dynamical 3-space parameter-free prediction.

The effective dimensionless distance is given by

$$d(z) = (1+z)\int_0^z \frac{H_0 dz'}{H(z')} \tag{11}$$

and the theory distance modulus is then defined by

$$\mu_{th}(z) = 5\log_{10}\big(d(z)\big) + m\,. \tag{12}$$

Because all the selected supernova have the same absolute magnitude, m is a constant whose value is determined by fitting the low z data.

Using the Hubble expansion (7) in (11) and (12) we obtain the middle curves (red) in Figs. 2 and the 3, yielding an excellent agreement with the supernovae and GRB data. Note that because $\frac{\alpha}{4}$ is so small it actually has negligible effect on these plots. Hence the dynamical 3-space gives an immediate account of the universe expansion data, and does not require the introduction of a cosmological constant or "dark energy", but which will be nevertheless discussed next.

When the energy density is not zero we need to take account of the dependence of $\rho(r,t)$ on the scale factor of the universe. In the usual manner we thus write

$$\rho(r,t) = \frac{\rho_m}{R(t)^3} + \frac{\rho_r}{R(t)^4} + \Lambda \tag{13}$$

for matter, EM radiation and the cosmological constant or "dark energy" Λ, respectively, where the matter and radiation is approximated by a spatially uniform (i.e independent of r) equivalent matter density. We argue here that Λ — the dark energy density, like dark matter, is an unnecessary concept. Then (4) becomes for $R(t)$

$$\frac{\ddot{R}}{R} + \frac{\alpha}{4}\frac{\dot{R}^2}{R^2} = -\frac{4\pi G}{3}\left(\frac{\rho_m}{R^3} + \frac{\rho_r}{R^4} + \Lambda\right) \tag{14}$$

giving

$$\dot{R}^2 = \frac{8\pi G}{3}\left(\frac{\rho_m}{R} + \frac{\rho_r}{R^2} + \Lambda R^2\right) - \frac{\alpha}{2}\int \frac{\dot{R}^2}{R}dR\,. \tag{15}$$

In terms of $\dot{R}^2$ this has the solution

$$\dot{R}^2 = \frac{8\pi G}{3}\left(\frac{\rho_m}{(1-\frac{\alpha}{2})R} + \frac{\rho_r}{(1-\frac{\alpha}{4})R^2} + \frac{\Lambda R^2}{(1+\frac{\alpha}{4})} + bR^{-\frac{\alpha}{2}}\right) \tag{16}$$

which is easily checked by substitution into (15), where b is an arbitrary integration constant. Finally we obtain from (16)

$$t(R) = \int_{R_0}^{R} \frac{dR}{\sqrt{\frac{8\pi G}{3}\left(\frac{\rho_m}{R} + \frac{\rho_r}{R^2} + \Lambda R^2 + bR^{-\alpha/2}\right)}} \tag{17}$$

where now we have re-scaled parameters $\rho_m \to \rho_m/(1-\frac{\alpha}{2})$, $\rho_r \to \rho_r/(1-\frac{\alpha}{4})$ and $\Lambda \to \Lambda/(1+\frac{\alpha}{4})$. When $\rho_m = \rho_r = \Lambda = 0$, (17) reproduces the expansion in (6), and so the density terms in (16) give the modifications to the dominant purely spatial expansion, which we have noted above already gives an excellent account of the data.

From (17) we then obtain

$$H(z)^2 = H_0{}^2(\Omega_m\,(1+z)^3 + \Omega_r(1+z)^4 + \\ + \Omega_\Lambda + \Omega_s(1+z)^{2+\alpha/2}) \quad (18)$$

with

$$\Omega_m + \Omega_r + \Omega_\Lambda + \Omega_s = 1\,. \quad (19)$$

Using the Hubble function (18) in (11) and (12) we obtain the plots in Figs. 2 and 3 for four cases:

(i) $\Omega_m = 0$, $\Omega_r = 0$, $\Omega_\Lambda = 1$, $\Omega_s = 0$, i.e a pure "dark energy" driven expansion,

(ii) $\Omega_m = 1$, $\Omega_r = 0$, $\Omega_\Lambda = 0$, $\Omega_s = 0$ showing that a matter only expansion is not a good account of the data,

(iii) from a least squares fit with $\Omega_s = 0$ we find $\Omega_m = 0.28$, $\Omega_r = 0$, $\Omega_\Lambda = 0.68$ which led to the suggestion that the "dark energy" effect was needed to fix the poor fit from (ii), and finally

(iv) $\Omega_m = 0$, $\Omega_r = 0$, $\Omega_\Lambda = 0$, $\Omega_s = 1$, as noted above, that the spatial expansion dynamics alone gives a good account of the data.

Of course the EM radiation term Ω_r is non-zero but small and determines the expansion during the baryogenesis initial phase, as does the spatial dynamics expansion term because of the α dependence. If the age of the universe is inferred to be some 14Gyrs for case (iii) then, as seen in Fig. 1, the age of the universe is changed to some 14.7Gyr for case (iv). We see that the one-parameter best-fit "dark energy"-matter curve essentially converges on the no-parameter dynamical 3-space result.

4 Conclusions

There is extensive evidence for a dynamical 3-space, with the minimal dynamical equation now known and confirmed by numerous experimental and observational data. As well we have shown that this equation has a Hubble expanding 3-space solution that in a parameter-free manner manifestly fits the recent supernova data, and in doing so reveals that "dark energy", like "dark matter", is an unnecessary notion. The Hubble solution leads to a uniformly expanding universe, and so without acceleration: the claimed acceleration is merely an artifact related to the unnecessary "dark energy" notion. This result gives an older age for the universe of some 14.7Gyr, and resolves as well various problems such as the fine tuning problem, the horizon problem and other difficulties in the current modelling of the universe.

Submitted on June 20, 2007
Accepted on June 25, 2007

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Dynamical 3-Space: Alternative Explanation of the "Dark Matter Ring"

Reginald T. Cahill

School of Chemistry, Physics and Earth Sciences, Flinders University, Adelaide 5001, Australia

E-mail: Reg.Cahill@flinders.edu.au

NASA has claimed the discovery of a "Ring of Dark Matter" in the galaxy cluster CL 0024+17, see Jee M.J. *et al.* arXiv:0705.2171, based upon gravitational lensing data. Here we show that the lensing can be given an alternative explanation that does not involve "dark matter". This explanation comes from the new dynamics of 3-space. This dynamics involves two constant G and α — the fine structure constant. This dynamics has explained the bore hole anomaly, spiral galaxy flat rotation speeds, the masses of black holes in spherical galaxies, gravitational light bending and lensing, all without invoking "dark matter", and also the supernova redshift data without the need for "dark energy".

1 Introduction

Jee *et al.* [1] claim that the analysis of gravitational lensing data from the HST observations of the galaxy cluster CL 0024+17 demonstrates the existence of a "dark matter ring". While the lensing is clearly evident, as an observable phenomenon, it does not follow that this must be caused by some undetected form of matter, namely the putative "dark matter". Here we show that the lensing can be given an alternative explanation that does not involve "dark matter". This explanation comes from the new dynamics of 3-space [2, 3, 4, 5, 6]. This dynamics involves two constant G and α — the fine structure constant. This dynamics has explained the bore hole anomaly, spiral galaxy flat rotation speeds, the masses of black holes in spherical galaxies, gravitational light bending and lensing, all without invoking "dark matter". The 3-space dynamics also has a Hubble expanding 3-space solution that explains the supernova redshift data without the need for "dark energy" [8]. The issue is that the Newtonian theory of gravity [9], which was based upon observations of planetary motion in the solar system, missed a key dynamical effect that is not manifest in this system. The consequences of this failure has been the invoking of the fix-ups of "dark matter" and "dark energy". What is missing is the 3-space self-interaction effect. Experimental and observational data has shown that the coupling constant for this self-interaction is the fine structure constant, $\alpha \approx 1/137$, to within measurement errors. It is shown here that this 3-space self-interaction effect gives a direct explanation for the reported ring-like gravitational lensing effect.

2 3-space dynamics

As discussed elsewhere [2, 8] a deeper information — theoretic *Process Physics* has an emergent structured 3-space, where the 3-dimensionality is partly modelled at a phenomenological level by embedding the time- dependent structure in an E^3 or S^3 embedding space. This embedding space is not real — it serves to coordinatise the structured 3-space, that is, to provide an abstract frame of reference. Assuming the simplest dynamical description for zero-vorticity spatial velocity field $\mathbf{v}(\mathbf{r},t)$, based upon covariant scalars we obtain at lowest order [2]

$$\nabla\cdot\left(\frac{\partial\mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{v}\right)+\frac{\alpha}{8}\left((\mathrm{tr}D)^2-\mathrm{tr}(D^2)\right)=-4\pi G\rho\,, \quad (1)$$

$$\nabla\times\mathbf{v}=\mathbf{0}\,,\qquad D_{ij}=\frac{1}{2}\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right), \quad (2)$$

where $\rho(\mathbf{r},t)$ is the matter and EM energy density expressed as an effective matter density. In Process Physics quantum matter are topological defects in the structured 3-spaces, but here it is sufficient to give a simple description in terms of an effective density.

We see that there are two constants G and α. G turns out to be Newton's gravitational constant, and describes the rate of non-conservative flow of 3-space into matter, and α is revealed by experiment to be the fine structure constant. Now the acceleration $\mathbf{a}$ of the dynamical patterns of 3-space is given by the Euler convective expression

$$\mathbf{a}(\mathbf{r},t)=\lim_{\Delta t\to 0}\frac{\mathbf{v}\big(\mathbf{r}+\mathbf{v}(\mathbf{r},t)\Delta t,t+\Delta t\big)-\mathbf{v}(\mathbf{r},t)}{\Delta t}= \\ =\frac{\partial\mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{v} \quad (3)$$

and this appears in the first term in (1). As shown in [3] the acceleration of quantum matter $\mathbf{g}$ is identical to this acceleration, apart from vorticity and relativistic effects, and so the gravitational acceleration of matter is also given by (3). Eqn. (1) is highly non-linear, and indeed non-local. It exhibits a range of different phenomena, and as has been shown the α term is responsible for all those effects attributed to the undetected and unnecessary "dark matter". For example, outside of a spherically symmetric distribution of matter, of total mass M, we find that one solution of (1) is the velocity in-flow field

$$\mathbf{v}(\mathbf{r}) = -\hat{\mathbf{r}}\sqrt{\frac{2GM(1+\frac{\alpha}{2}+\ldots)}{r}} \qquad (4)$$

and then the the acceleration of (quantum) matter, from (3), induced by this in-flow is

$$\mathbf{g}(\mathbf{r}) = -\hat{\mathbf{r}}\,\frac{GM(1+\frac{\alpha}{2}+\ldots)}{r^2} \qquad (5)$$

which is Newton's Inverse Square Law of 1687 [9], but with an effective mass $M(1+\frac{\alpha}{2}+\ldots)$ that is different from the actual mass M.

In general because (1) is a scalar equation it is only applicable for vorticity-free flows $\nabla\times\mathbf{v}=\mathbf{0}$, for then we can write $\mathbf{v}=\nabla u$, and then (1) can always be solved to determine the time evolution of $u(\mathbf{r},t)$ given an initial form at some time t_0. The α-dependent term in (1) and the matter acceleration effect, now also given by (3), permits (1) to be written in the form

$$\nabla\cdot\mathbf{g} = -4\pi G\rho - 4\pi G\rho_{DM}\,, \qquad (6)$$

$$\rho_{DM}(\mathbf{r},t) \equiv \frac{\alpha}{32\pi G}\left((\mathrm{tr}D)^2 - \mathrm{tr}(D^2)\right), \qquad (7)$$

which is an effective "matter" density that would be required to mimic the α-dependent spatial self-interaction dynamics. Then (6) is the differential form for Newton's law of gravity but with an additional non-matter effective matter density. So we label this as ρ_{DM} even though no matter is involved [4, 5]. This effect has been shown to explain the so-called "dark matter" effect in spiral galaxies, bore hole g anomalies, and the systematics of galactic black hole masses.

The spatial dynamics is non-local. Historically this was first noticed by Newton who called it action-at-a-distance. To see this we can write (1) as an integro-differential equation

$$\frac{\partial\mathbf{v}}{\partial t} = -\nabla\left(\frac{\mathbf{v}^2}{2}\right) + G\int d^3r'\,\frac{\rho_{DM}(\mathbf{r}',t)+\rho(\mathbf{r}',t)}{|\mathbf{r}-\mathbf{r}'|^3}\,(\mathbf{r}-\mathbf{r}')\,. \qquad (8)$$

This shows a high degree of non-locality and non-linearity, and in particular that the behaviour of both ρ_{DM} and ρ manifest at a distance irrespective of the dynamics of the intervening space. This non-local behaviour is analogous to that in quantum systems and may offer a resolution to the horizon problem.

2.1 Spiral galaxy rotation anomaly

Eqn (1) gives also a direct explanation for the spiral galaxy rotation anomaly. For a non-spherical system numerical solutions of (1) are required, but sufficiently far from the centre, where we have $\rho=0$, we find an exact non-perturbative two-parameter class of analytic solutions

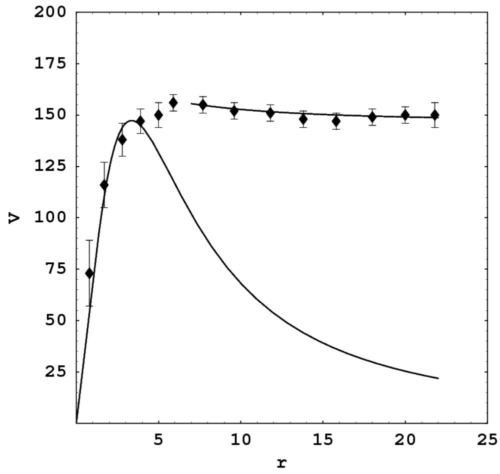

Fig. 1: Data shows the non-Keplerian rotation-speed curve v_O for the spiral galaxy NGC 3198 in km/s plotted against radius in kpc/h. Lower curve is the rotation curve from the Newtonian theory for an exponential disk, which decreases asymptotically like $1/\sqrt{r}$. The upper curve shows the asymptotic form from (11), with the decrease determined by the small value of α. This asymptotic form is caused by the primordial black holes at the centres of spiral galaxies, and which play a critical role in their formation. The spiral structure is caused by the rapid in-fall towards these primordial black holes.

$$\mathbf{v}(\mathbf{r}) = -\,\hat{\mathbf{r}}K\left(\frac{1}{r}+\frac{1}{R_s}\left(\frac{R_s}{r}\right)^{\frac{\alpha}{2}}\right)^{1/2} \qquad (9)$$

where K and R_s are arbitrary constants in the $\rho=0$ region, but whose values are determined by matching to the solution in the matter region. Here R_s characterises the length scale of the non-perturbative part of this expression, and K depends on α, G and details of the matter distribution. From (5) and (9) we obtain a replacement for the Newtonian "inverse square law"

$$\mathbf{g}(\mathbf{r}) = -\,\hat{\mathbf{r}}\,\frac{K^2}{2}\left(\frac{1}{r^2}+\frac{\alpha}{2rR_s}\left(\frac{R_s}{r}\right)^{\frac{\alpha}{2}}\right). \qquad (10)$$

The 1st term, $1/r^2$, is the Newtonian part. The 2nd term is caused by a "black hole" phenomenon that (1) exhibits. This manifests in different ways, from minimal supermassive black holes, as seen in spherical star systems, from globular clusters to spherical galaxies for which the black hole mass is predicted to be $M_{BH}=\alpha M/2$, as confirmed by the observational datas [2, 4, 5, 6, 7], to primordial supermassive black holes as seen in spiral galaxies as described by (9); here the matter spiral is caused by matter in-falling towards the primordial black hole.

The spatial-inflow phenomenon in (9) is completely different from the putative "black holes" of General Relativity — the new "black holes" have an essentially $1/r$ force law, up to $\mathrm{O}(\alpha)$ corrections, rather than the usual Newtonain and

Fig. 2: The "dark matter" density extracted by deconvolution of the gravitational lensing data for galaxy cluster CL 0024+17, see Jee M.J. *et al.* arXiv:0705.2171. Picture credit: NASA, ESA, M.J. Jee and H.C. Ford (John Hopkins University). The "dark matter" density has been superimposed on a HST image of the cluster. The axis of "symmetry" is perpendicular to the planer of this image. The gravitational lensing is caused by two galaxy clusters that have undergone collision. It is claimed herein that the lensing is associated with the 3-space interaction of these two "nearby" galaxy clusters, and not by the fact that they had collided, as claimed in [1]. The effect it is claimed, herein, is caused by the spatial in-flows into the black holes within the galaxies.

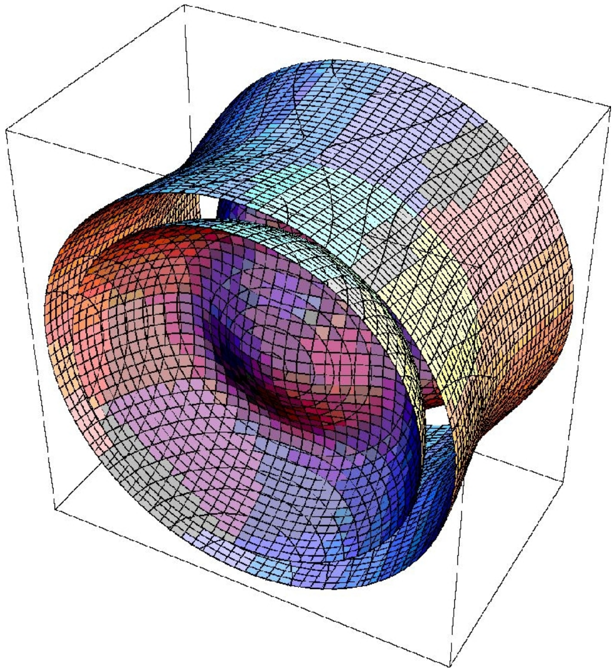

Fig. 3: Plot showing two constant value surfaces of $\Delta\rho_{DM}(\mathbf{r})$ from (19). We have modelled the system with two galaxies located on the axis of symmetry, but outside of the range of the plot. This plot shows the effects of the interfering spatial in-flows generating an effective "dark matter" density, as a spatial self-interaction effect. This "dark matter" density is that required to reproduce the gravitational acceleration if we used Newton's law of gravity. This phenomenon is caused by the α-dependent dynamics in (1), essentially a quantum-space effect. Viewed along the axis of symmetry this shell structure would appear as a ring-like structure, as seen in Fig. 2.

GR $1/r^2$ law. The centripetal acceleration relation for circular orbits $v_\circ(r) = \sqrt{rg(r)}$ gives a "universal rotation-speed curve"

$$v_\circ(r) = \frac{K}{2}\left(\frac{1}{r} + \frac{\alpha}{2R_s}\left(\frac{R_s}{r}\right)^{\frac{\alpha}{2}}\right)^{1/2}. \qquad (11)$$

Because of the α dependent part this rotation-velocity curve falls off extremely slowly with r, as is indeed observed for spiral galaxies. An example is shown in Fig. 1. It was the inability of the Newtonian and Einsteinian gravity theories to explain these observations that led to the notion of "dark matter".

For the spatial flow in (9) we may compute the effective dark matter density from (7)

$$\tilde{\rho}_{DM}(r) = \frac{(1-\alpha)\alpha}{16\pi G}\frac{K^2}{R_s^3}\left(\frac{R_s}{r}\right)^{2+\alpha/2}. \qquad (12)$$

It should be noted that the Newtonian component of (9) does not contribute, and that $\tilde{\rho}_{DM}(\mathbf{r})$ is exactly zero in the limit $\alpha \to 0$. So supermassive black holes and the spiral galaxy rotation anomaly are all α-dynamics phenomena.

2.2 Gravitational lensing

The spatial velocity field may be observed on the cosmological scale by means of the light bending and lensing effect. But first we must generalise the Maxwell equations so that the electric and magnetic fields are excitations of the dynamical 3-space, and not of the embedding space:

$$\nabla\times\mathbf{E} = -\mu\left(\frac{\partial\mathbf{H}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{H}\right), \quad \nabla\cdot\mathbf{E} = \mathbf{0}, \qquad (13)$$

$$\nabla\times\mathbf{H} = \epsilon\left(\frac{\partial\mathbf{E}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{E}\right), \quad \nabla\cdot\mathbf{H} = \mathbf{0}, \qquad (14)$$

which was first suggested by Hertz in 1890, but with $\mathbf{v}$ being a constant vector field. As easily determined the speed of EM radiation is $c = \frac{1}{\sqrt{\epsilon\mu}}$ wrt to the dynamical space, and not wt to the embedding space as in the original form of Maxwell's equations, and as light-speed anisotropy experiment have indicated [2]. The time-dependent and inhomogeneous velocity field causes the refraction of EM radiation. This can be computed by using the Fermat least-time approximation. Then the EM trajectory $\mathbf{r}(t)$ is determined by minimising the elapsed

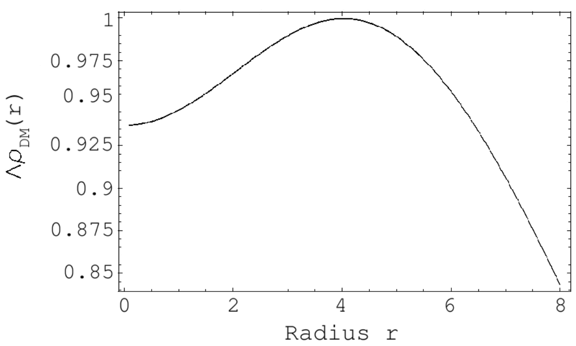

Fig. 4: Plot of $\triangle\rho_{DM}(\mathbf{r})$ from (19) in a radial direction from a mid-point on the axis joining the two galaxies.

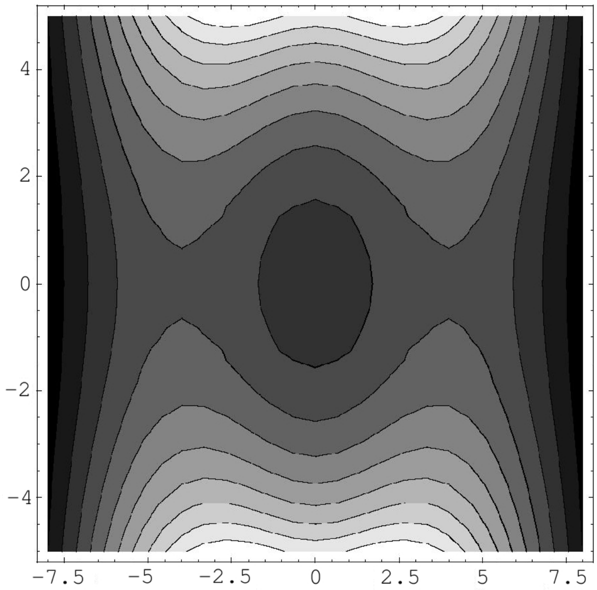

Fig. 5: Plot of $\triangle\rho_{DM}(\mathbf{r})$ from (19) in the plane containing the two galaxies. The two galaxies are located at +10 and -10, i.e above and below the vertical in this contour plot. This plot shows the effects of the interfering in-flows.

travel time:

$$\tau = \int_{s_i}^{s_f} \frac{ds \left|\frac{d\mathbf{r}}{ds}\right|}{|c\hat{\mathbf{v}}_R(s) + \mathbf{v}(\mathbf{r}(s), \mathbf{t}(s))|} \tag{15}$$

$$\mathbf{v}_R = \left(\frac{d\mathbf{r}}{dt} - \mathbf{v}(\mathbf{r}, \mathbf{t})\right) \tag{16}$$

by varying both $\mathbf{r}(s)$ and $t(s)$, finally giving $\mathbf{r}(t)$. Here s is a path parameter, and $\mathbf{v}_R$ is a 3-space tangent vector for the path. As an example, the in-flow in (4), which is applicable to light bending by the sun, gives the angle of deflection

$$\delta = 2\,\frac{v^2}{c^2} = \frac{4GM(1 + \frac{\alpha}{2} + \ldots)}{c^2 d} + \ldots \tag{17}$$

where v is the in-flow speed at distance d and d is the impact parameter. This agrees with the GR result except for the α correction. Hence the observed deflection of 8.4×10^{-6} radians is actually a measure of the in-flow speed at the sun's surface, and that gives $v = 615$ km/s. These generalised Maxwell equations also predict gravitational lensing produced by the large in-flows from (9) associated with the new "black holes" in galaxies. So again this effect permits the direct observation of the these black hole effects with their non inverse-square-law accelerations.

3 Galaxy Cluster lensing

It is straightforward to analyse the gravitational lensing predicted by a galaxy cluster, with the data from CL 0024+17 of particular interest. However rather than compute the actual lensing images, we shall compute the "dark matter" effective density from (7), and compare that with the putative "dark matter" density extracted from the actual lensing data in [1]. To that end we need to solve (1) for two reasonably close galaxies, located at positions $\mathbf{R}$ and $-\mathbf{R}$. Here we look for a perturbative modification of the 3-space in-flows when the two galaxies are nearby. We take the velocity field in 1st approximation to be the superposition

$$\mathbf{v}(\mathbf{r}) \approx \mathbf{v}(\mathbf{r} - \mathbf{R}) + \mathbf{v}(\mathbf{r} + \mathbf{R})\,, \tag{18}$$

where the RHS $\mathbf{v}$'s are from (9).

Substituting this in (1) will then generate an improved solution, keeping in mind that (1) is non-linear, and so this superposition cannot be exact. Indeed it is the non-linearity effect which it is claimed herein is responsible for the ring-like structure reported in [1]. Substituting (18) in (7) we may compute the change in the effective "dark matter" density caused by the two galaxies interfering with the in-flow into each separately, i.e.

$$\triangle\rho_{DM}(\mathbf{r}) = \rho_{DM}(\mathbf{r}) - \tilde{\rho}_{DM}(\mathbf{r} - \mathbf{R}) - \tilde{\rho}_{DM}(\mathbf{r} + \mathbf{R}) \tag{19}$$

$\tilde{\rho}_{DM}(\mathbf{r} \pm \mathbf{R})$ are the the effective "dark matter" densities for one isolated galaxy in (12). Several graphical representations of $\triangle\rho_{DM}(\mathbf{r})$ are given in Figs. 3, 4 and 5. We seen that viewed along the line of the two galaxies the change in the effective "dark matter" density has the form of a ring, in particular one should compare the predicted effective "dark matter" density in Fig. 3 with that found by deconvoluting the gravitaitaional lensing data shown in shown Fig. 2.

4 Conclusions

We have shown that the dynamical 3-space theory gives a direct account of the observed gravitational lensing caused by two galaxy clusters, which had previously collided, but that the ring-like structure is not related to that collision, contrary to the claims in [1]. The distinctive lensing effect is caused by interference between the two spatial in-flows, resulting in EM refraction which appears to be caused by the presence of a "matter" having the form of a ringed-shell structure, exactly comparable to the observed effect. This demonstrates yet another success of the new dynamical theory of 3-space, which like the bore hole, black hole and spiral galaxy rotation effects all reveal the dynamical consequences of the α-dependent term in (1). This amounts to a totally different understanding of the nature of space, and a completely different account of gravity. As shown in [3] gravity is a quantum effect where the quantum waves are refracted by the 3-space, and that analysis also gave a first derivation of the equivalence principle. We see again that "dark matter" and "dark energy" are spurious concepts required only because Newtonian gravity, and *ipso facto* GR, lacks fundamental processes of a dynamical 3-space — they are merely *ad hoc* fix-ups. We have shown elsewhere [7] that from (1) and the generalised Dirac equation we may show that a curved spacetime formalism may be introduced that permits the determination of the quantum matter geodesics, but that in general the spacetime metric does not satisfy the Hilbert-Einstein equations, as of course GR lacks the α-dependent dynamics. This induced spacetime has no ontological significance. At a deeper level the occurrence of α in (1) suggests that 3-space is actually a quantum system, and that (1) is merely a phenomenological description of that at the "classical" level. In which case the α-dependent dynamics amounts to the detection of quantum space and quantum gravity effects, although clearly not of the form suggested by the quantisation of GR.

Submitted on May 21, 2007
Accepted on June 25, 2007

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Quantum Spin Transport in Mesoscopic Interferometer

Walid A. Zein, Adel H. Phillips and Omar A. Omar

Faculty of Engineering, Ain Shams University, Cairo, Egypt
E-mail: adel_phillips@yahoo.com

Spin-dependent conductance of ballistic mesoscopic interferometer is investigated. The quantum interferometer is in the form of ring, in which a quantum dot is embedded in one arm. This quantum dot is connected to one lead via tunnel barrier. Both Aharonov-Casher and Aharonov-Bohm effects are studied. Our results confirm the interplay of spin-orbit coupling and quantum interference effects in such confined quantum systems. This investigation is valuable for spintronics application, for example, quantum information processing.

1 Introduction

The flexibility offered by semiconductor spintronics [1] is anticipated to lead to novel devices and may eventually become used for quantum information processing. Another advantage offered by spin systems in semiconductors is their long coherence times [2, 3]. In recent years, much attention has been devoted towards the interplay of the spin-orbit interaction and quantum interference effects in confined semiconductor heterostructures [4, 5, 6]. Such interplay can be exploited as a mean to control and manipulate the spin degree of freedom at mesoscopic scale useful for phase-coherent spintronics applications.

Since the original proposal of the spin field effect transistor (SFET) [7] by Datta and Das, many proposals have appeared based on intrinsic spin splitting properties of semiconductors associated with the Rashba spin-orbit interaction [8, 9, 10].

In the present paper, a quantum interference effect in coherent Aharonov-Casher ring is investigated. In such devices quantum effects are affecting transport properties.

2 The model

The mesoscopic device proposed in the present paper is in the form of quantum dot embedded in one arm of the Aharonov-Casher interferometer. This interferometer is connected to two conducting leads. The form of the confining potential in such spintronics device is modulated by an external gate electrode, allowing for direct control of the electron spin-orbit interaction. The main feature of the electron transport through such device is that the difference in the Aharonov-Casher phase of the electrons traveling clockwise and counterclockwise directions produces spin-sensitive interference effects [11, 12]. The quantum transport of the electrons occurs in the presence of Rashba spin-orbit coupling [13] and the influence of an external magnetic field. With the present proposed mesoscopic device, we can predict that the spin polarized current through such device is controlled via gate voltage.

The Hamiltonian, $\hat{H}$, describing the quantum transport through the present studied device could be written in the form as [14]

$$\hat{H} = \frac{P^2}{2m^*} + V(r) + \hat{H}_{soc}\,, \tag{1}$$

where $\hat{H}_{soc}$ is the Hamiltonian due to the spin-orbit coupling and is expressed as

$$\hat{H}_{soc} = \frac{\hbar^2}{2m^* a^2}\left(-i\,\frac{\partial}{\partial\varphi} + \frac{\omega_{soc}\, m^*\, a^2}{\hbar}\,\sigma_r\right), \tag{2}$$

where $\omega_{soc} = \frac{\alpha}{\hbar a}$ and it is called the frequency associated with the spin-orbit coupling, α is the strength of the spin-orbit coupling, a is the radius of the Aharonov-Casher ring and σ_r is the radial part of the Pauli matrices which expressed in the components of Pauli matrices σ_x, σ_y as

$$\begin{aligned} \sigma_r &= \sigma_x \cos\varphi + \sigma_y \sin\varphi\,, \\ \sigma_\varphi &= \sigma_y \cos\varphi - \sigma_x \sin\varphi\,. \end{aligned} \tag{3}$$

The parameter φ, Eq. (3) represents the phase difference of electrons passing through the upper and the lower arms of the ring. In Eq. (1), $V(r)$ is the effective potential for transmission of electrons through the quantum dot which depends, mainly, on the tunnel barrier between the quantum dot and the lead. Applying external magnetic field, B, normal to the plane of the device, then the Aharonov-Bohm phase picked up by an electron encircling this magnetic flux is given by

$$\Phi_{AB} = \frac{\pi\, e\, B\, a^2}{\hbar}\,. \tag{4}$$

Then the Hamiltonian, $\hat{H}_{soc}$, due to the spin-orbit coupling Eq. (2) will take the form

$$H'_{soc} = \frac{\hbar^2}{2m^* a^2}\left(-i\,\frac{\partial}{\partial\varphi} - \frac{\Phi_{AB}}{2\pi} - \frac{\omega_{soc}\, m^*\, a^2}{\hbar}\,\sigma_r\right). \tag{5}$$

Now in order to study the transport properties of the present quantum system, we have to solve Schrödinger equation and finding the eigenfunctions for this system as follows

$$\hat{H}\,\Psi = E\,\Psi\,. \tag{6}$$

The solution of Eq. (6) consists of four eigenfunctions [14], where $\Psi_L(x)$ is the eigenfunction for transmission through the left lead, $\Psi_R(x)$-for the right lead, $\Psi_{up}(\theta)$-for the upper arm of the ring and $\Psi_{low}(\theta)$-for the lower arm of the ring. Their forms will be as

$$\Psi_L(x) = \sum_\sigma \left[A\,e^{ikx} + B\,e^{-ikx}\right]\,\chi^\sigma(\pi)\,, \quad x \in [-\infty, 0]\,, \tag{7a}$$

$$\Psi_R(x) = \sum_\sigma \left[C\,e^{ikx'} + \,D\,e^{-ikx'}\right]\,\chi^\sigma(0)\,, \quad x \in [0, \infty]\,, \tag{7b}$$

$$\Psi_{up}(\varphi) = \sum_{\sigma,\mu} F_\mu\,e^{in'^\sigma_\mu\varphi}\,\chi^\sigma(\varphi)\,, \quad \varphi \in [0, \pi]\,, \tag{7c}$$

$$\Psi_{low}(\varphi) \;=\; \sum_{\sigma,\mu} G_\mu\,e^{in^\sigma_\mu\varphi}\,\chi^\sigma(\varphi)\,, \quad \varphi \in [\pi, 2\pi]\,. \tag{7d}$$

The mutually orthogonal spinors $\psi^\sigma_n(\varphi)$ are expressed in terms of the eigenvectors $\begin{pmatrix}1\\0\end{pmatrix}$, $\begin{pmatrix}0\\1\end{pmatrix}$ of the Pauli matrix σ_z as

$$\chi^{(1)}_n(\varphi) = \begin{pmatrix}\cos\frac{\theta}{2}\\ e^{i\varphi}\sin\frac{\theta}{2}\end{pmatrix}, \tag{7}$$

$$\chi^{(2)}_n(\varphi) = \begin{pmatrix}\sin\frac{\theta}{2}\\ -e^{i\varphi}\cos\frac{\theta}{2}\end{pmatrix}, \tag{8}$$

where the angle θ [15] is given by

$$\theta = 2\tan^{-1}\frac{\Omega - \sqrt{\Omega^2 + \omega^2_{soc}}}{\omega_{soc}} \tag{9}$$

in which Ω is given by

$$\Omega = \frac{\hbar}{2m^*\,a^2}\,. \tag{10}$$

The parameters n'^σ_μ and n^σ_μ are expressed respectively as

$$n'^\sigma_\mu = \mu\,k'\,a - \varphi + \frac{\Phi_{AB}}{2\pi} + \frac{\Phi^\sigma_{AC}}{2\pi}\,, \tag{11}$$

$$n^\sigma_\mu = \mu\,k\,a - \varphi + \frac{\Phi_{AB}}{2\pi} + \frac{\Phi^\sigma_{AC}}{2\pi}\,, \tag{12}$$

where $\mu = \pm 1$ corresponding to the spin up and spin down of transmitted electrons, Φ_{AB} is given by Eq. (4). The term Φ_{AC} represents the Aharonov-Casher phase and is given by

$$\Phi^{(\mu)}_{AC} = -\pi\left[1 + \frac{(-1)^\mu(\omega^2_{soc} + \Omega^2)^{1/2}}{\Omega}\right]. \tag{13}$$

The wave numbers k', k are given respectively by

$$k' = \sqrt{\frac{2m^*\,E}{\hbar^2}}\,, \tag{14}$$

$$k = \sqrt{\frac{2m^*}{\hbar^2}\left(V_d + e\,V_g + \frac{N^2 e^2}{2C} + E_F - E\right)}\,, \tag{15}$$

where V_d is the barrier height, V_g is the gate voltage, N is the number of electrons entering the quantum dot, C is the total capacitance of the quantum dot, m^* is the effective mass of electrons with energy, E, and charge, e, and E_F is the Fermi energy.

The conductance, G, for the present investigated device will be calculated using landauer formula [16] as

$$G = \frac{2e^2\sin\varphi}{h}\sum_{\mu=1,2}\int dE\left(-\frac{\partial f_{FD}}{\partial E}\right)|\,\Gamma_\mu(E)|^2\,, \tag{16}$$

where f_{FD} is the Fermi-Dirac distribution is function and $|\,\Gamma_\mu(E)|^2$ is tunneling probability. This tunneling probability could be obtained by applying the Griffith boundary conditions [15, 17, 18], which states that the eigenfunctions (Eqs. 7a, 7b, 7c, 7d) are continuous and that the current density is conserved at each intersection. Then the expression for $\Gamma_\mu(E)$ is given by

$$\Gamma_\mu(E) = \frac{8i\,\cos\frac{\Phi_{AB}+\Phi^{(\mu)}_{AC}}{2}\,\sin(\pi k a)}{4\cos(2\pi k' a) + 4\cos\left(\Phi_{AB} + \Phi^{(\mu)}_{AC}\right) + 4i\,\sin(2\pi k' a)}\,. \tag{17}$$

3 Results and discussion

In order to investigate the quantum spin transport characteristics through the present device, we solve Eqs. (17, 18) numerically. We use the heterostructures as InGaAs/InAlAs.

We calculate the conductance, G, at different both magnetic field and the ω_{soc} which depends on the Rashba spin-orbit coupling strength. The main features of our obtained results are:

1. Figs. 1 and Fig. 2 show the dependence of the conductance on the magnetic field, B, for small and large values of B at different ω_{soc}.
2. Fig. 3 shows the dependence of the conductance on the parameter ω_{soc} at different values of B.

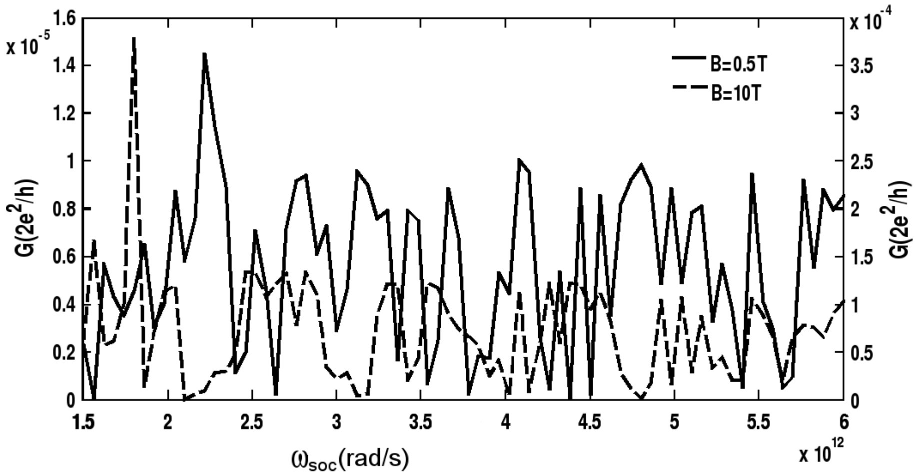

Fig. 3: The dependence of conductance on ω_{soc} at different values of B.

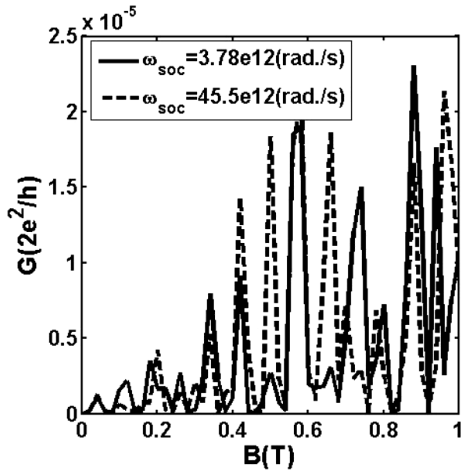

Fig. 1: The dependence of conductance on B at different ω_{soc} (small B).

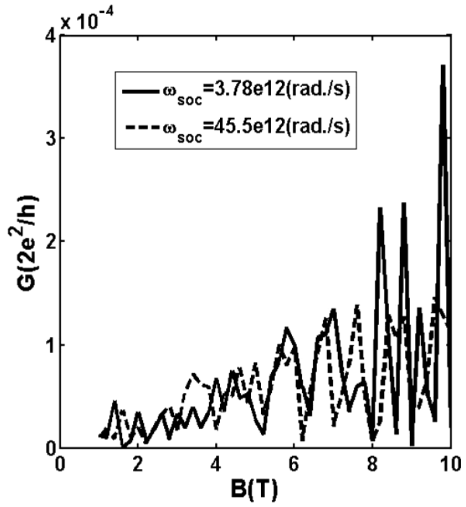

Fig. 2: The dependence of conductance on B at different ω_{soc} (large B).

From the figures we observe a quasi-periodic oscillations in the conductance (Fig. 1), and takes the form of satellite peaks. While for large values of B, the oscillations behave completely different from those in case of small values of B. The oscillatory behavior of $G(\omega_{soc})$ shows a wide peaks and in some ranges of ω_{soc}, there is a splitting in the peaks.

The obtained results could be explained as follows: The oscillatory behavior of the conductance with B and ω_{soc} could be due to spin-sensitive quantum-interference effects caused by the difference in the Aharonov-Casher phase accumulated by the opposite spin states. Also the quantum interference effects in the present device could be due to Aharonov-Bohm effect. Our results are found concordant with those in the literatures [4, 5, 15, 19].

4 Conclusions

In the present paper an expression for the conductance has been deduced for the investigated mesoscopic device. The spin transport in such coherent device is investigated taking into consideration both Aharonov-Casher and Aharonov-Bohm effects in the quantum dot connected to conducting lead via a tunnel barrier. The present results are valuable for employing such devices in phase coherent spintronics applications.

Submitted on August 15, 2007
Accepted on August 20, 2007

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Some Remarks on Ricci Flow and the Quantum Potential

Robert Carroll

University of Illinois, Urbana, IL 61801, USA

E-mail: rcarroll@math.uiuc.edu

We indicate some formulas connecting Ricci flow and Perelman entropy to Fisher information, differential entropy, and the quantum potential. There is a known relation involving the Schroedinger equation in a Weyl space where the Weyl-Ricci curvature is proportional to the quantum potential. The quantum potential in turn is related to Fisher information which is given via the Perelman entropy functional arising from a differential entropy under Ricci flow. These relations are written out and seem to suggest connections between quantum mechanics and Ricci flow.

1 Formulas involving Ricci flow

Certain aspects of Perelman's work on the Poincaré conjecture have applications in physics and we want to suggest a few formulas in this direction; a fuller exposition will appear in a book in preparation [8]. We go first to [13, 24–28, 33, 39] and simply write down a few formulas from [28, 39] here with minimal explanation. Thus one has Perelman's functional ($\dot{\mathcal{R}}$ is the Riemannian Ricci curvature)

$$\mathfrak{F} = \int_M (\dot{\mathcal{R}} + |\nabla f|^2)\exp(-f)dV \tag{1.1}$$

and a so-called Nash entropy $(\mathbf{1A})$ $N(u) = \int_M u\log(u)dV$ where $u = \exp(-f)$. One considers Ricci flows with $\delta g \sim \partial_t g = h$ and for $(\mathbf{1B})$ $\Box^* u = -\partial_t u - \Delta u + \dot{\mathcal{R}} u = 0$ (or equivalently $\partial_t f + \Delta f - |\nabla f|^2 + \dot{\mathcal{R}} = 0$) it follows that $\int_M \exp(-f)dV = 1$ is preserved and $\partial_t N = \mathfrak{F}$. Note the Ricci flow equation is $\partial_t g = -2Ric$. Extremizing $\mathfrak{F}$ via $\delta\mathfrak{F} \sim \sim \partial_t \mathfrak{F} = 0$ involves $Ric + Hess(f) = 0$ or $R_{ij} + \nabla_i\nabla_j f = 0$ and one knows also that

$$\left.\begin{aligned} &\partial_t N = \int_M (|\nabla f|^2 + \dot{\mathcal{R}})\exp(-f)\,dV = \mathfrak{F};\\ &\partial_t \mathfrak{F} = 2\int_M |Ric + Hess(f)|^2 \exp(-f)\,dV. \end{aligned}\right. \tag{1.2}$$

2 The Schrödinger equation and WDW

Now referring to [3–5, 7–12, 15, 16, 18–23, 29–32, 35–38, 40] for details we note first the important observation in [39] that $\mathfrak{F}$ is in fact a Fisher information functional. Fisher information has come up repeatedly in studies of the Schrödinger equation (SE) and the Wheeler-deWitt equation (WDW) and is connected to a differential entropy correspondingto the Nash entropy above (cf. [4, 7, 18, 19]). The basic ideas involve (using 1-D for simplicity) a quantum potential Q such that $\int_M PQ\,dx \sim \mathfrak{F}$ arising from a wave function $\psi = R\exp(iS/\hbar)$ where $Q = -(\hbar^2/2m)(\Delta R/R)$ and $P \sim |\psi|^2$ is a probability density. In a WDW context for example one can develop a framework

$$\left.\begin{aligned} &Q = cP^{-1/2}\,\partial(GP^{1/2});\\ &\int QP = c\int P^{1/2}\,\partial(GP^{1/2})\,\mathfrak{D}h\,dx \to\\ &\to -c\int \partial P^{1/2}\,G\partial P^{1/2}\,\mathfrak{D}h\,dx \end{aligned}\right\} \tag{2.1}$$

where G is an expression involving the deWitt metric $G_{ijk\ell}(h)$. In a more simple minded context consider a SE in 1-D $i\hbar\partial_t\psi = -(\hbar^2/2m)\,\partial_x^2\psi + V\psi$ where $\psi = R\exp(iS/\hbar)$ leads to the equations

$$\left.\begin{aligned} &S_t + \frac{1}{2m}S_x^2 + Q + V = 0;\\ &\partial_t R^2 + \frac{1}{m}(R^2 S_x)_x = 0:\; Q = -\frac{\hbar^2}{2m}\frac{R_{xx}}{R}. \end{aligned}\right\} \tag{2.2}$$

In terms of the exact uncertainty principle of Hall and Reginatto (see [21, 23, 34] and cf. also [4, 6, 7, 31, 32]) the quantum Hamiltonian has a Fisher information term $c\int dx\,(\nabla P\cdot\nabla P/2mP)$ added to the classical Hamiltonian (where $P = R^2 \sim |\psi|^2$) and a simple calculation gives

$$\begin{aligned} \int PQ\,d^3x \sim -\frac{\hbar^2}{8m}\int\left[2\Delta P - \frac{1}{P}|\nabla P|^2\right]d^3x =\\ = \frac{\hbar^2}{8m}\int\frac{1}{P}|\nabla P|^2\,d^3x\,. \end{aligned} \tag{2.3}$$

In the situation of (2.1) the analogues to Section 1 involve $(\partial \sim \partial_x)$

$$\left.\begin{aligned} &P \sim e^{-f}; \qquad P' \sim P_x \sim -f'e^{-f};\\ &Q \sim e^{f/2}\,\partial(G\partial e^{-f/2});\;\; PQ \sim e^{-f/2}\,\partial(G\partial e^{-f/2});\\ &\int PQ \to -\int \partial e^{-f/2}\,G\partial e^{-f/2} \sim -\int \partial P^{1/2} G\partial P^{1/2}. \end{aligned}\right\} \tag{2.4}$$

In the context of the SE in Weyl space developed in [1, 2, 4, 7, 10, 11, 12, 35, 36, 40] one has a situation $|\psi|^2 \sim R^2 \sim \sim P \sim \hat{\rho} = \rho/\sqrt{g}$ with a Weyl vector $\vec{\phi} = -\nabla\log(\hat{\rho})$ and a quantum potential

$$Q \sim -\frac{\hbar^2}{16m}\left[\dot{\mathcal{R}} + \frac{8}{\sqrt{\hat{\rho}}}\frac{1}{\sqrt{g}}\,\partial_i\left(\sqrt{g}\,g^{ik}\partial_k\sqrt{\hat{\rho}}\right)\right] = = -\frac{\hbar^2}{16m}\left[\dot{\mathcal{R}} + \frac{8}{\sqrt{\hat{\rho}}}\,\Delta\sqrt{\hat{\rho}}\right] \tag{2.5}$$

(recall $\mathrm{div}\,\mathrm{grad}(U) = \Delta U = (1/\sqrt{g})\,\partial_m(\sqrt{g}\,g^{mn}\partial_n U)$. Here the Weyl-Ricci curvature is $(\mathbf{2A})$ $\mathcal{R} = \dot{\mathcal{R}} + \mathcal{R}_w$ where

$$\mathcal{R}_w = 2|\vec{\phi}|^2 - 4\nabla\cdot\vec{\phi} = 8\,\frac{\Delta\sqrt{\hat{\rho}}}{\sqrt{\hat{\rho}}} \tag{2.6}$$

and $Q = -(\hbar^2/16m)\,\mathcal{R}$. Note that

$$-\nabla\cdot\vec{\phi} \sim -\Delta\log(\hat{\rho}) \sim -\frac{\Delta\hat{\rho}}{\hat{\rho}} + \frac{|\nabla\hat{\rho}|^2}{\hat{\rho}^2} \tag{2.7}$$

and for $\exp(-f) = \hat{\rho} = u$

$$\int \hat{\rho}\,\nabla\cdot\vec{\phi}\,dV = \int\left[-\Delta\hat{\rho} + \frac{|\nabla\hat{\rho}|^2}{\hat{\rho}}\right]dV \tag{2.8}$$

with the first term in the last integral vanishing and the second providing Fisher information again. Comparing with Section 1 we have analogues $(\mathbf{2B})$ $G \sim (R + |\vec{\phi}|^2)$ with $\vec{\phi} = = -\nabla\log(\hat{\rho}) \sim \nabla f$ to go with (2.4). Clearly $\hat{\rho}$ is basically a probability concept with $\int \hat{\rho}\,dV = 1$ and Quantum Mechanics (QM) (or rather perhaps Bohmian mechanics) seems to enter the picture through the second equation in (2.2), namely $(\mathbf{2C})$ $\partial_t\hat{\rho} + (1/m)\,\mathrm{div}\,(\hat{\rho}\,\nabla S) = 0$ with $p = mv = \nabla S$, which must be reconciled with (**1B**) (i.e. $(1/m)\,\mathrm{div}(u\nabla S) = = \Delta u - \dot{\mathcal{R}}u$). In any event the term $G = \dot{\mathcal{R}} + |\vec{\phi}|^2$ can be written as $(\mathbf{2D})$ $\dot{\mathcal{R}} + \mathcal{R}_w + (|\vec{\phi}|^2 - \mathcal{R}_w) = \alpha Q + (4\nabla\cdot\vec{\phi} - |\vec{\phi}|^2)$ which leads to $(\mathbf{2E})$ $\mathfrak{F} \sim \alpha\int_M QP\,dV + \beta\int|\vec{\phi}|^2 P dV$ putting Q directly into the picture and suggesting some sort of quantum mechanical connection.

REMARK 2.1. We mention also that Q appears in a fascinating geometrical role in the relativistic Bohmian format following [3, 15, 37, 38] (cf. also [4, 7] for survey material). Thus e.g. one can define a quantum mass field via

$$\mathfrak{M}^2 = m^2\exp(Q) \sim m^2(1+Q); \qquad Q \sim \frac{-\hbar^2}{c^2m^2}\frac{\Box(\sqrt{\rho})}{\sqrt{\rho}} \sim \frac{\alpha}{6}\,\mathcal{R}_w \tag{2.9}$$

where ρ refers to an appropriate mass density and $\mathfrak{M}$ is in fact the Dirac field β in a Weyl-Dirac formulation of Bohmian quantum gravity. Further one can change the 4-D Lorentzian metric via a conformal factor $\Omega^2 = \mathfrak{M}^2/m^2$ in the form $\tilde{g}_{\mu\nu} = = \Omega^2 g_{\mu\nu}$ and this suggests possible interest in Ricci flows etc. in conformal Lorentzian spaces (cf. here also [14]). We refer to [3, 15] for another fascinating form of the quantum potential as a mass generating term and intrinsic self energy. ■

NOTE. Publication information for items below listed by archive numbers can often be found on the net listing. ■

Submitted on June 27, 2007
Accepted on June 29, 2007

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The Little Heat Engine: Heat Transfer in Solids, Liquids and Gases

Pierre-Marie Robitaille

Dept. of Radiology, The Ohio State University, 130 Means Hall, 1654 Upham Drive, Columbus, Ohio 43210, USA

E-mail: robitaille.1@osu.edu

In this work, an introductory exposition of the laws of thermodynamics and radiative heat transfer is presented while exploring the concepts of the ideal solid, the lattice, and the vibrational, translational, and rotational degrees of freedom. Analysis of heat transfer in this manner helps scientists to recognize that the laws of thermal radiation are strictly applicable only to the ideal solid. On the Earth, such a solid is best represented by either graphite or soot. Indeed, certain forms of graphite can approach perfect absorption over a relatively large frequency range. Nonetheless, in dealing with heat, solids will eventually sublime or melt. Similarly, liquids will give way to the gas phase. That thermal conductivity eventually decreases in the solid signals an inability to further dissipate heat and the coming breakdown of Planck's law. Ultimately, this breakdown is reflected in the thermal emission of gases. Interestingly, total gaseous emissivity can decrease with increasing temperature. Consequently, neither solids, liquids, or gases can maintain the behavior predicted by the laws of thermal emission. Since the laws of thermal emission are, in fact, not universal, the extension of these principles to non-solids constitutes a serious overextension of the work of Kirchhoff, Wien, Stefan and Planck.

The question now is wherein the mistake consists and how it can be removed.

Max Planck, Philosophy of Physics, 1936.

While it is true that the field of thermodynamics can be complex [1–8] the basic ideas behind the study of heat (or energy) transfer remain simple. Let us begin this study with an ideal solid, S_1, in an empty universe. S_1 contains atoms arranged in a regular array called a "lattice" (see Figure 1). Bonding electrons may be present. The nuclei of each atom act as weights and the bonding electrons as springs in an oscillator model. Non-bonding electrons may also be present, however in an ideal solid these electrons are not involved in carrying current. By extension, S_1 contains no electronic conduction bands. The non-bonding electrons may be involved in Van der Waals (or contact) interactions between atoms. Given these restraints, it is clear that S_1 is a non-metal.

Ideal solids do not exist. However, graphite provides a close approximation of such an object. Graphite is a black, carbon-containing, solid material. Each carbon atom within graphite is bonded to 3 neighbors. Graphite is black because it very efficiently absorbs light which is incident upon its surface. In the 1800's, scientists studied objects made from graphite plates. Since the graphite plates were black, these objects became known as "blackbodies". By extension, we will therefore assume that S_1, being an ideal solid, is also a perfect blackbody. That is to say, S_1 can perfectly absorb any light incident on its surface.

Let us place our ideal solid, S_1, in an imaginary box. The walls of this box have the property of not permitting any heat to be transferred from inside the box to the outside world and vice versa. When an imaginary partition has the property of not permitting the transfer of heat, mass, and light, we say that the partition is adiabatic. Since, S_1 is alone inside the adiabatic box, no light can strike its surface (sources of light do not exist). Let us assume that S_1 is in the lowest possible energy state. This is the rest energy, E_{rest}. For our ideal solid, the rest energy is the sum of the relativistic energy, E_{rel}, and the energy contained in the bonds of the solid, E_{bond}. The relativistic energy is given by Einstein's equation, $E = mc^2$. Other than relativistic and bonding energy, S_1 contains no other energy (or heat). Simplistically speaking, it is near 0 Kelvin, or absolute zero.

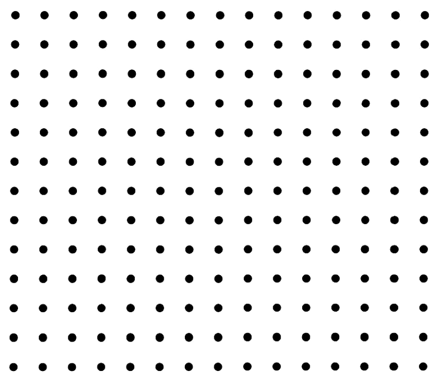

Fig. 1: Schematic representation of the ideal solid, S_1. The atoms are arranged in a regular array, or "lattice".

That absolute zero exists is expressed in the form of the 3rd law of thermodynamics, the last major law of heat transfer to be formulated. This law is the most appropriate starting point for our discussion. Thus, an ideal solid contain-

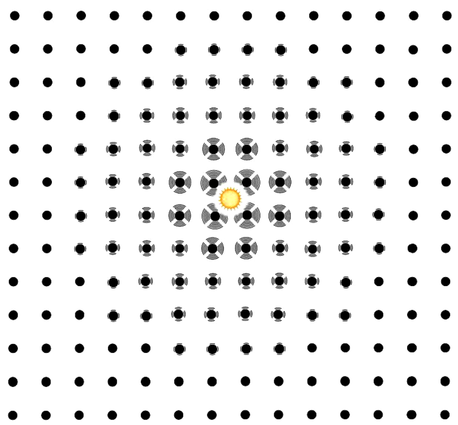

Fig. 2: Depiction of the Little Heat Engine at the center of the lattice. The atoms near this heat source move about their absolute location, such that they experience no net displacement over time. The vibrational degrees of freedom are slowly being filled.

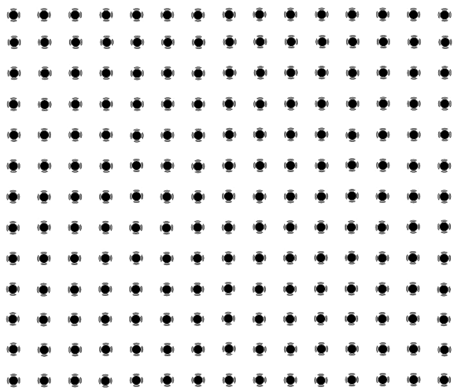

Fig. 3: The Little Heat Engine is turned off and the heat introduced into the lattice begins to equilibrate throughout the solid.

ing no heat energy is close to absolute zero as defined by the 3rd law of thermodynamics. In such a setting, the atoms that make up the solid are perfectly still. Our universe has a total energy (E_{total}) equal to the rest mass of the solid: $E_{total} = E_{solid} = E_{rest} = E_{rel} + E_{bond}$.

Now, let us imagine that there is a hypothetical little heat engine inside S_1. We chose an engine rather than a source to reflect the fact that work is being done as we ponder this problem. However, to be strictly correct, a source of heat could have been invoked. For now, we assume that our little heat engine is producing hypothetical work and it is also operating at a single temperature. It is therefore said to be isothermal. As it works, the little heat engine releases heat into its environment.

It is thus possible to turn on this hypothetical little heat engine and to start releasing heat inside our solid. However, where will this heat go? We must introduce some kind of "receptacle" to accept the heat. This receptacle will be referred to as a "degree of freedom". The first degrees of freedom that we shall introduce are found in the vibration of the atoms about their absolute location, such that there is no net displacement of the atoms over time. The heat produced by our little heat engine will therefore begin to fill the vibrational degrees of freedom and the atoms in its vicinity will start vibrating. When this happens, the bonds of the solid begin to act as little springs. Let us turn on the heat engine for just a little while and then turn it off again. Now we have introduced a certain quantity of heat (or energy) inside the solid. This heat is in the immediate vicinity of the little heat engine (see Figure 2). As a result, the atoms closest to the heat engine begin to vibrate reflecting the fact that they have been heated. The total amount of energy contained in the vibrational degrees of freedom will be equal to E_{vib}.

Since the little heat engine has been turned off, the heat produced will now start to equilibrate within the solid (see Figure 3). Thus, the area nearest the little heat engine becomes colder (the atoms nearest the heat engine slow down their vibration) and the areas away from our little engine heat up (they increase their vibration). As this happens, S_1 is moving towards thermal equilibrium. That is, it is becoming isothermal — moving to a single uniform temperature. In this state, all the atoms in S_1 share equally in the energy stored in the vibrational degrees of freedom. The driving force for reaching this thermal equilibrium is contained in the 2nd law of thermodynamics. This law states that heat must always move from hotter to colder regions in an irreversible manner.

That heat flows in an irreversible manner is the central theme of the 2nd law of thermodynamics. Indeed, no matter what mechanism will be invoked to transfer heat in nature, it will always be true that the macroscopic transfer of heat occurs in an irreversible manner.

So far, S_1 is seeking to reach a uniform temperature or thermal equilibrium. For our ideal solid, thermal equilibrium can only be achieved through thermal conduction which in turn is supported by energy contained in the vibrational degrees of freedom. Thermal conduction is the process whereby heat energy is transferred within an object without the absolute displacement of atoms or molecules. If the little heat engine was kept on, then thermal conduction would constantly be trying to bring our solid to thermal equilibrium. If there were no processes other than thermal conduction, and the engine was turned off, eventually one would think that the entire solid would come to a single new temperature and thermal equilibrium would be achieved. At this stage, our universe would have a total energy equal to that contained in the rest energy ($E_{rel} + E_{bond}$) and in the vibrational degrees of freedom ($E_{total} = E_{solid} = E_{rest} + E_{vib}$).

However, even though our little heat engine has been turned off, thermal equilibrium cannot be reached in this scenario. This is because there is another means of dissipating heat available to the solid. Thus, as the solid is heated, it dissipates some of the energy contained in its vibrational degrees of freedom into our universe in an effort to cool down. This is accomplished by converting some of the energy contained in the vibrational degrees of freedom into light!

The light that objects emit in an attempt to cool down is called thermal radiation. The word thermal comes in because we are dealing with heat. The word radiation comes from the fact that it is light (or radiation) which is being emitted.

This light is emitted at many different frequencies (see Figure 4). We represent the total amount of energy in this emission as E_{em}. Emission of light provides another means of dealing with heat. Thus, the emission of light joins vibration in providing for our stationary non-metallic solid the only degrees of freedom to which it can ever have access. However, the energy of emission becomes a characteristic of our universe and not of the solid. Thus, the universe now has a total energy given by $E_{total} = E_{solid} + E_{em}$. As for the solid, it still has an energy equal only to that stored as rest energy and that contained in the vibrational degrees of freedom, $E_{solid} = E_{rest} + E_{vib}$. However, note that since all the heat energy of the solid was initially contained in its vibrational degrees of freedom, the energy of emission (E_{em}) must be related to the energy contained in E_{vib} at the time of emission.

As stated above, light has the property that it cannot cross an adiabatic partition. Consequently, the light produced by heating the solid becomes trapped in our virtual box. If we kept our adiabatic walls close to the solid, eventually thermal equilibrium would be achieved between the solid and the radiation. In this scenario, the solid would be constantly emitting and absorbing radiation. Under a steady state regimen, all of the atoms in the solid would be sharing equally in the energy contained in the vibrational degrees of freedom. However, let us make the box large for now, so that it will take the light many years to reach the walls of the box and be reflected back towards the solid. For all purposes then, the light that the solid emits cannot return and hit the surface of the solid.

Up to this point, by turning on our little heat engine, we have been able to discuss two important processes. The first is thermal conduction. Thermal conduction is that process which tries to bring the internal structure of the solid to thermal equilibrium. In our ideal solid, the vibrations of the atoms are the underlying support for this process. The second process is thermal radiation (also called radiative emission). Through radiative emission, the solid is trying to come to thermal equilibrium with the outside world. There are only two means for an ideal solid to deal with heat. It can strive to achieve internal thermal equilibrium through thermal conduction supported by the vibrations of its atoms and it can dissipate some of the energy contained in its vibrational degrees of freedom to the outside world through thermal radiation.

For an ideal solid, the light emitted in an attempt to reach or maintain thermal equilibrium will contain a continuous range of frequencies (see Figure 4). The intensity of the light at any given frequency will be given by the well known

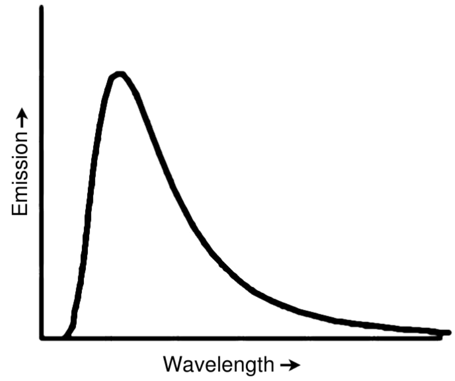

Fig. 4: The light that objects emit in an attempt to cool down is called thermal radiation. Emission of light provides another means of dealing with heat. The emission is continuous over all frequencies for our ideal solid, S_1.

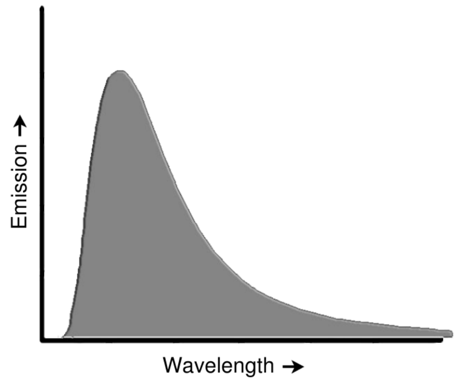

Fig. 5: For the ideal solid, S_1, the total emission (area under the curve) is proportional to the fourth power of the temperature as dictated by Stefan's law of thermal emission.

Planckian relation [9]

$$B_\nu(T) = \frac{\varepsilon_\nu}{\kappa_\nu} = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT} - 1}\,.$$

Planck's equation states that the light produced, at a frequency ν, by a blackbody (or an ideal solid), B_ν, depends only on two variables: temperature, T, and the frequency, ν. All the other terms in this equation are constants (h = Planck's constant, k = Boltzman's constant, c = speed of light). This equation tells us that the nature of light produced is dependent only on the temperature of the solid and on the frequency of interest. The fact that the light emitted by an ideal solid was dependent only on temperature and frequency was first highlighted by Gustav Robert Kirchhoff in the mid-1800's. Kirchhoff's formulation became know as Kirchhoff's Law of Thermal Radiation [10, 11]

$$B_\nu(T) = \frac{\varepsilon_\nu}{\kappa_\nu}\,.$$

In this equation, ε_ν represents the ability of the blackbody to emit light (emissivity) and κ_ν represents its ability

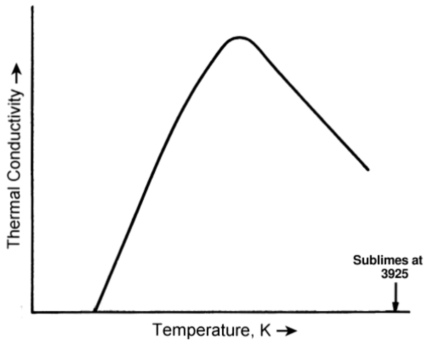

Fig. 6: Thermal conductivity for pyrolytic graphite (parallel to the layer planes) increases to a maximum and then begins to decrease. Eventually, graphite sublimes at 3925 K. Adapted from reference 15, volume 2, *Thermal Conductivity of Nonmetallic Solids*, 1970.

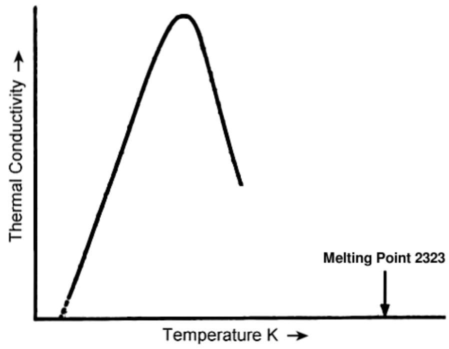

Fig. 7: Thermal conductivity for sapphire (Al_2O_3) increases to a maximum and then decreases. Eventually, sapphire melts at 2323 K. Adapted from reference 15, volume 2, *Thermal Conductivity of Nonmetallic Solids*, 1970.

to absorb light (opacity) at a given frequency. As mentioned above, an ideal solid is a blackbody, or a perfect absorber of light ($\kappa_\nu = 1$). As such, this equation states that the manner in which a blackbody emits or absorbs light at a given frequency depends exclusively on its temperature. The function, f, contained in Kirchhoff's Law $B_\nu = \frac{\varepsilon_\nu}{\kappa_\nu} = f(T, \nu)$ was elucidated by Max Planck as shown in the first equation above. It is for this reason that Kirchhoff's equation constitutes the left hand portion of Planck's equation [9]. As a result, any work by Kirchhoff on this topic is critical to our understanding of Planck's work [9, 10, 11].

It has also been observed that the amount of light that our ideal solid will produce, or the total emission (see the area under the curve in Figure 5), is proportional to the fourth power of the solid's temperature. This is known as Stefan's law of emission ($\varepsilon = \sigma T^4$), where ε represents total emission and Stefan's constant, σ , is equal to 5.67051×10^{-8} Watts/($m^2 K^4$) [12]. Note that Stefan's law of emission reveals a pronounced increase in the production of light, with temperature. Thus, as the temperature of the solid increases, thermal radiation can greatly increase to accommodate the increased requirement for heat dissipation. If the solid is at room temperature, this light will be emitted at infrared frequencies, that is, just below the portion of the electromagnetic spectrum that is visible to the human eye. Indeed, this emitted light at room temperatures can be viewed with a thermal or infrared camera of the type used by the military to see at night.

Interestingly for S_1, the frequency of light at which the maximal emission occurs (ν_{max}) is directly related to the temperature $\nu_{max}/c = T$). This is known as Wien's law of displacement [13].

Let us turn on our little heat engine once again. As the little heat engine releases more heat into solid, it becomes apparent that thermal conductivity increases only approximately linearly with temperature. In fact, as temperature is increased for many real solids, thermal conductivity actually may initially increase to a maximum and then suddenly begin to decrease (see Figure 6 for graphite and Figure 7 for sapphire or Al_2O_3) [14]. Since the vibrational degrees of freedom are central to both thermal conduction and emission, one can only gather that the vibrational degrees of freedom simply become incapable of dealing with more heat (see Figure 8). Herein lies a problem for maintaining the solid phase. As temperature is increased, there is a greater difficulty of dealing with the internal flow of heat within the solid. The solid must begin to search for a new degree of freedom.

The next available means of dealing with heat lies in breaking bonds that link up the atoms forming the ideal solid. As these bonds begin to break, the atoms (or the molecules) gain the ability to change their average location. New degrees of freedom are born, namely, the translational and rotational degrees of freedom. Interestingly, these new degrees of freedom are associated with both the flow of heat and mass.

With the arrival of the translational and rotational degrees of freedom, S_1 is transformed into one of two possibilities. It can either melt — giving rise to the liquid phase, L_1; or, it can sublime — giving rise directly to the gas phase, G_1. Graphite, perhaps the closest material to an ideal solid, sublimes (see Figure 6) and never melts. Whereas sapphire or Al_2O_3 melts (see Figure 7). In any case, as a solid is being converted to a liquid or a gas, the absolute amount of rest energy is changing, because bonds are being broken ($E_{bond} \to 0$).

Since many solids melt giving rise to L_1, let us turn our attention first to this situation. We assume that unlike graphite, our ideal solid can in fact melt. Thus, as more heat is pumped into S_1, the temperature will no longer rise. Rather, the solid S_1 will simply slowly be converted to the liquid L_1. The melting point has been reached (see Figure 9 and Figure 10) and the liquid created (see Figure 11).

Since L_1 has just been created, let us turn off our little heat engine once again. The liquid L_1 at this stage, much like S_1 of old, is still capable of sustaining thermal conduction as an internal means of trying to reach thermal equilibrium through the vibrational degrees of freedom. However, the absolute level of thermal conduction is often more than 100 times lower than in the solid [8, 15]. The liquid L_1 also

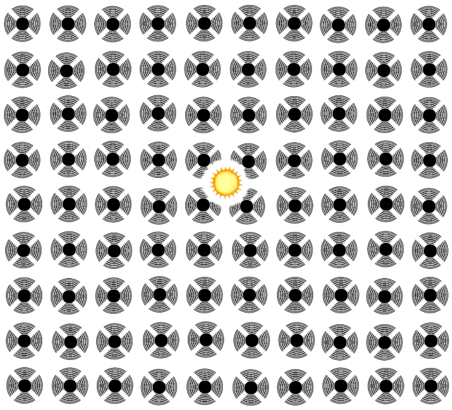

Fig. 8: The Little Heat Engine has introduced so much heat into the lattice that the vibrational degrees of freedom become full. The solid must search for a new way to deal with the continued influx of heat.

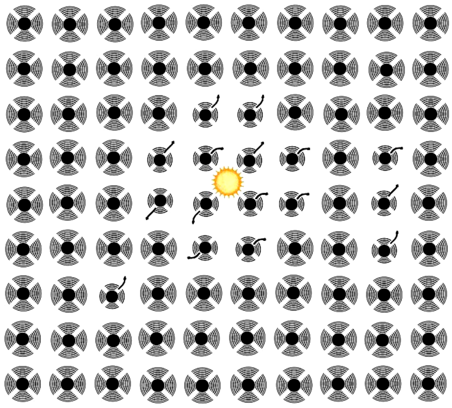

Fig. 9: As The Little Heat Engine continues to heat the lattice, the melting point is eventually reached. The solid, S_1, begins to melt as the translational and rotational degrees of freedom start to be filled.

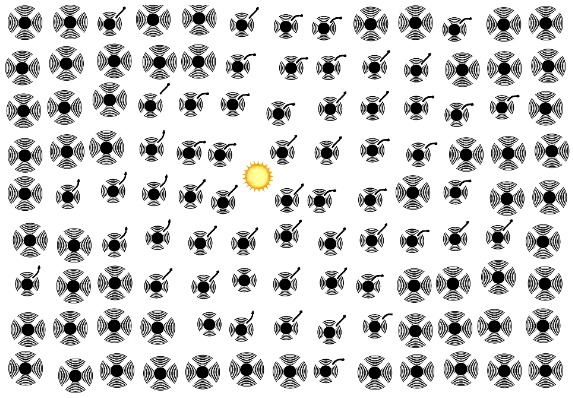

Fig. 10: As The Little Heat Engine continues to heat the lattice, melting continues. The regular array of the solid lattice is being replaced by the fleeting lattice of the liquid.

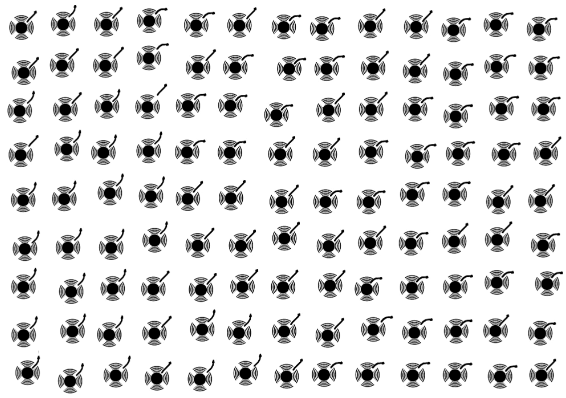

Fig. 11: The Little Heat Engine is turned off and melting of S_1 into L_1 is completed. The regular solid lattice is now completely replaced with the fleeting lattice of the liquid. The individual atoms now experience absolute displacement in position over time.

has access to thermal radiation as a means of dissipating heat to the outside world.

However, within L_1, a new reality has taken hold. The requirements placed on conduction and radiative emission for heat dissipation have now been relaxed for the liquid and, mass transfer becomes a key means of dissipating heat within such an object. Indeed, internal convection, the physical displacement (or flow) of atoms or molecules, can now assist thermal conduction in the process of trying to reach internal thermal equilibrium. Convection is a direct result of the arrival of the translational degrees of freedom. The driving force for this process once again is the 2nd law of thermodynamics and the physical phenomenon involved is expressed in kinetic energy of motion. Thus, through internal convection, currents are set up within the liquid, whose sole purpose is an attempt at thermal equilibrium. As convection currents form, the bonds that make up the liquid are constantly in the process of breaking and reforming. Like thermal conduction, the process of internal convection changes approximately linearly with temperature. For its part, L_1 now has three means of dealing with heat transfer: conduction, convection (internal), and thermal radiation (external). The total energy of the universe is now expressed as $E_{total} = E_{liquid} + E_{emission}$. The energy within the liquid is divided between the rest energy and the energy flowing through the vibrational, translational, and rotational degrees of freedom, $E_{liquid} = E_{rest} + E_{vib} + E_{trans} + E_{rot}$. Thermal conduction and radiative emission remain tied to the energy associated with the vibrational degrees of freedom, while convection becomes associated with the energy within the translational and rotational degrees of freedom. The added heat energy contained within the liquid is now partitioned amongst three separate degrees of freedom: $E_{vib} + E_{trans} + E_{rot}$.

At this stage, the little heat engine can be turned on again. Very little is known regarding thermal emission from liquids. However, it appears that when confronted with increased inflow of heat, the liquid responds in a very different way. Indeed, this is seen in its thermal emission. Thus, while thermal emission in the solid increased with the fourth power of the temperature, thermal emissivity in a liquid increases little, if

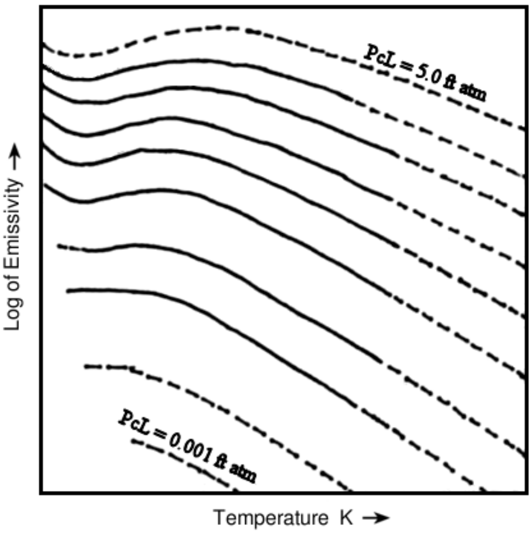

Fig. 12: The total emissivity for CO_2 at 1 atmosphere for various pressure path lengths. Note that emissivity can actually drop significantly with increasing temperature. Gases are unable to follow Stefan's Law. Adapted from reference 17.

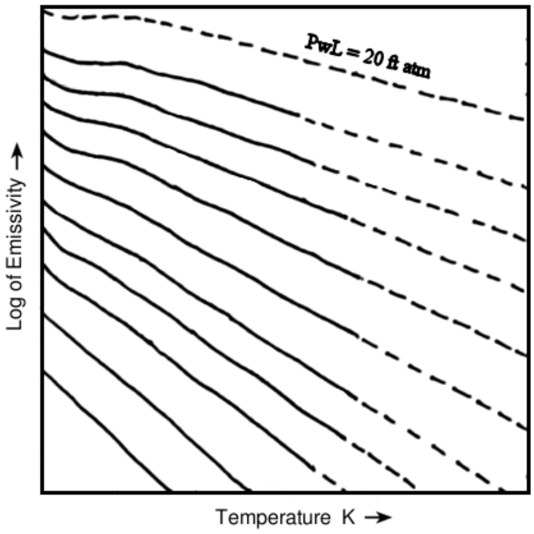

Fig. 13: The total emissivity of water vapor for various pressure path lengths. Note that the total emissivity can actually drop significantly with increasing temperature. Gases are unable to follow Stefan's Law. Adapted from reference 17.

at all, with temperature [8, 15]. Indeed, total thermal emission may actually decrease. Stefan's law does not hold in a liquid. That is because new degrees of freedom, namely the translational and rotational degrees of freedom (and its associated convection), have now been introduced into the problem. Since the vibrational degrees of freedom are no longer exclusively in control of the situation, Stefan's law fails.

It has already been noted that thermal conduction is eventually unable to deal with increased heat in the solid. In liquids, it is often observed that thermal conductivity changes only slightly with temperature and often decreases [8, 14, 15]. At the same time, it is clear that thermal radiation does not increase with temperature in the liquid. One can only surmise that convection rapidly becomes a dominant means of dealing with heat transfer in the liquid phase. This can be seen by examining the viscosity of the liquid. Thus, the viscosity of liquids decreases with temperature and the liquid flows better at higher temperature [14]. This is a direct reflection that an increasing percentage of bonds within the liquid are being broken in order to accommodate the increased flow of heat, or energy, into the translational degrees of freedom.

Let us now return to our little heat engine. Since the little heat engine has been left on, as it continues to heat L_1, a point will be reached where internal conduction and convection along with thermal radiation can no longer accommodate the increase in heat. At that point, a new process must arise to carry heat away. Thus, with an internal structure weakened by broken bonds, individual atoms or molecules are now free to carry mass and heat directly away from the liquid in the form of kinetic energy of motion. The liquid L_1 enters the gas phase becoming G_1. This is exactly analogous to what occurred previously for the solid with sublimation. The liquid L_1 has now reached the boiling point. While it boils, its temperature will no longer increase. Rather, it is simply being slowly converted from the liquid L_1 to the gas G_1. According to the kinetic theory of gases, the molecules of the gas are traveling at a particular average velocity related to the temperature of the gas at a given pressure. It is our adiabatic partitions that have ensured that we can speak of pressure. The fact that the gas molecules are moving is a reflection of the convection within the gas which, in turn, is an expression of the translational degrees of freedom. Let us turn off our little heat engine for a moment in order to analyze what has just transpired.

In the gas G_1, individual molecules are not attached to each other but are free to move about. This is once again a reflection of the translational degrees of freedom. G_1 can have either a molecular nature (it is made up of individual molecules) or an atomic nature (it is made up of individual atoms). For now, let us make the assumption that S_1 was selected such that a diatomic molecular gas, G_1, is produced. Let us also assume that our diatomic molecular gas will be made up of two different types of atoms. Note that we are deviating slightly from the requirements of an ideal solid in order to deal with molecular gases. Once in the gas phase, the molecular gas can also invoke rotational degrees of freedom. Therefore, the molecular gas G_1 has energy partitioned amongst its available degrees of freedom, $E_{gas} = E_{rest} + E_{vib} + E_{trans} + E_{rot}$. Note that in the molecular gas the E_{rest} term decreases, reflecting the breakdown of S_1 and L_1 into the gas G_1 (less energy is now contained in E_{bond}). From above, we now see that the total energy in the universe is $E_{total} = E_{gas} + E_{em} = E_{rest} + E_{vib} + E_{trans} + E_{rot} + E_{em}$. The molecular gas will still be able to emit ra-

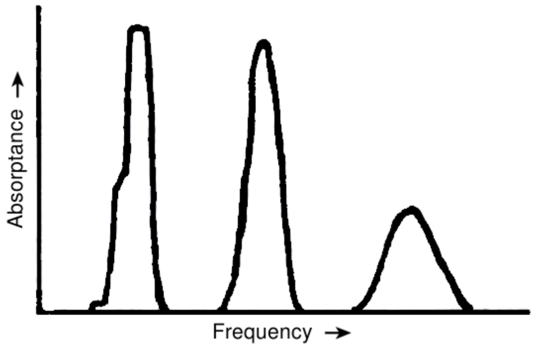

Fig. 14: Schematic representation of the absorptance for CO_2 at 830 K and 10 atmospheres for a path length of 0.388 meters. The gas absorbs in discrete bands and not in a continuous fashion as previously observed for a solid. Adapted from reference 8.

diation, typically in the microwave or infrared region of the electromagnetic spectrum.

It is now time to turn our little heat engine on again. As more heat is generated, the gas will increase the average kinetic energy of motion of its constituent molecules. Nonetheless, thermal conduction within G_1 is now at least 10 times lower than was the case for the liquid [15]. Most importantly the total radiative emissivity for the molecular gas at constant pressure actually begins to drop dramatically with increased temperature [7, 16]. We can speak of constant pressure when we do not permit the adiabatic walls of our imaginary box to move. If we now move in our adiabatic walls we increase the pressure on the gas and the emissivity will increase, corresponding to a higher apparent temperature.

Nonetheless, it should be noted that the total emissivity for a gas at constant pressure can actually drop significantly with increasing temperature (see Figure 12 and Figure 13). Consequently, we can see that Stefan's law does not hold for gases [7]. In fact, thermal emission for the diatomic gas (like CO and NO) occurs in discrete bands of the electromagnetic spectrum and in a manner not simply related to temperature (see Figure 14, Figure 15 and Figure 16) [8, 16]. The situation becomes even more interesting if the gas is not molecular, but rather monatomic in nature (like Ar or He for instance). In that case, when moving from the liquid to the gas phase, G_1 looses both its rotational, and more importantly, its vibrational, degrees of freedom, $E_{bond} = E_{vib} = E_{rot} = 0$. Neglecting electronic emission, which typically occurs in the ultra violet or visible range, a monatomic gas cannot emit significant radiation in the microwave and infrared regions. Indeed, for such a gas, Stefan's law no longer has any real meaning.

It is now clear that relative to S_1 (and even L_1), the molecular gas G_1 is unable to dissipate its heat effectively to the outside world in response to increased temperature. Indeed, since thermal emission can drop dramatically with temperature for molecular gases, as temperature is increased, a greater fraction of the heat energy must be dealt with by the transla-

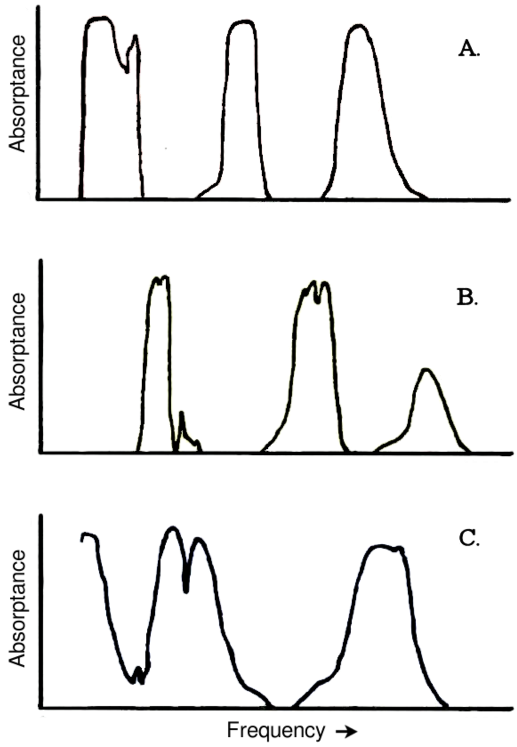

Fig. 15: Schematic representation of the absorptance for carbon dioxide (A: CO_2 partial pressure path product = 3.9 atm m, temperature = 1389 K, total pressure = 10 atm, partial pressure = 10 atm), water (B: H_2O partial pressure path product = 3.9 atm m, temperature = 1389 K, total pressure = 10 atm, partial pressure = 10 atm), and methane (C: CH_4 partial pressure path product = 3.9 atm m, temperature = 1389 K, total pressure = 10 atm, partial pressure = 10 atm). Note that for gases absorbance is not continuous and occurs in discrete bands. Adapted from reference 8.

tional and rotational degrees of freedom. If the gas is made up of molecules as is the case for G_1, then as more heat is pumped into the gas by our little engine, the gas molecules will eventually break apart into their constituent atoms. The gas then adopts the nature of monatomic gases as mentioned above with $E_{bond} = E_{vib} = E_{rot} = 0$. As more heat is pumped into the system, electronic transitions within each atom becomes more and more important. If the little heat engine is not stopped, much like what happened in the case of the solid and the liquid, the atomic gas will no longer be able to deal with the increased heat. Eventually, the electrons gain enough energy to start emitting radiation in the visible or ultra-violet range. As the little heat engine continues to generate heat, the electrons will gain enough energy to become free of the nucleus and a final new state is born — the plasma. The discussion of heat flow in plasmas is beyond our scope at this stage. Suffice it to say that if the little heat engine continues to operate, still another process would occur, namely nuclear reactions.

It is now time to finally turn off our little heat engine. We have learned a lot with this little device and so it is somewhat

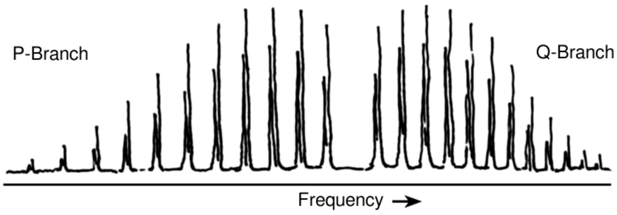

Fig. 16: Vibrational-rotational spectrum of hydrochloric acid at room temperature. The spectrum reveals the presence of the two isotropic form, $H^{35}Cl$ and $H^{37}Cl$. Adapted from reference 18.

sad to state that it can live only in our imagination. This is because our little heat engine violates the 1st law of thermodynamics. That law states that there must be conservation of energy. Namely, energy cannot be created or destroyed. However, when Einstein introduced relativity he demonstrated that $E = mc^2$. Thus, it is actually possible to convert mass into energy and vise versa. As a result, after Einstein, the 1st law of thermodynamics had to be modified. Consequently, the 1st law of thermodynamics now states that there must be conservation of mass and energy. Theoretically, these two entities could be freely interchanged with one another.

For a moment in closing however, let us return to our initial solid S_1. Of course, in the real world our solid is not in an isolated universe. Other solids, liquids (like our oceans) and gases (like our atmosphere) also exist. How do these affect our solid? In order to understand this, let us now bring two other solids into our adiabatic box. We will assume that these two solids, denoted "S_2" and "S_3", are in thermal equilibrium with each other. That is to say that, if "S_2" is placed in direct contact with "S_3" no net heat will flow between these objects. Now, if we now place solid "S_1" in contact with solid "S_2", we will discover one of two things. Either solid "S_1" is in thermal equilibrium with solid "S_2", or it is not in equilibrium. If it is in equilibrium with S_2, then by the 0th law of thermodynamics, it must also be in equilibrium with S_3. If on the other hand the solid S_1 it is not in equilibrium with S_2, then S_1, S_2 and S_3 will all move to a new thermal equilibrium with each other. If they are not in direct physical contact, this can only occur through thermal emission. However, if they are in direct contact, then they can use the much more efficient means of conduction to reach thermal equilibrium. If in turn we substitute a liquid or a gas for one of the solids, then convection can also be used to reach thermal equilibrium amongst all the objects. This is provided of course that the solids remain in physical contact with the gas or liquid. In the real universe therefore, all of the matter is simultaneously trying to reach thermal equilibrium with all other matter. The 2nd law of thermodynamics is governing this flow of heat. Most importantly, this process on a macroscopic scale is irreversible.

But now what of our little heat engine? Would it not be nice to bring it back? Perhaps we can! That is because, for our solar system, it is our Sun, and its internal energy, which is the ultimate source of energy. Therefore our Sun becomes for us a local little heat engine. As for the stars, they become other local heat engines, in a universe constantly striving for thermal equilibrium.

Author's comment on The Little Heat Engine:

The Little Heat Engine is telling us that the internal processes involved in heat transfer cannot be ignored. However, modern courses in classical thermodynamics often neglect the internal workings of the system. In large part, this is because the fathers of thermodynamics (men like Kirchhoff, Gibbbs and Claussius) did not yet have knowledge of the internal workings of the system. As such, they had no choice but to treat the entire system.

In this essay, it becomes apparent that Stefan's Law of thermal emission does not hold for liquids and gases. This is a reflection that these two states of matter have other available degrees of freedom. For instance, if Stefan's Law had held, solids would have no need to melt. They could keep dealing with heat easily, simply by emitting photons in a manner proportional with the fourth power of the temperature. However, the drop in thermal conductivity observed in the solid heralds the breakdown of Stefan's law and the ensuing change in phase. The Little Heat Engine is telling us that statistical thermodynamics must be applied when dealing with thermal emission. The Little Heat Engine is a constant reminder that universality does not exist in thermal radiation. The only materials which approach the blackbody on the Earth are generally made of either graphite or soot. The application, by astrophysics, of the laws of blackbody radiation [9–13] to the Sun [19, 20] and to unknown signals [21] irrespective of the phase of origin constitutes a serious overextension of these laws. Experimental physics has well established that there is no universality and that the laws of thermal radiation are properly restricted to the solid [22, 23].

Acknowledgments

The author acknowledges Karl Bedard and Luc Robitaille for assistance in preparing the figures and Christophe Robitaille is recognized for his computing skills.

Submitted on June 29, 2007
Accepted on July 02, 2007

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SPECIAL REPORT

A Four-Dimensional Continuum Theory of Space-Time and the Classical Physical Fields

Indranu Suhendro

Department of Physics, Karlstad University, Karlstad 651 88, Sweden

E-mail: spherical_symmetry@yahoo.com

In this work, we attempt to describe the classical physical fields of gravity, electromagnetism, and the so-called intrinsic spin (chirality) in terms of a set of fully geometrized constitutive equations. In our formalism, we treat the four-dimensional space-time continuum as a deformable medium and the classical fields as intrinsic stress and spin fields generated by infinitesimal displacements and rotations in the space-time continuum itself. In itself, the unifying continuum approach employed herein may suggest a possible unified field theory of the known classical physical fields.

1 Introduction

Many attempts have been made to incorporate the so-called standard (Hookean) linear elasticity theory into general relativity in the hope to describe the dynamics of material bodies in a fully covariant four-dimensional manner. As we know, many of these attempts have concentrated solely on the treatment of material bodies as linearly elastic continua and not quite generally on the treatment of space-time itself as a linearly elastic, deformable continuum. In the former case, taking into account the gravitational field as the only intrinsic field in the space-time continuum, it is therefore true that the linearity attributed to the material bodies means that the general consideration is limited to weakly gravitating objects only. This is because the curvature tensor is in general quadratic in the the so-called connection which can be said to represent the displacement field in the space-time manifold. However, in most cases, it is enough to consider an infinitesimal displacement field only such that the linear theory works perfectly well. However, for the sake of generality, we need not assume only the linear behavior of the properly-stressed space-time continuum (and material bodies) such that the possible limiting consequences of the linear theory can be readily overcome whenever it becomes necessary. Therefore, in the present work, we shall both consider both the linear and non-linear formulations in terms of the response of the space-time geometry to infinitesimal deformations and rotations with intrinsic generators.

A few past attempts at the full description of the elastic behavior of the space-time geometry in the presence of physical fields in the language of general relativity have been quite significant. However, as standard general relativity describes only the field of gravity in a purely geometric fashion, these past attempts have generally never gone beyond the simple reformulation of the classical laws of elasticity in the presence of gravity which means that these classical laws of elasticity have merely been referred to the general four-dimensional curvilinear coordinates of Riemannian geometry, nothing more. As such, any possible interaction between the physical fields (e.g., the interaction between gravity and electromagnetism) has not been investigated in detail.

In the present work, we develop a fully geometrized continuum theory of space-time and the classical physical fields in which the actions of these physical fields contribute directly to the dynamics of the space-time geometry itself. In this model, we therefore assume that a physical field is directly associated with each and every point in the region of space-time occupied by the field (or, a material body in the case of gravity). This allows us to describe the dynamics of the space-time geometry solely in terms of the translational and rotational behavior of points within the occupied region. Consequently, the geometric quantities (objects) of the space-time continuum (e.g., curvature) are directly describable in terms of purely kinematic variables such as displacement, spin, velocity, acceleration, and the particle symmetries themselves.

As we have said above, at present, for the sake of simplicity, we shall assume the inherently elastic behavior of the space-time continuum. This, I believe, is adequate especially in most cosmological cases. Such an assumption is nothing but intuitive, especially when considering the fact that we do not fully know the reality of the constituents of the fabric of the Universe yet. As such, the possible limitations of the present theory, if any, can be neglected considerably until we fully understand how the fabric of the space-time continuum is actually formed and how the properties of individual elementary particles might contribute to this formation.

2 The fundamental geometric properties of a curved manifold

Let us present the fundamental geometric objects of an n-dimensional curved manifold. Let $\omega_a = \frac{\partial X^i}{\partial x^a} E_i = \partial_a X^i E_i$ (the Einstein summation convention is assumed throughout this work) be the covariant (*frame*) basis spanning the n-dimensional base manifold $\mathbb{C}^\infty$ with local coordinates $x^a = x^a\left(X^k\right)$. The contravariant (*coframe*) basis θ^b is then given via the orthogonal projection $\left\langle\theta^b, \omega_a\right\rangle = \delta^b_a$, where δ^b_a are the components of the Kronecker delta (whose value is unity if the indices coincide or null otherwise). The set of linearly independent local directional derivatives $E_i = \frac{\partial}{\partial X^i} = \partial_i$ gives the coordinate basis of the locally flat tangent space $\mathbb{T}_x(\mathbb{M})$ at a point $x \in \mathbb{C}^\infty$. Here $\mathbb{M}$ denotes the topological space of the so-called n-tuples $h\,(x) = h\left(x^1, \ldots, x^n\right)$ such that relative to a given chart $(U, h\,(x))$ on a neighborhood U of a local coordinate point, our $\mathbb{C}^\infty$-differentiable manifold itself is a topological space. The dual basis to E_i spanning the locally flat cotangent space $\mathbb{T}^*_x(\mathbb{M})$ will then be given by the differential elements dX^k via the relation $\left\langle dX^k, \partial_i\right\rangle = \delta^k_i$. In fact and in general, the *one-forms* dX^k indeed act as a linear map $\mathbb{T}_x(\mathbb{M}) \to \mathbb{R}$ when applied to an arbitrary vector field $F \in \mathbb{T}_x(\mathbb{M})$ of the explicit form $F = F^i \frac{\partial}{\partial X^i} = f^a \frac{\partial}{\partial x^a}$. Then it is easy to see that $F^i = F\,X^i$ and $f^a = F\,x^a$, from which we obtain the usual transformation laws for the contravariant components of a vector field, i.e., $F^i = \partial_a X^i f^a$ and $f^i = \partial_i\, x^a F^i$, relating the localized components of F to the general ones and vice versa. In addition, we also see that $\left\langle dX^k, F\right\rangle = FX^k = F^k$.

The components of the symmetric metric tensor $g = g_{ab}\,\theta^a \otimes \theta^b$ of the base manifold $\mathbb{C}^\infty$ are readily given by

$$g_{ab} = \langle\omega_a, \omega_b\rangle$$

satisfying

$$g_{ac}\, g^{bc} = \delta^b_a$$

where $g^{ab} = \left\langle\theta^a, \theta^b\right\rangle$. It is to be understood that the covariant and contravariant components of the metric tensor will be used to raise and the (component) indices of vectors and tensors.

The components of the metric tensor

$$g\,(x_N) = \eta_{ik}\, dX^i \otimes dX^k$$

describing the locally flat tangent space $\mathbb{T}_x(\mathbb{M})$ of rigid frames at a point $x_N = x_N\,(x^a)$ are given by

$$\eta_{ik} = \langle E_i, E_k\rangle = \text{diag}\,(\pm1, \pm1, \ldots, \pm1)\,.$$

In four dimensions, the above may be taken to be the components of the Minkowski metric tensor, i.e., $\eta_{ik} = \langle E_i, E_k\rangle = \text{diag}\,(1, -1, -1, -1)$.

Then we have the expression

$$g_{ab} = \eta_{ik}\,\partial_a X^i\,\partial_b X^k.$$

The line-element of $\mathbb{C}^\infty$ is then given by

$$ds^2 = g = g_{ab}\left(\partial_i\, x^a\,\partial_k x^b\right) dX^i \otimes dX^k$$

where $\theta^a = \partial_i\, x^a dX^i$.

Given the existence of a local coordinate transformation via $x^i = x^i\,(\bar{x}^\alpha)$ in $\mathbb{C}^\infty$, the components of an arbitrary tensor field $T \in \mathbb{C}^\infty$ of rank (p, q) transform according to

$$T^{ab\ldots g}_{cd\ldots h} = T^{\alpha\beta\ldots\lambda}_{\mu\nu\ldots\eta}\,\partial_\alpha\, x^a\,\partial_\beta\, x^b \ldots \partial_\lambda\, x^g\,\partial_c\,\bar{x}^\mu\,\partial_d\,\bar{x}^\nu \ldots \partial_h\,\bar{x}^\eta.$$

Let $\delta^{i_1 i_2\ldots i_p}_{j_1 j_2\ldots j_p}$ be the components of the generalized Kronecker delta. They are given by

$$\delta^{i_1 i_2\ldots i_p}_{j_1 j_2\ldots j_p} = \in_{j_1 j_2\ldots j_p} \in^{i_1\ldots i_p} = \det\begin{pmatrix} \delta^{i_1}_{j_1} & \delta^{i_2}_{j_1} & \ldots & \delta^{i_p}_{j_1} \\ \delta^{i_1}_{j_2} & \delta^{i_2}_{j_2} & \ldots & \delta^{i_p}_{j_2} \\ \ldots & \ldots & \ldots & \ldots \\ \delta^{i_1}_{j_p} & \delta^{i_2}_{j_p} & \ldots & \delta^{i_p}_{j_p} \end{pmatrix}$$

where $\in_{j_1 j_2\ldots j_p} = \sqrt{\det\,(g)}\,\epsilon_{j_1 j_2\ldots j_p}$ and $\in^{i_1 i_2\ldots i_p} = \frac{\epsilon^{i_1 i_2\ldots i_p}}{\sqrt{\det(g)}}$ are the covariant and contravariant components of the completely anti-symmetric Levi-Civita permutation tensor, respectively, with the ordinary permutation symbols being given as usual by $\epsilon_{j_1 j_2\ldots j_q}$ and $\epsilon^{i_1 i_2\ldots i_p}$. Again, if ω is an arbitrary tensor, then the object represented by

$$^*\omega_{j_1 j_2\ldots j_p} = \frac{1}{p!}\,\delta^{i_1 i_2\ldots i_p}_{j_1 j_2\ldots j_p}\,\omega_{i_1 i_2\ldots i_p}$$

is completely anti-symmetric.

Introducing a generally asymmetric connection Γ via the covariant derivative

$$\partial_b\,\omega_a = \Gamma^c_{ab}\,\omega_c$$

i.e.,

$$\Gamma^c_{ab} = \langle\theta^c, \partial_b\omega_a\rangle = \Gamma^c_{(ab)} + \Gamma^c_{[ab]}$$

where the round index brackets indicate symmetrization and the square ones indicate anti-symmetrization, we have, by means of the local coordinate transformation given by $x^a = x^a\,(\bar{x}^\alpha)$ in $\mathbb{C}^\infty$

$$\partial_b\, e^\alpha_a = \Gamma^c_{ab}\, e^\alpha_c - \bar{\Gamma}^\alpha_{\beta\lambda}\, e^\beta_a\, e^\lambda_b$$

where the tetrads of the *moving frames* are given by $e^\alpha_a = \partial_a\bar{x}^\alpha$ and $e^a_\alpha = \partial_\alpha x^a$. They satisfy $e^a_\alpha e^\alpha_b = \delta^a_b$ and $e^\alpha_a e^a_\beta = \delta^\alpha_\beta$. In addition, it can also be verified that

$$\partial_\beta\, e^a_\alpha = \bar{\Gamma}^\lambda_{\alpha\beta}\, e^a_\lambda - \Gamma^a_{bc}\, e^b_\alpha\, e^c_\beta\,,$$

$$\partial_b\, e^a_\alpha = e^a_\lambda\,\bar{\Gamma}^\lambda_{\alpha\beta}\, e^\beta_b - \Gamma^a_{cb}\, e^c_\alpha\,.$$

We know that Γ is a non-tensorial object, since its components transform as

$$\Gamma^c_{ab} = e^c_\alpha\,\partial_b e^\alpha_a + e^c_\alpha\,\bar{\Gamma}^\alpha_{\beta\lambda}\, e^\beta_a\, e^\lambda_b\,.$$

However, it can be described as a kind of displacement field since it is what makes possible a comparison of vectors from point to point in $\mathbb{C}^\infty$. In fact the relation $\partial_b\,\omega_a = \Gamma^c_{ab}\,\omega_c$

defines the so-called metricity condition, i.e., the change (during a displacement) in the basis can be measured by the basis itself. This immediately translates into

$$\nabla_c\, g_{ab} = 0$$

where we have just applied the notion of a covariant derivative to an arbitrary tensor field T:

$$\begin{aligned}\nabla_m T^{ab\ldots g}_{cd\ldots h} &= \partial_m T^{ab\ldots g}_{cd\ldots h} + \Gamma^a_{pm} T^{pb\ldots g}_{cd\ldots h} + \Gamma^b_{pm} T^{ap\ldots g}_{cd\ldots h} + \ldots \\ &\ldots + \Gamma^g_{pm} T^{ab\ldots p}_{cd\ldots h} - \Gamma^p_{cm} T^{ab\ldots g}_{pd\ldots h} - \Gamma^p_{dm} T^{ab\ldots g}_{cp\ldots h} - \ldots \\ &\ldots - \Gamma^p_{hm} T^{ab\ldots g}_{cd\ldots p}\end{aligned}$$

such that $(\partial_m T)^{ab\ldots g}_{cd\ldots h} = \nabla_m\, T^{ab\ldots g}_{cd\ldots h}$.

The condition $\nabla_c g_{ab} = 0$ can be solved to give

$$\Gamma^c_{ab} = \frac{1}{2}\, g^{cd} \left(\partial_b\, g_{da} - \partial_d\, g_{ab} + \partial_a\, g_{bd}\right) + \\ + \Gamma^c_{[ab]} - g^{cd}\left(g_{ae}\Gamma^e_{[db]} + g_{be}\Gamma^e_{[da]}\right)$$

from which it is customary to define

$$\Delta^c_{ab} = \frac{1}{2}\, g^{cd} \left(\partial_b\, g_{da} - \partial_d\, g_{ab} + \partial_a\, g_{bd}\right)$$

as the Christoffel symbols (symmetric in their two lower indices) and

$$K^c_{ab} = \Gamma^c_{[ab]} - g^{cd}\left(g_{ae}\Gamma^e_{[db]} + g_{be}\Gamma^e_{[da]}\right)$$

as the components of the so-called cotwist tensor (anti-symmetric in the first two mixed indices).

Note that the components of the twist tensor are given by

$$\Gamma^a_{[bc]} = \frac{1}{2}\, e^a_\alpha \left(\partial_c\, e^\alpha_b - \partial_b\, e^\alpha_c + e^\beta_b\, \bar{\Gamma}^\alpha_{\beta c} - e^\beta_c\, \bar{\Gamma}^\alpha_{\beta b}\right)$$

where we have set $\bar{\Gamma}^\alpha_{\beta c} = \bar{\Gamma}^\alpha_{\beta\lambda} e^\lambda_c$, such that for an arbitrary scalar field Φ we have

$$\left(\nabla_a \nabla_b - \nabla_b \nabla_a\right)\Phi = 2\Gamma^c_{[ab]} \nabla_c\, \Phi\,.$$

The components of the curvature tensor R of $\mathbb{C}^\infty$ are then given via the relation

$$\begin{aligned}\left(\nabla_q \nabla_p - \nabla_p \nabla_q\right) T^{ab\ldots s}_{cd\ldots r} &= T^{ab\ldots s}_{wd\ldots r} R^w{}_{cpq} + T^{ab\ldots s}_{cw\ldots r} R^w{}_{dpq} + \\ &+ \ldots + T^{ab\ldots s}_{cd\ldots w} R^w{}_{rpq} - T^{wb\ldots s}_{cd\ldots r} R^a{}_{wpq} - T^{aw\ldots s}_{cd\ldots r} R^b{}_{wpq} - \\ &- \ldots - T^{ab\ldots w}_{cd\ldots r} R^s{}_{wpq} - 2\Gamma^w_{[pq]} \nabla_w\, T^{ab\ldots s}_{cd\ldots r}\end{aligned}$$

where

$$\begin{aligned}R^d{}_{abc} &= \partial_b \Gamma^d_{ac} - \partial_c \Gamma^d_{ab} + \Gamma^e_{ac}\Gamma^d_{eb} - \Gamma^e_{ab}\Gamma^d_{ec} \\ &= B^d{}_{abc}\left(\Delta\right) + \hat{\nabla}_b\, K^d_{ac} - \hat{\nabla}_c\, K^d_{ab} + K^e_{ac} K^d_{eb} - K^e_{ab} K^d_{ec}\,.\end{aligned}$$

where $\hat{\nabla}$ denotes covariant differentiation with respect to the Christoffel symbols alone, and where

$$B^d{}_{abc}\left(\Delta\right) = \partial_b \Delta^d_{ac} - \partial_c \Delta^d_{ab} + \Delta^e_{ac}\Delta^d_{eb} - \Delta^e_{ab}\Delta^d_{ec}$$

are the components of the Riemann-Christoffel curvature tensor of $\mathbb{C}^\infty$.

From the components of the curvature tensor, namely, $R^d{}_{abc}$, we have (using the metric tensor to raise and lower indices)

$$R_{ab} \equiv R^c{}_{acb} = B_{ab}\left(\Delta\right) + \hat{\nabla}_c K^c_{ab} - K^c_{ad} K^d_{cb} - \\ - 2\hat{\nabla}_b\, \Gamma^c_{[ac]} + 2K^c_{ab}\Gamma^d_{[cd]}$$

$$R \equiv R^a{}_a = B\left(\Delta\right) - 4\, g^{ab}\, \hat{\nabla}_a\, \Gamma^c_{[bc]} - \\ - 2\, g^{ac}\, \Gamma^b_{[ab]}\Gamma^d_{[cd]} - K_{abc} K^{acb}$$

where $B_{ab}\left(\Delta\right) \equiv B^c_{acb}\left(\Delta\right)$ are the components of the symmetric Ricci tensor and $B\left(\Delta\right) \equiv B^a_a\left(\Delta\right)$ is the Ricci scalar. Note that $K_{abc} \equiv g_{ad} K^d_{bc}$ and $K^{acb} \equiv g^{cd} g^{be} K^a_{de}$.

Now since

$$\Gamma^b_{ba} = \Delta^b_{ba} = \Delta^b_{ab} = \partial_a \left(\ln\, \sqrt{\det\left(g\right)}\right)$$

$$\Gamma^b_{ab} = \partial_a \left(\ln\, \sqrt{\det\left(g\right)}\right) + 2\Gamma^b_{[ab]}$$

we see that for a continuous metric determinant, the so-called homothetic curvature vanishes:

$$H_{ab} \equiv R^c{}_{cab} = \partial_a\, \Gamma^c_{cb} - \partial_b\, \Gamma^c_{ca} = 0\,.$$

Introducing the traceless Weyl tensor W, we have the following decomposition theorem:

$$R^d{}_{abc} = W^d_{abc} + \frac{1}{n-2}\left(\delta^d_b R_{ac} + g_{ac} R^d{}_b - \delta^d_c R_{ab} - g_{ab} R^d{}_c\right) + \\ + \frac{1}{\left(n-1\right)\left(n-2\right)}\left(\delta^d_c\, g_{ab} - \delta^d_b\, g_{ac}\right) R$$

which is valid for $n > 2$. For $n = 2$, we have

$$R^d{}_{abc} = K_G \left(\delta^d_b\, g_{ac} - \delta^d_c\, g_{ab}\right)$$

where

$$K_G = \frac{1}{2}\, R$$

is the Gaussian curvature of the surface. Note that (in this case) the Weyl tensor vanishes.

Any n-dimensional manifold (for which $n > 1$) with constant sectional curvature R and vanishing twist is called an Einstein space. It is described by the following simple relations:

$$R^d{}_{abc} = \frac{1}{n(n-1)}\left(\delta^d_b\, g_{ac} - \delta^d_c\, g_{ab}\right) R\,,$$

$$R_{ab} = \frac{1}{n}\, g_{ab}\, R\,.$$

In the above, we note especially that

$$R^d{}_{abc} = B^d{}_{abc}(\Delta),$$

$$R_{ab} = B_{ab}(\Delta),$$

$$R = B(\Delta).$$

Furthermore, after some lengthy algebra, we obtain, in general, the following *generalized* Bianchi identities:

$$R^a{}_{bcd} + R^a{}_{cdb} + R^a{}_{dbc} = -2\big(\partial_d \Gamma^a_{[bc]} + \partial_b \Gamma^a_{[cd]} + \partial_c \Gamma^a_{[db]} + \Gamma^a_{eb}\Gamma^e_{[cd]} + \Gamma^a_{ec}\Gamma^e_{[db]} + \Gamma^a_{ed}\Gamma^e_{[bc]}\big),$$

$$\nabla_e R^a{}_{bcd} + \nabla_c R^a{}_{bde} + \nabla_d R^a{}_{bec} = 2\big(\Gamma^f_{[cd]} R^a{}_{bfe} + \Gamma^f_{[de]} R^a{}_{bfc} + \Gamma^f_{[ec]} R^a{}_{bfd}\big),$$

$$\nabla_a \left(R^{ab} - \frac{1}{2} g^{ab} R\right) = 2 g^{ab} \Gamma^c_{[da]} R^d{}_c + \Gamma^a_{[cd]} R^{cdb}{}_a$$

for any metric-compatible manifold endowed with both curvature and twist.

In the last of the above set of equations, we have introduced the generalized Einstein tensor, i.e.,

$$G_{ab} \equiv R_{ab} - \frac{1}{2} g_{ab} R$$

In particular, we also have the following specialized identities, i.e., the *regular* Bianchi identities:

$$B^a{}_{bcd} + B^a{}_{cdb} + B^a{}_{dbc} = 0,$$

$$\hat{\nabla}_e B^a{}_{bcd} + \hat{\nabla}_c B^a{}_{bde} + \hat{\nabla}_d B^a{}_{bec} = 0,$$

$$\hat{\nabla}_a \left(B^{ab} - \frac{1}{2} g^{ab} B\right) = 0.$$

In general, these hold in the case of a symmetric, metric-compatible connection. Non-metric differential geometry is beyond the scope of our present consideration.

We now define the so-called Lie derivative which can be used to define a diffeomorphism invariant in $\mathbb{C}^\infty$. for a vector field U and a tensor field T, both arbitrary, the invariant derivative represented (in component notation) by

$$L_U T^{ab\ldots g}_{cd\ldots h} = \partial_m T^{ab\ldots g}_{cd\ldots h} U^m + T^{ab\ldots g}_{md\ldots h} \partial_c U^m + T^{ab\ldots g}_{cm\ldots h} \partial_d U^m + \ldots + T^{ab\ldots g}_{cd\ldots m} \partial_h U^m - T^{mb\ldots g}_{cd\ldots h} \partial_m U^a - T^{am\ldots g}_{cd\ldots. h} \partial_m U^b - \ldots - T^{ab\ldots m}_{cd\ldots h} \partial_m U^g$$

defines the Lie derivative of T with respect to U. With the help of the twist tensor and the relation

$$\partial_b U^a = \nabla_b U^a - \Gamma^a_{cb} U^c = \nabla_b U^a - \big(\Gamma^a_{bc} - 2\Gamma^a_{[bc]}\big) U^c$$

we can write

$$L_U T^{ab\ldots g}_{cd\ldots h} = \nabla_m T^{ab\ldots g}_{cd\ldots h} U^m + T^{ab\ldots g}_{md\ldots h} \nabla_c U^m + T^{ab\ldots g}_{cm\ldots h} \nabla_d U^m + \ldots + T^{ab\ldots g}_{cd\ldots m} \nabla_h U^m - T^{mb\ldots g}_{cd\ldots h} \nabla_m U^a - T^{am\ldots g}_{cd\ldots. h} \nabla_m U^b - \ldots - T^{ab\ldots m}_{cd\ldots h} \nabla_m U^g + 2\Gamma^a_{[mp]} T^{mb\ldots g}_{cd\ldots h} U^p + 2\Gamma^b_{[mp]} T^{am\ldots g}_{cd\ldots h} U^p + \ldots + 2\Gamma^g_{[mp]} T^{ab\ldots m}_{cd\ldots h} U^p - 2\Gamma^m_{[cp]} T^{ab\ldots g}_{md\ldots h} U^p + 2\Gamma^m_{[dp]} T^{ab\ldots g}_{cm\ldots h} U^p - \ldots - 2\Gamma^m_{[hp]} T^{ab\ldots g}_{cd\ldots m} U^p.$$

Hence, noting that the components of the twist tensor, namely, $\Gamma^i_{[kl]}$, indeed transform as components of a tensor field, it is seen that the $L_U T^{ij\ldots s}_{kl\ldots r}$ do transform as components of a tensor field. Apparently, the beautiful property of the Lie derivative (applied to an arbitrary tensor field) is that it is connection-independent even in a curved manifold.

We will need the identities derived in this Section later on.

3 The generalized four-dimensional linear constitutive field equations

We shall now present a four-dimensional linear continuum theory of the classical physical fields capable of describing microspin phenomena in addition to the gravitational and electromagnetic fields. By microspin phenomena, we mean those phenomena generated by rotation of points in the four-dimensional space-time manifold (continuum) $\mathbb{S}^4$ with local coordinates x^μ in the manner described by the so-called Cosserat continuum theory.

We start with the following constitutive equation in four dimensions:

$$T^{\mu\nu} = C^{\mu\nu}{}_{\rho\sigma} D^{\rho\sigma} = \frac{1}{\kappa}\left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R\right)$$

where now the Greek indices run from 0 to 3. In the above equation, $T^{\mu\nu}$ are the contravariant components of the generally asymmetric energy-momentum tensor, $C^{\mu\nu}_{\rho\sigma}$ are the mixed components of the generalized four-dimensional elasticity tensor, $D^{\rho\sigma}$ are the contravariant components of the four-dimensional displacement gradient tensor, $R^{\mu\nu}$ are the contravariant components of the generalized (asymmetric) four-dimensional Ricci curvature tensor, $\kappa = -8\pi$ is the Einstein coupling constant (in geometrized units), and $R = R^\mu_\mu$ is the generalized Ricci four-dimensional curvature scalar.

Furthermore, we can decompose our four-dimensional elasticity tensor into its holonomic and anholonomic parts as follows:

$$C^{\mu\nu}{}_{\rho\sigma} = A^{\mu\nu}{}_{\rho\sigma} + B^{\mu\nu}{}_{\rho\sigma}$$

where

$$A^{\mu\nu}{}_{\rho\sigma} = A^{(\mu\nu)}{}_{(\rho\sigma)} = A^{\mu\nu}{}_{\rho\sigma}$$

$$B^{\mu\nu}{}_{\rho\sigma} = B^{[\mu\nu]}{}_{[\rho\sigma]} = B^{\mu\nu}{}_{\rho\sigma}$$

such that

$$C^{\mu\nu}{}_{\rho\sigma} = C_{\rho\sigma}{}^{\mu\nu} \, .$$

Therefore, we can express the fully covariant components of the generalized four-dimensional elasticity tensor in terms of the covariant components of the symmetric metric tensor $g_{\mu\nu}$ (satisfying, as before, $g_{\nu\sigma}g^{\mu\sigma} = \delta^{\mu}_{\nu}$) as

$$C_{\mu\nu\rho\sigma} = \alpha\, g_{\mu\nu}\, g_{\rho\sigma} + \beta\, g_{\mu\rho} g_{\nu\sigma} + \gamma\, g_{\mu\sigma} g_{\nu\rho} =$$

$$= \alpha\, g_{\mu\nu}\, g_{\rho\sigma} + \lambda \left(g_{\mu\rho} g_{\nu\sigma} + g_{\mu\sigma} g_{\nu\rho} \right) + \omega \left(g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho} \right)$$

where α, β, γ, λ, and ω are constitutive invariants that are not necessarily constant. It is therefore seen that

$$A_{\mu\nu\rho\sigma} = \alpha\, g_{\mu\nu}\, g_{\rho\sigma} + \lambda \left(g_{\mu\rho} g_{\nu\sigma} + g_{\mu\sigma} g_{\nu\rho} \right)$$

$$B_{\mu\nu\rho\sigma} = \omega \left(g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho} \right)$$

An infinitesimal displacement (diffeomorphism) in the space-time manifold $\mathbb{S}^4$ from an initial point P to a neighboring point Q is given as usual by

$$x^{\mu}\,(Q) = x^{\mu}(P) + \xi^{\mu}$$

where ξ^{μ} are the components of the four-dimensional infinitesimal displacement field vector. The generally asymmetric four-dimensional displacement gradient tensor is then given by

$$D_{\mu\nu} = \nabla_{\nu}\, \xi_{\mu} \, .$$

The decomposition $D_{\mu\nu} = D_{(\mu\nu)} + D_{[\mu\nu]}$ and the supplementary infinitesimal point-rotation condition $\Gamma^{\alpha}_{[\mu\nu]}\xi^{\mu} = 0$ allow us to define the symmetric four-dimensional displacement ("dilation") tensor by

$$\Phi_{\mu\nu} = D_{(\mu\nu)} = \frac{1}{2} \left(\nabla_{\mu}\xi_{\nu} + \nabla_{\nu}\xi_{\mu} \right) = \frac{1}{2}\, L_{\xi}\, g_{\mu\nu}$$

from which the "dilation" scalar is given by

$$\Phi = \Phi^{\mu}{}_{\mu} = D^{\mu}{}_{\mu} = \frac{1}{2}\, g^{\mu\nu}\, L_{\xi}\, g_{\mu\nu} = \nabla_{\mu}\, \xi^{\mu}$$

as well as the anti-symmetric four-dimensional intrinsic spin (vorticity) tensor by

$$\omega_{\mu\nu} = D_{[\mu\nu]} = \frac{1}{2} \left(\nabla_{\nu}\, \xi_{\mu} - \nabla_{\mu}\, \xi_{\nu} \right).$$

Let us now decompose the four-dimensional infinitesimal displacement field vector as follows:

$$\xi^{\mu} = \partial^{\mu} F + \psi^{\mu}.$$

Here the continuous scalar function F represents the integrable part of the four-dimensional macroscopic displacement field vector while the remaining parts are given by ψ^{μ} via

$$\psi^{\mu} = \sigma^{\mu} + \phi^{\mu} + 2\,\bar{e}\,\varphi^{\mu}$$

where σ^{μ} are the components of the non-integrable four-dimensional macroscopic displacement field vector, ϕ^{μ} are the components of the four-dimensional microscopic (micropolar) intrinsic spin vector, $\bar{e}$ is a constant proportional to the electric charge, and φ^{μ} are the components of the electromagnetic four-potential vector. We assume that in general σ^{μ}, ϕ^{μ}, and φ^{μ} are linearly independent of each other.

The intrinsic four-dimensional macroscopic spin ("angular momentum") tensor is then given by

$$\Omega_{\mu\nu} = \frac{1}{2} \left(\nabla_{\nu}\, \sigma_{\mu} - \nabla_{\mu}\, \sigma_{\nu} \right).$$

Likewise, the intrinsic four-dimensional microscopic (micropolar) spin tensor is given by

$$S_{\mu\nu} = \frac{1}{2} \left(\nabla_{\nu}\, \phi_{\mu} - \nabla_{\mu}\, \phi_{\nu} \right).$$

Note that this tensor vanishes when the points are not allowed to rotate such as in conventional (standard) cases.

Meanwhile, the electromagnetic field tensor is given by

$$F_{\mu\nu} = \nabla_{\nu}\, \varphi_{\mu} - \nabla_{\mu}\, \varphi_{\nu} \, .$$

In this case, we especially note that, by means of the condition $\Gamma^{\alpha}_{[\mu\nu]}\xi^{\mu} = 0$, the above expression reduces to the usual Maxwellian relation

$$F_{\mu\nu} = \partial_{\nu}\varphi_{\mu} - \partial_{\mu}\varphi_{\nu} \, .$$

We can now write the intrinsic spin tensor as

$$\omega_{\mu\nu} = \Omega_{\mu\nu} + S_{\mu\nu} + \bar{e}F_{\mu\nu} \, .$$

Hence the full electromagnetic content of the theory becomes visible. We also see that our space-time continuum can be considered as a dynamically polarizable medium possessing chirality. As such, the gravitational and electromagnetic fields, i.e., the familiar classical fields, are intrinsic geometric objects in the theory.

Furthermore, from the cotwist tensor, let us define a geometric spin vector via

$$A_{\mu} \equiv K^{\sigma}_{\mu\sigma} = 2\Gamma^{\sigma}_{[\mu\sigma]} .$$

Now, in a somewhat restrictive case, in connection with the spin fields represented by σ^{μ}, ϕ^{μ}, and φ^{μ}, the selection

$$A_{\mu} = c_1\sigma_{\mu} + c_2\phi_{\mu} + 2\,\bar{e}\,c_3\varphi_{\mu} = \in\, \psi_{\mu}$$

i.e.,

$$\in\, = \frac{c_1\sigma_{\mu} + c_2\phi_{\mu} + 2\,\bar{e}\,c_3\varphi_{\mu}}{\sigma_{\mu} + \phi_{\mu} + 2\,\bar{e}\,\varphi_{\mu}}$$

will directly attribute the cotwist tensor to the intrinsic spin fields of the theory. However, we would in general expect the intrinsic spin fields to remain in the case of a semi-symmetric connection, for which $A_{\mu} = 0$ and so we cannot carry this proposition any further.

At this point, we see that the holonomic part of the generalized four-dimensional elasticity tensor given by $A_{\mu\nu\rho\sigma}$ is responsible for (centrally symmetric) gravitational phenomena while the anholonomic part given by $B_{\mu\nu\rho\sigma}$ owes its existence to the (con)twist tensor which is responsible for the existence of the intrinsic spin fields in our consideration.

Furthermore, we see that the components of the energy-momentum tensor can now be expressed as

$$T_{\mu\nu} = \alpha\, g_{\mu\nu}\,\Phi + \beta\, D_{\mu\nu} + \gamma\, D_{\nu\mu}\,.$$

In other words,

$$T_{(\mu\nu)} = \alpha\, g_{\mu\nu}\,\Phi + (\beta+\gamma)\,\Phi_{\mu\nu}\,,$$

$$T_{[\mu\nu]} = (\beta-\gamma)\,\omega_{\mu\nu}\,.$$

Alternatively,

$$T_{(\mu\nu)} = \frac{1}{2}\,\alpha\, g_{\mu\nu} g^{\alpha\beta} L_\xi\, g_{\alpha\beta} + \frac{1}{2}\,(\beta+\gamma)\, L_\xi\, g_{\mu\nu}\,,$$

$$T_{[\mu\nu]} = (\beta-\gamma)\,(\Omega_{\mu\nu} + S_{\mu\nu} + \bar{e}\, F_{\mu\nu})\,.$$

We may note that, in a sense analogous to that of the ordinary mechanics of continuous media, the generally asymmetric character of the energy-momentum tensor means that a material object in motion is generally subject to distributed body couples.

We also have

$$T = T^{\mu}_{\ \mu} = (4\alpha+\beta+\gamma)\,\Phi = -\frac{1}{\kappa}\,R\,.$$

Let us briefly relate our description to the standard material description given by general relativity. For this purpose, let us assume that the intrinsic spin fields other than the electromagnetic field are negligible. If we denote the material density and the pressure by ρ and p, respectively, then it can be directly verified that

$$\Phi = \frac{\rho - 4p}{4\alpha+\beta+\gamma}$$

is a solution to the ordinary expression

$$T_{(\mu\nu)} = \rho\, u_\mu u_\nu - p g_{\mu\nu} - {} - \frac{1}{4\pi}\left(F_{\mu\sigma}F^{\sigma}_{\ \nu} - \frac{1}{4}\, g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}\right)$$

where u_μ are the covariant components of the unit velocity vector. This is true whether the electromagnetic field is present or not since the (symmetric) energy-momentum tensor of the electromagnetic field given by

$$J_{\mu\nu} = -\frac{1}{4\pi}\left(F_{\mu\sigma}F^{\sigma}_{\ \nu} - \frac{1}{4}\, g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}\right)$$

is traceless.

At this point, however, we may note that the covariant divergence

$$\nabla_\mu T^{\mu\nu} = g^{\mu\nu}\nabla_\mu(\alpha\Phi) + \beta\nabla_\mu D^{\mu\nu} + {} + \gamma\nabla_\mu D^{\nu\mu} + D^{\mu\nu}\nabla_\mu\beta + D^{\nu\mu}\nabla_\mu\gamma$$

need not vanish in general since

$$\nabla_\mu T^{\mu\nu} = \frac{1}{\kappa}\nabla_\mu\left(R^{\mu\nu} - \frac{1}{2}\, g^{\mu\nu} R\right) = {} = \frac{1}{\kappa}\left(2 g^{\mu\nu}\,\Gamma^{\rho}_{[\sigma\mu]} R^{\sigma}_{\rho} + \Gamma^{\lambda}_{[\rho\sigma]} R^{\rho\sigma\nu}_{\ \ \ \ \lambda}\right).$$

In an isotropic, homogeneous Universe, for which the constitutive invariants $\alpha, \beta, \gamma, \lambda$, and ω are constant, the above expression reduces to

$$\nabla_\mu T^{\mu\nu} = \alpha\, g^{\mu\nu}\nabla_\mu\Phi + \beta\nabla_\mu D^{\mu\nu} + \gamma\nabla_\mu D^{\nu\mu}.$$

If we require the above divergence to vanish, however, we see that the motion described by this condition is still more general than the pure geodesic motion for point-particles.

Still in the case of an isotropic, homogeneous Universe, possibly on large cosmological scales, then our expression for the energy-momentum tensor relates the generalized Ricci curvature scalar directly to the "dilation" scalar. In general, we have

$$R = -\kappa\,(4\alpha+\beta+\gamma)\,\Phi = -\kappa\Lambda\Phi = -\frac{1}{2}\,\kappa\Lambda\, g^{\mu\nu} L_\xi\, g_{\mu\nu}\,.$$

Now, for the generalized Ricci curvature tensor, we obtain the following asymmetric constitutive field equation:

$$R_{\mu\nu} = \kappa\left(T_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, T\right) = \kappa\,(\theta\, g_{\mu\nu} + \beta\, D_{\mu\nu} + \gamma)$$

where

$$\theta = -\frac{1}{2}\,(2\alpha+\beta+\gamma)\,\Phi\,.$$

In other words,

$$R_{(\mu\nu)} = \kappa\big(\theta\, g_{\mu\nu} + (\beta+\gamma)\,\Phi_{\mu\nu}\big),$$

$$R_{[\mu\nu]} = \kappa\,(\beta-\gamma)\,\omega_{\mu\nu}\,.$$

Inserting the value of κ, we can alternatively write

$$R_{(\mu\nu)} = -8\pi\left(\theta\, g_{\mu\nu} + \frac{1}{2}\,(\beta+\gamma)\, L_\xi g_{\mu\nu}\right)$$

$$R_{[\mu\nu]} = -8\pi\,(\beta-\gamma)\,(\Omega_{\mu\nu} + S_{\mu\nu} + \bar{e}F_{\mu\nu})\,.$$

Hence, the correspondence between the generalized Ricci curvature tensor and the physical fields in our theory becomes complete. The present theory shows that in a curved space-time with a particular spherical symmetry and in a flat Minkowski space-time (both space-times are solutions to the equation $\Phi_{\mu\nu} = 0$, i.e., $L_\xi\, g_{\mu\nu} = 0$) it is in general still possible for the spin fields to exist. One possible geometry that complies with such a space-time symmetry is the geometry of distant parallelism with vanishing space-time curvature (but non-vanishing Riemann-Christoffel curvature) and non-vanishing twist.

Now let us recall that in four dimensions, with the help of the Weyl tensor W, we have the decomposition

$$R_{\mu\nu\rho\sigma} = W_{\mu\nu\rho\sigma} + \\ + \frac{1}{2}\left(g_{\mu\rho}R_{\nu\sigma} + g_{\nu\sigma}R_{\mu\rho} - g_{\mu\sigma}R_{\nu\rho} - g_{\nu\rho}R_{\mu\sigma}\right) + \\ + \frac{1}{6}\left(g_{\mu\sigma}g_{\nu\rho} - g_{\mu\rho}g_{\nu\sigma}\right) R\,.$$

We obtain, upon setting $\bar{\alpha} = \frac{1}{2}\kappa\theta$, $\bar{\beta} = \frac{1}{2}\kappa\beta$, $\bar{\gamma} = \frac{1}{2}\kappa\gamma$, and $\bar{\lambda} = \frac{1}{6}\kappa\Lambda$

$$R_{\mu\nu\rho\sigma} = W_{\mu\nu\rho\sigma} + 2\bar{\alpha}\left(g_{\mu\rho}\,g_{\nu\sigma} - g_{\mu\sigma}\,g_{\nu\rho}\right) + \\ + \bar{\beta}\left(g_{\mu\rho}\,D_{\nu\sigma} + g_{\nu\sigma}\,D_{\mu\rho} - g_{\mu\sigma}\,D_{\nu\rho} - g_{\nu\rho}\,D_{\mu\sigma}\right) + \\ + \bar{\gamma}\left(g_{\mu\rho}\,D_{\sigma\nu} + g_{\nu\sigma}\,D_{\rho\mu} - g_{\mu\sigma}\,D_{\rho\nu} - g_{\nu\rho}\,D_{\sigma\mu}\right) + \\ + \bar{\lambda}\left(g_{\mu\rho}\,g_{\nu\sigma} - g_{\mu\sigma}\,g_{\nu\rho}\right)\Phi\,.$$

Therefore, in terms of the anholonomic part of the generalized elasticity tensor, we have

$$R_{\mu\nu\rho\sigma} = W_{\mu\nu\rho\sigma} + 2\,\frac{\bar{\alpha}}{\omega}\,B_{\mu\nu\rho\sigma} + \\ + \bar{\beta}\left(g_{\mu\rho}\,D_{\nu\sigma} + g_{\nu\sigma}\,D_{\mu\rho} - g_{\mu\sigma}\,D_{\nu\rho} - g_{\nu\rho}\,D_{\mu\sigma}\right) + \\ + \bar{\gamma}\left(g_{\mu\rho}\,D_{\sigma\nu} + g_{\nu\sigma}\,D_{\rho\mu} - g_{\mu\sigma}\,D_{\rho\nu} - g_{\nu\rho}\,D_{\sigma\mu}\right) + \\ + \bar{\lambda}\left(g_{\mu\rho}\,g_{\nu\sigma} - g_{\mu\sigma}\,g_{\nu\rho}\right)\Phi\,.$$

In the special case of a pure gravitational field, the twist of the space-time continuum vanishes. In this situation our intrinsic spin fields vanish and consequently, we are left simply with

$$R_{\mu\nu\rho\sigma} = W_{\mu\nu\rho\sigma} + \\ + \frac{1}{2}\left(\bar{\beta} + \bar{\gamma}\right)\left(g_{\mu\rho}\,D_{\nu\sigma} + g_{\nu\sigma}\,D_{\mu\rho} - g_{\mu\sigma}\,D_{\nu\rho} - g_{\nu\rho}\,D_{\mu\sigma}\right) + \\ + \bar{\lambda}\left(g_{\mu\rho}\,g_{\nu\sigma} - g_{\mu\sigma}\,g_{\nu\rho}\right)\Phi\,.$$

In standard general relativity, this gives the explicit form of the Riemann-Christoffel curvature tensor in terms of the Lie derivative $L_\xi\, g_{\mu\nu} = 2\Phi_{\mu\nu}$. For a space-time satisfying the symmetry $L_\xi\, g_{\mu\nu} = 0$, we simply have $R_{\mu\nu\rho\sigma} = W_{\mu\nu\rho\sigma}$, i.e., the space-time is devoid of material sources or "empty". This condition is relatively weaker than the case of a space-time with constant sectional curvature, $R = \text{const.}$ for which the Weyl tensor vanishes.

4 The generalized four-dimensional non-linear constitutive field equations

In reference to the preceding section, let us now present, in a somewhat concise manner, a non-linear extension of the formulation presented in the preceding section. The resulting non-linear constitutive field equations will therefore not be limited to weak fields only. In general, it can be shown that the full curvature tensor contains terms quadratic in the displacement gradient tensor and this gives us the reason to express the energy-momentum tensor which is quadratic in the displacement gradient tensor.

We start with the non-linear constitutive field equation

$$T^{\mu\nu} = C^{\mu\nu}{}_{\rho\sigma}D^{\rho\sigma} + K^{\mu\nu}{}_{\rho\sigma\lambda\eta}D^{\rho\sigma}D^{\lambda\eta} = \frac{1}{\kappa}\left(R^{\mu\nu} - \frac{1}{2}\,g^{\mu\nu}R\right)$$

where

$$K_{\mu\nu\rho\sigma\lambda\eta} = a_1\,g_{\mu\nu}\,g_{\rho\sigma}\,g_{\lambda\eta} + a_2\,g_{\mu\nu}\,g_{\rho\lambda}\,g_{\sigma\eta} + \\ + a_3 g_{\mu\nu}\,g_{\rho\eta}\,g_{\sigma\lambda} + a_4\,g_{\rho\sigma}\,g_{\mu\lambda}\,g_{\nu\eta} + a_5\,g_{\rho\sigma}\,g_{\mu\eta}\,g_{\nu\lambda} + \\ + a_6\,g_{\lambda\eta}\,g_{\mu\rho}\,g_{\nu\sigma} + a_7\,g_{\lambda\eta}\,g_{\mu\sigma}\,g_{\nu\rho} + a_8\,g_{\mu\lambda}\,g_{\nu\rho}\,g_{\sigma\eta} + \\ + a_9\,g_{\mu\lambda}\,g_{\nu\sigma}\,g_{\rho\eta} + a_{10}\,g_{\mu\eta}\,g_{\nu\rho}\,g_{\sigma\lambda} + a_{11}\,g_{\mu\eta}\,g_{\nu\sigma}\,g_{\rho\lambda} + \\ + a_{12}\,g_{\nu\lambda}\,g_{\mu\rho}\,g_{\sigma\eta} + a_{13}\,g_{\nu\lambda}\,g_{\mu\sigma}\,g_{\rho\eta} + a_{14}\,g_{\nu\eta}\,g_{\mu\rho}\,g_{\sigma\lambda} + \\ + a_{15}\,g_{\nu\eta}\,g_{\mu\sigma}\,g_{\rho\lambda}$$

where the fifteen constitutive invariants $a_1,\ a_2,\ \ldots,\ a_{15}$ are not necessarily constant.

We shall set

$$K_{\mu\nu\rho\sigma\lambda\eta} = K_{\rho\sigma\mu\nu\lambda\eta} = K_{\lambda\eta\mu\nu\rho\sigma} = K_{\mu\nu\lambda\eta\rho\sigma}\,.$$

Letting

$$K_{\mu\nu\rho\sigma\lambda\eta} = P_{\mu\nu\rho\sigma\lambda\eta} + Q_{\mu\nu\rho\sigma\lambda\eta}\,,$$

$$P_{\mu\nu\rho\sigma\lambda\eta} = P_{(\mu\nu)(\rho\sigma)(\lambda\eta)}\,,$$

$$Q_{\mu\nu\rho\sigma\lambda\eta} = Q_{[\mu\nu][\rho\sigma][\lambda\eta]}\,,$$

we have

$$P_{\mu\nu\rho\sigma\lambda\eta} = P_{\rho\sigma\mu\nu\lambda\eta} = P_{\lambda\eta\mu\nu\rho\sigma} = P_{\mu\nu\lambda\eta\rho\sigma}\,,$$

$$Q_{\mu\nu\rho\sigma\lambda\eta} = Q_{\rho\sigma\mu\nu\lambda\eta} = Q_{\lambda\eta\mu\nu\rho\sigma} = Q_{\mu\nu\lambda\eta\rho\sigma}\,.$$

Introducing the eleven constitutive invariants $b_1,\ b_2,\ \ldots,\ b_{11}$, we can write

$$K_{\mu\nu\rho\sigma\lambda\eta} = b_1 g_{\mu\nu} g_{\rho\sigma} g_{\lambda\eta} + b_2 g_{\mu\nu}\left(g_{\rho\lambda} g_{\sigma\eta} + g_{\rho\eta} + g_{\sigma\lambda}\right) + \\ + b_3 g_{\mu\nu}\left(g_{\rho\lambda} g_{\sigma\eta} - g_{\rho\eta} g_{\sigma\lambda}\right) + b_4 g_{\rho\sigma}\left(g_{\mu\lambda} g_{\nu\eta} + g_{\mu\eta} g_{\nu\lambda}\right) + \\ + b_5 g_{\rho\sigma}\left(g_{\mu\lambda} g_{\nu\eta} - g_{\mu\eta} g_{\nu\lambda}\right) + b_6 g_{\lambda\eta}\left(g_{\mu\rho} g_{\nu\sigma} + g_{\mu\sigma} g_{\nu\rho}\right) + \\ + b_7 g_{\lambda\eta}\left(g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho}\right) + b_8 g_{\mu\lambda}\left(g_{\nu\rho} g_{\sigma\eta} + g_{\nu\sigma} g_{\rho\eta}\right) + \\ + b_9 g_{\mu\lambda}\left(g_{\nu\rho} g_{\sigma\eta} - g_{\nu\sigma} g_{\rho\eta}\right) + b_{10} g_{\nu\lambda}\left(g_{\mu\rho} g_{\sigma\eta} + g_{\mu\sigma} g_{\rho\eta}\right) + \\ + b_{11} g_{\nu\lambda}\left(g_{\mu\rho} g_{\sigma\eta} - g_{\mu\sigma} g_{\rho\eta}\right).$$

The energy-momentum tensor is therefore given by

$$T_{\mu\nu} = \left(\alpha\,\Phi + b_1\,\Phi^2 + 2\,b_2\,\Phi_{\rho\sigma}\,\Phi^{\rho\sigma} + 2\,b_3\,\omega_{\rho\sigma}\,\omega^{\rho\sigma}\right) g_{\mu\nu} + \\ + \beta\,D_{\mu\nu} + \gamma\,D_{\nu\mu} + 2\left(b_4 + b_6\right)\Phi\,\Phi_{\mu\nu} + \\ + 2\left(b_5 + b_7\right)\Phi\,\omega_{\mu\nu} + 2\,b_8 D^{\rho}{}_{\mu}\,\Phi_{\nu\rho} + 2\,b_9 D^{\rho}{}_{\mu}\,\omega_{\nu\rho} + \\ + 2\,b_{10} D^{\rho}{}_{\nu}\,\Phi_{\mu\rho} + 2\,b_{11} D^{\rho}{}_{\nu}\,\omega_{\mu\rho}\,.$$

In other words,

$$T_{(\mu\nu)} = \left(\alpha\Phi + b_1\Phi^2 + 2b_2\Phi_{\rho\sigma}\Phi^{\rho\sigma} + 2b_3\,\omega_{\rho\sigma}\omega^{\rho\sigma}\right) g_{\mu\nu} + \\ + (\beta+\gamma)\,\Phi_{\mu\nu} 2\,(b_4+b_6)\,\Phi\Phi_{\mu\nu} + (b_8+b_{10}) \times \\ \times \left(D^\rho{}_\mu\Phi_{\nu\rho} + D^\rho{}_\nu\Phi_{\mu\rho}\right) + (b_9+b_{11})\left(D^\rho{}_\mu\omega_{\nu\rho} + D^\rho{}_\nu\omega_{\mu\rho}\right),$$

$$T_{[\mu\nu]} = (\beta-\gamma)\,\omega_{\mu\nu} + 2\,(b_4+b_6)\,\Phi\omega_{\mu\nu} + (b_8+b_{10}) \times \\ \times \left(D^\rho{}_\mu\Phi_{\nu\rho} - D^\rho{}_\nu\Phi_{\mu\rho}\right) + (b_9+b_{11})\left(D^\rho{}_\mu\omega_{\nu\rho} - D^\rho{}_\nu\omega_{\mu\rho}\right).$$

We also have

$$T = \mu_1\,\Phi + \mu_2\,\Phi^2 + \mu_3\,\Phi_{\mu\nu}\,\Phi^{\mu\nu} + \mu_4\,\omega_{\mu\nu}\,\omega^{\mu\nu}$$

where we have set

$$\mu_1 = 4\alpha + \beta + \gamma\,,$$
$$\mu_2 = 4b_1 + 2\,(b_4+b_6)\,,$$
$$\mu_3 = 8b_2 + 2\,(b_8+b_{10})\,,$$
$$\mu_4 = 8b_3 + 2\,(b_9-b_{11})\,,$$

for the sake of simplicity.

For the generalized Ricci curvature tensor, we obtain

$$R_{\mu\nu} = \kappa\Big\{\left(c_1\Phi + c_2\Phi^2 + c_3\Phi_{\rho\sigma}\Phi^{\rho\sigma} + c_4\omega_{\rho\sigma}\omega^{\rho\sigma}\right) g_{\mu\nu} + \\ + c_5 D_{\mu\nu} + c_6 D_{\nu\mu} + c_7\Phi\Phi_{\mu\nu} + c_8\Phi\omega_{\mu\nu} + c_9 D^\rho{}_\mu\Phi_{\nu\rho} + \\ + c_{10}D^\rho{}_\mu\omega_{\nu\rho} + c_{11}D^\rho{}_\nu\Phi_{\mu\rho} + c_{12}D^\rho{}_\nu\omega_{\mu\rho}\Big\}$$

where

$$c_1 = -\frac{1}{2}\,(2\alpha+\beta+\gamma)\,, \qquad c_7 = 2\,(b_4+b_6)\,,$$
$$c_2 = -\,(b_1+b_4+b_6)\,, \qquad c_8 = 2\,(b_5+b_7)\,,$$
$$c_3 = -\,(2b_2+b_8+b_{10})\,, \qquad c_9 = 2b_8\,,$$
$$c_4 = -\,(2b_3+b_9-b_{11})\,, \qquad c_{10} = 2b_9\,,$$
$$c_5 = \beta\,, \qquad c_{11} = 2b_{10}\,,$$
$$c_6 = \gamma\,, \qquad c_{12} = 2b_{11}\,,$$

i.e.,

$$R_{(\mu\nu)} = \kappa\Big\{\left(c_1\Phi + c_2\,\Phi^2 + c_3\,\Phi_{\rho\sigma}\,\Phi^{\rho\sigma} + \right. \\ \left. + c_4\,\omega_{\rho\sigma}\,\omega^{\rho\sigma}\right) g_{\mu\nu} + (c_5+c_6)\,\Phi_{\mu\nu} + c_7\Phi\,\Phi_{\mu\nu} + \\ + \frac{1}{2}\,(c_9+c_{11})\left(D^\rho{}_\mu\,\Phi_{\nu\rho} + D^\rho{}_\nu\,\Phi_{\mu\rho}\right) + \\ + \frac{1}{2}\,(c_{10}+c_{12})\left(D^\rho{}_\mu\,\omega_{\nu\rho} + D^\rho{}_\nu\,\omega_{\mu\rho}\right)\Big\},$$

$$R_{[\mu\nu]} = \kappa\Big\{(c_5-c_6)\,\omega_{\mu\nu} + c_8\,\Phi\,\omega_{\mu\nu} + \\ + \frac{1}{2}\,(c_9+c_{11})\left(D^\rho{}_\mu\,\Phi_{\nu\rho} - D^\rho{}_\nu\,\Phi_{\mu\rho}\right) + \\ + \frac{1}{2}\,(c_{10}+c_{11})\left(D^\rho{}_\mu\,\omega_{\nu\rho} - D^\rho{}_\nu\,\omega_{\mu\rho}\right)\Big\}.$$

The generalized Ricci curvature scalar is then

$$R = \kappa\left(h_1\,\Phi + h_2\,\Phi^2 + h_3\,\Phi_{\mu\nu}\,\Phi^{\mu\nu} + h_4\,\omega_{\mu\nu}\,\omega^{\mu\nu}\right)$$

where

$$h_1 = 4c_1 + c_5 + c_6\,,$$
$$h_2 = 4c_2 + c_5\,,$$
$$h_3 = 4c_3 + c_9 + c_{11}\,,$$
$$h_4 = 4c_4 + c_{10} + c_{12}\,.$$

Finally, we obtain, for the curvature tensor, the following expression:

$$R_{\mu\nu\rho\sigma} = W_{\mu\nu\rho\sigma} + \\ + \left(f_1\,\Phi + f_2\,\Phi^2 + f_3\,\Phi_{\lambda\eta}\,\Phi^{\lambda\eta} + f_4\,\omega_{\lambda\eta}\,\omega^{\lambda\eta}\right) \times \\ \times (g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}) + (\bar\beta + f_5\Phi)\left(g_{\mu\rho}\Phi_{\nu\sigma} + g_{\nu\sigma}\Phi_{\mu\rho} - \right. \\ \left. - g_{\mu\sigma}\Phi_{\nu\rho} - g_{\nu\rho}\Phi_{\mu\sigma}\right) + (\bar\beta + f_6\Phi)\left(g_{\mu\rho}\omega_{\nu\sigma} + g_{\nu\sigma}\omega_{\mu\rho} - \right. \\ \left. - g_{\mu\sigma}\omega_{\nu\rho} - g_{\nu\rho}\omega_{\mu\sigma}\right) + \bar\gamma\left(g_{\mu\rho}D_{\sigma\nu} + g_{\nu\sigma}D_{\rho\mu} - \right. \\ \left. - g_{\mu\sigma}D_{\rho\nu} - g_{\nu\rho}D_{\sigma\mu}\right) + f_7\left(D^\lambda{}_\nu\Phi_{\sigma\lambda}\,g_{\mu\rho} + \right. \\ \left. + D^\lambda{}_\mu\Phi_{\rho\lambda}\,g_{\nu\sigma} - D^\lambda{}_\nu\Phi_{\rho\lambda}\,g_{\mu\sigma} - D^\lambda{}_\mu\Phi_{\sigma\lambda}\,g_{\nu\rho}\right) + \\ + f_8\left(D^\lambda{}_\nu\,\omega_{\sigma\lambda}\,g_{\mu\rho} + D^\lambda{}_\mu\,\omega_{\rho\lambda}\,g_{\nu\sigma} - D^\lambda{}_\nu\,\omega_{\rho\lambda}\,g_{\mu\sigma} - \right. \\ \left. - D^\lambda{}_\mu\,\omega_{\sigma\lambda}\,g_{\nu\rho}\right) + f_9\left(D^\lambda{}_\sigma\,\Phi_{\nu\lambda}\,g_{\mu\rho} + D^\lambda{}_\rho\,\Phi_{\mu\lambda}\,g_{\nu\sigma} - \right. \\ \left. - D^\lambda{}_\rho\,\Phi_{\nu\lambda}\,g_{\mu\sigma} - D^\lambda{}_\sigma\,\Phi_{\mu\lambda}g_{\nu\rho}\right) + f_{10}\left(D^\lambda{}_\sigma\,\omega_{\nu\lambda}\,g_{\mu\rho} + \right. \\ \left. + D^\lambda{}_\rho\,\omega_{\mu\lambda}\,g_{\nu\sigma} - D^\lambda{}_\rho\,\omega_{\nu\lambda}\,g_{\mu\sigma} - D^\lambda{}_\sigma\,\omega_{\mu\lambda}\,g_{\nu\rho}\right)$$

where

$$f_1 = c_1 = \bar\alpha + \bar\lambda\,, \qquad f_6 = c_8\,,$$
$$f_2 = \left(1 - \frac{2}{3}\,\kappa\right) c_2 + \frac{1}{6}\,\kappa\,c_7\,, \qquad f_7 = c_9\,,$$
$$f_3 = \left(1 - \frac{2}{3}\,\kappa\right) c_3 + \frac{1}{6}\,\kappa\,(c_9+c_{11})\,, \qquad f_8 = c_{10}\,,$$
$$f_4 = \left(1 - \frac{2}{3}\,\kappa\right) c_4 + \frac{1}{6}\,\kappa\,(c_{10}-c_{12})\,, \qquad f_9 = c_{11}\,,$$
$$f_5 = c_7\,, \qquad f_{10} = c_{12}\,.$$

At this point, the apparent main difficulty lies in the fact that there are too many constitutive invariants that need to be exactly determined. As such, the linear theory is comparatively preferable since it only contains three constitutive invariants. However, by presenting the most general structure of the non-linear continuum theory in this section, we have acquired a quite general picture of the most general behavior of the space-time continuum in the presence of the classical fields.

5 The equations of motion

Let us now investigate the local translational-rotational motion of points in the space-time continuum $\mathbb{S}^4$. Consider an infinitesimal displacement in the manner described in the preceding section. Keeping the initial position fixed, the unit velocity vector is given by

$$u^\mu = \frac{d\xi^\mu}{ds} = \frac{dx^\mu}{ds}\,,$$
$$1 = g_{\mu\nu} u^\mu u^\nu\,,$$

such that, at any proper time given by the world-line s, the parametric representation

$$d\xi^\mu = u^\mu\left(x^\alpha, s\right) ds$$

describes space-time curves whose tangents are everywhere directed along the direction of a particle's motion. As usual, the world-line can be parametrized by a scalar ς via $s = a\varsigma + b$, where a and b are constants of motion.

The local equations of motion along arbitrary curves in the space-time continuum $\mathbb{S}^4$ can be described by the quadruplet of unit space-time vectors (u, v, w, z) orthogonal to each other where the first three unit vectors, or the triplet (u, v, w), may be defined as (a set of) local tangent vectors in the (three-dimensional) hypersurface $\Sigma\,(t)$ such that the unit vector z is normal to it. More explicitly, the hypersurface $\Sigma\,(t)$ is given as the time section $t = x^0 = \text{const}$ of $\mathbb{S}^4$. This way, the equations of motion will be derived by generalizing the ordinary Frenet equations of orientable points along an arbitrary curve in three-dimensional Euclidean space, i.e., by recasting them in a four-dimensional manner. Of course, we will also include effects of microspin generated by the twist of space-time.

With respect to the anholonomic space-time basis $\omega_\mu = \omega_\mu\big(x^\alpha(X^k)\big) = e^i_\mu \frac{\partial}{\partial X^i}$, we can write

$$u = u^\mu\,\omega_\mu\,,$$
$$v = v^\mu\,\omega_\mu\,,$$
$$w = w^\mu\,\omega_\mu\,,$$
$$z = z^\mu\,\omega_\mu\,,$$

we obtain, in general, the following set of equations of motion of points, i.e., point-like particles, along an arbitrary curve ℓ in the space-time continuum $\mathbb{S}^4$:

$$\frac{Du^\mu}{Ds} = \phi\, v^\mu\,,$$
$$\frac{Dv^\mu}{Ds} = \tau\, w^\mu - \phi\, u^\mu\,,$$
$$\frac{Dw^\mu}{Ds} = \tau\, v^\mu + \varphi\, z^\mu\,,$$
$$\frac{Dz^\mu}{Ds} = \varphi w^\mu\,,$$

where the operator $\frac{D}{Ds} = u^\mu \nabla_\mu$ represents the absolute covariant derivative. In the above equations we have introduced the following invariants:

$$\phi = \left(g_{\mu\nu}\frac{Du^\mu}{Ds}\frac{Du^\nu}{Ds}\right)^{1/2},$$
$$\tau = \epsilon_{\mu\nu\rho\sigma}\, u^\mu v^\nu \frac{Dv^\rho}{Ds}\, z^\sigma,$$
$$\varphi = \left(g_{\mu\nu}\frac{Dz^\mu}{Ds}\frac{Dz^\nu}{Ds}\right)^{1/2}.$$

In particular, we note that, the twist scalar τ measures the twist of the curve ℓ in $\mathbb{S}^4$ due to microspin.

At this point, we see that our equations of motion describe a "minimal" geodesic motion (with intrinsic spin) when $\phi = 0$. In other words, if

$$\frac{Du^\mu}{Ds} = 0\,,$$
$$\frac{Dv^\mu}{Ds} = \tau\, w^\mu,$$
$$\frac{Dw^\mu}{Ds} = \tau\, v^\mu + \varphi z^\mu,$$
$$\frac{Dz^\mu}{Ds} = \varphi w^\mu.$$

However, in general, any material motion in $\mathbb{S}^4$ will not follow the condition $\phi = 0$. This is true especially for the motion of a physical object with structure. In general, any physical object can be regarded as a collection of points (with different orientations) obeying our general equations of motion. It is therefore clear that $\phi \neq 0$ for a moving finite physical object (with structure) whose material points cannot be homogeneously oriented.

Furthermore, it can be shown that the gradient of the unit velocity vector can be decomposed according to

$$\nabla_\nu\, u_\mu = \alpha_{\mu\nu} + \beta_{\mu\nu} + \frac{1}{6}\, h_{\mu\nu}\,\bar{\theta} + u_\nu a_\mu$$

where

$$h_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu\,,$$

$$\alpha_{\mu\nu} = \frac{1}{4}\, h^\alpha_\mu h^\beta_\nu \left(\nabla_\alpha\, u_\beta + \nabla_\beta\, u_\alpha\right) = \frac{1}{4}\, h^\alpha_\mu h^\beta_\nu \left(\hat{\nabla}_\alpha\, u_\beta + \hat{\nabla}_\beta\, u_\alpha\right) - \frac{1}{2}\, h^\alpha_\mu h^\beta_\nu K^\sigma_{(\alpha\beta)} u_\sigma\,,$$

$$\beta_{\mu\nu} = \frac{1}{4}\, h^\alpha_\mu h^\beta_\nu \left(\nabla_\alpha\, u_\beta - \nabla_\beta\, u_\alpha\right) = \frac{1}{4}\, h^\alpha_\mu h^\beta_\nu \left(\hat{\nabla}_\alpha\, u_\beta - \hat{\nabla}_\beta\, u_\alpha\right) - \frac{1}{2} h^\alpha_\mu h^\beta_\nu K^\sigma_{[\alpha\beta]} u_\sigma\,,$$

$$\bar{\theta} = \nabla_\mu\, u^\mu,$$

$$a_\mu = \frac{Du_\mu}{Ds}\,.$$

Note that

$$h_{\mu\nu}\,u^{\nu} = \alpha_{\mu\nu}\,u^{\nu} = \beta_{\mu\nu}\,u^{\nu} = 0\,,$$

$$K^{\sigma}_{(\alpha\beta)} = -g^{\sigma\lambda}\left(g_{\alpha\eta}\,\Gamma^{\eta}_{[\lambda\beta]} + g_{\beta\eta}\,\Gamma^{\eta}_{[\lambda\alpha]}\right),$$

$$K^{\sigma}_{[\alpha\beta]} = \Gamma^{\sigma}_{[\alpha\beta]}\,.$$

Meanwhile, with the help of the identities

$$u^{\lambda}\nabla_{\nu}\nabla_{\lambda}\,u_{\mu} = \nabla_{\nu}\left(u^{\lambda}\nabla_{\lambda}\,u_{\mu}\right) - \left(\nabla_{\nu}\,u_{\lambda}\right)\left(\nabla^{\lambda}u_{\mu}\right) = \nabla_{\nu}\,a_{\mu} - \left(\nabla_{\nu}\,u_{\lambda}\right)\left(\nabla^{\lambda}u_{\mu}\right),$$

$$u^{\lambda}\left(\nabla_{\nu}\nabla_{\lambda} - \nabla_{\lambda}\nabla_{\nu}\right)u_{\mu} = R^{\sigma}{}_{\mu\lambda\nu}\,u_{\sigma}u^{\lambda} - 2\Gamma^{\sigma}_{[\lambda\nu]}u^{\lambda}\nabla_{\sigma}u_{\mu}\,,$$

we obtain

$$\frac{D\bar{\theta}}{Ds} = \nabla_{\mu}a^{\mu} - (\nabla_{\mu}u^{\nu})(\nabla_{\nu}u^{\mu}) - R_{\mu\nu}u^{\mu}u^{\nu} + 2\Gamma^{\sigma}_{[\mu\nu]}u^{\mu}\nabla_{\sigma}u^{\nu}$$

for the "rate of shear" of a moving material object with respect to the world-line.

6 The variational principle for the theory

Let us now derive the field equations of the present theory by means of the variational principle. Considering thermodynamic effects, in general, our theory can best be described by the following Lagrangian density:

$$\bar{L} = \bar{L}_1 + \bar{L}_2 + \bar{L}_3$$

where

$$\bar{L}_1 = \frac{1}{\kappa}\sqrt{\det(g)}\times\left(R^{\mu\nu}\left(\nabla_{\nu}\,\xi_{\mu} - D_{\mu\nu}\right) - \frac{1}{2}\left(\Phi - D^{\mu}{}_{\mu}\right)R\right),$$

$$\bar{L}_2 = \sqrt{\det(g)}\left(\frac{1}{2}\,C^{\mu\nu}{}_{\rho\sigma}\,D_{\mu\nu}D^{\rho\sigma} + \frac{1}{3}\,K^{\mu\nu}{}_{\rho\sigma\lambda\eta}\,D_{\mu\nu}D^{\rho\sigma}D^{\lambda\eta} - \Theta D^{\mu}{}_{\mu}\Delta T\right),$$

$$\bar{L}_3 = \sqrt{\det(g)}\;u^{\mu}\left(\nabla_{\mu}\,\xi_{\nu}\right)\left(f\xi^{\nu} - \rho u^{\nu}\right),$$

where Θ is a thermal coefficient, ΔT is (the change in) the temperature, and f is a generally varying scalar entity. Note that here we have only explicitly assumed that $\Phi = \nabla_{\mu}\,\xi^{\mu}$.

Alternatively, we can express $\bar{L}$ as follows:

$$\bar{L}_1 = \frac{1}{\kappa}\sqrt{\det(g)}\left(R^{\mu\nu} - \frac{1}{2}\,g^{\mu\nu}R\right)\left(\nabla_{\nu}\,\xi_{\mu} - D_{\mu\nu}\right).$$

Hence we have

$$\bar{L} = \sqrt{\det(g)}\Big\{T^{\mu\nu}\left(\nabla_{\nu}\,\xi_{\mu} - D_{\mu\nu}\right) + \frac{1}{2}\,C^{\mu\nu}{}_{\rho\sigma}\,D_{\mu\nu}D^{\rho\sigma} + \frac{1}{3}\,K^{\mu\nu}{}_{\rho\sigma\lambda\eta}\,D_{\mu\nu}D^{\rho\sigma}D^{\lambda\eta} - \Theta D^{\mu}{}_{\mu}\Delta T + u^{\mu}\left(\nabla_{\mu}\,\xi_{\nu}\right)\left(f\xi^{\nu} - \rho u^{\nu}\right)\Big\}.$$

We then arrive at the following invariant integral:

$$I = \int_{\mathbb{S}^4}\Big\{T^{\mu\nu}\left(\nabla_{(\nu}\,\xi_{\mu)} - \Phi_{\mu\nu}\right) + T^{\mu\nu}\left(\nabla_{(\nu}\,\xi_{\mu)} - \omega_{\mu\nu}\right) + \frac{1}{2}\,A^{\mu\nu}{}_{\rho\sigma}\,\Phi_{\mu\nu}\Phi^{\rho\sigma} + \frac{1}{2}\,B^{\mu\nu}{}_{\rho\sigma}\,\omega_{\mu\nu}\,\omega^{\rho\sigma} + \frac{1}{3}\,P^{\mu\nu}{}_{\rho\sigma\lambda\eta}\,\Phi_{\mu\nu}\,\Phi^{\rho\sigma}\,\Phi^{\lambda\eta} + \frac{1}{3}\,Q^{\mu\nu}{}_{\rho\sigma\lambda\eta}\,\omega_{\mu\nu}\,\omega^{\rho\sigma}\,\omega^{\lambda\eta} - \Theta D^{\mu}{}_{\mu}\Delta T + u^{\mu}\left(\nabla_{\mu}\,\xi_{\nu}\right)\left(f\,\xi^{\nu} - \rho u^{\nu}\right)\Big\}\,d\Sigma$$

where $d\Sigma = \sqrt{\det(g)}\;dx^0dx^1dx^2dx^3$ is the proper four-dimensional differential volume.

Writing $\bar{L} = \sqrt{\det(g)}\,L$ and employing the variational principle, we then have

$$\delta I = \int_{\mathbb{S}^4}\Big\{\frac{\partial L}{\partial T^{\mu\nu}}\,\delta T^{\mu\nu} + \frac{\partial L}{\partial \Phi^{\mu\nu}}\,\delta\Phi^{\mu\nu} + \frac{\partial L}{\partial \omega^{\mu\nu}}\,\delta\omega^{\mu\nu} + \frac{\partial L}{\partial\left(\nabla_{\mu}\,\xi_{\nu}\right)}\,\delta\left(\nabla_{\mu}\,\xi_{\nu}\right)\Big\}\,d\Sigma = 0\,.$$

Now

$$\int_{\mathbb{S}^4}\frac{\partial L}{\partial\left(\nabla_{\mu}\,\xi_{\nu}\right)}\,\delta\left(\nabla_{\mu}\,\xi_{\nu}\right)d\Sigma = \int_{\mathbb{S}^4}\nabla_{\mu}\left(\frac{\partial L}{\partial\left(\nabla_{\mu}\,\xi_{\nu}\right)}\,\delta\xi_{\nu}\right)d\Sigma - \int_{\mathbb{S}^4}\nabla_{\mu}\left(\frac{\partial L}{\partial\left(\nabla_{\mu}\,\xi_{\nu}\right)}\right)\delta\xi_{\nu}\,d\Sigma = -\int_{\mathbb{S}^4}\nabla_{\mu}\left(\frac{\partial L}{\partial\left(\nabla_{\mu}\,\xi_{\nu}\right)}\right)\delta\xi_{\nu}\,d\Sigma$$

since the first term on the right-hand-side of the first line is an absolute differential that can be transformed away on the boundary of integration by means of the divergence theorem. Hence we have

$$\delta I = \int_{\mathbb{S}^4}\Big\{\frac{\partial L}{\partial T^{\mu\nu}}\,\delta T^{\mu\nu} + \frac{\partial L}{\partial \Phi^{\mu\nu}}\,\delta\Phi^{\mu\nu} + \frac{\partial L}{\partial \omega^{\mu\nu}}\,\delta\omega^{\mu\nu} - \nabla_{\mu}\left(\frac{\partial L}{\partial\left(\nabla_{\mu}\,\xi_{\nu}\right)}\right)\delta\xi_{\nu}\Big\}\,d\Sigma = 0$$

where each term in the integrand is independent of the others. We may also note that the variations $\delta T^{\mu\nu}$, $\delta\Phi^{\mu\nu}$, $\delta\omega^{\mu\nu}$, and $\delta\xi_{\nu}$ are arbitrary.

From $\frac{\partial L}{\partial T^{\mu\nu}} = 0$, we obtain

$$\Phi_{\mu\nu} = \nabla_{(\nu}\,\xi_{\mu)}\,,$$

$$\omega_{\mu\nu} = \nabla_{[\nu}\,\xi_{\mu]}\,,$$

i.e., the covariant components of the "dilation" and intrinsic spin tensors, respectively.

From $\frac{\partial L}{\partial \Phi^{\mu\nu}} = 0$, we obtain

$$T^{(\mu\nu)} = \frac{1}{\kappa}\left(R^{(\mu\nu)} - \frac{1}{2}\,g^{\mu\nu}R\right) = A^{\mu\nu}{}_{\rho\sigma}\,\Phi^{\rho\sigma} + P^{\mu\nu}{}_{\rho\sigma\lambda\eta}\,\Phi^{\rho\sigma}\Phi^{\lambda\eta} - \Theta g^{\mu\nu}\Delta T$$

i.e., the symmetric contravariant components of the energy-momentum tensor.

In other words,

$$T^{\mu\nu} = \frac{1}{\kappa}\left(R^{\mu\nu} - \frac{1}{2}\,g^{\mu\nu}R\right) =$$
$$= C^{\mu\nu}{}_{\rho\sigma}\,D^{\rho\sigma} + K^{\mu\nu}{}_{\rho\sigma\lambda\eta}\,D^{\rho\sigma}D^{\lambda\eta} - \Theta\,g^{\mu\nu}\Delta T\,.$$

Finally, we now show in detail that the fourth variation yields an important equation of motion. We first see that

$$\frac{\partial L}{\partial\left(\nabla_\mu\,\xi_\nu\right)} = T^{\mu\nu} + u^\mu\left(f\xi^\nu - \rho\,u^\nu\right).$$

Hence

$$\nabla_\mu\left(\frac{\partial L}{\partial\left(\nabla_\mu\,\xi_\nu\right)}\right) = \nabla_\mu\,T^{\mu\nu} + \nabla_\mu\left(f\,u^\mu\right)\xi^\nu +$$
$$+ f\,u^\mu\,\nabla_\mu\,\xi^\nu - \nabla_\mu\left(\rho u^\mu\right)u^\nu - \rho u^\mu\nabla_\mu\,u^\nu.$$

Let us define the "extended" shear scalar and the mass current density vector, respectively, via

$$l = \nabla_\mu\left(fu^\mu\right),$$

$$J^\mu = \rho\,u^\mu.$$

We can now readily identify the acceleration vector and the body force per unit mass, respectively, by

$$a^\mu = u^\nu\,\nabla_\nu\,a^\mu = \frac{Du^\mu}{Ds}\,,$$

$$b^\mu = \frac{1}{\rho}\left(l\,\xi^\mu + f\left(1 - \nabla_\nu\,J^\nu\right)u^\mu\right).$$

In the conservative case, the condition $\nabla_\mu\,J^\mu = 0$ gives

$$\frac{D\rho}{Ds} = -\,\rho\,\nabla_\mu\,u^\mu.$$

In the weak-field limit for which $u^\mu = \left(1, u^A\right)$ (where $A = 1, 2, 3$) we obtain the ordinary continuity equation,

$$\frac{\partial\rho}{\partial t} + \nabla_A\left(\rho\,u^A\right) = 0\,.$$

Finally, we have

$$\int\limits_{\mathbb{S}^4}\left(\nabla_\mu\,T^{\mu\nu} + \rho\,b^\nu - \rho\,a^\nu\right)\delta\xi_\nu d\Sigma = 0$$

i.e., the equation of motion

$$\nabla_\mu\,T^{\mu\nu} = \rho\,\left(a^\nu - b^\nu\right)$$

or

$$\nabla_\mu\left(R^{\mu\nu} - \frac{1}{2}\,g^{\mu\nu}R\right) = \kappa\,\rho\left(a^\nu - b^\nu\right).$$

If we restrict our attention to point-like particles, the body force vanishes since it cannot act on a structureless (zero-dimensional) object. And since the motion is geodesic, i.e., $a^\mu = 0$, we have the conservation law

$$\nabla_\mu\,T^{\mu\nu} = 0\,.$$

In this case, this conservation law is true regardless of whether the energy-momentum tensor is symmetric or not.

Let us now discuss the so-called couple stress, i.e., the couple per unit area which is also known as the distributed moment. We denote the couple stress tensor by the second-rank tensor field M. In analogy to the linear constitutive relations relating the energy-momentum tensor to the displacement gradient tensor, we write

$$M^{\mu\nu} = J^{\mu\nu}{}_{\rho\sigma}\,L^{\rho\sigma} + H^{\mu\nu}{}_{\rho\sigma\lambda\eta}\,L^{\rho\sigma}L^{\lambda\eta}$$

where

$$J_{\mu\nu\rho\sigma} = E_{\mu\nu\rho\sigma} + F_{\mu\nu\rho\sigma}\,,$$

$$H_{\mu\nu\rho\sigma\lambda\eta} = U_{\mu\nu\rho\sigma\lambda\eta} + V_{\mu\nu\rho\sigma\lambda\eta}\,.$$

These are assumed to possess the same symmetry properties as $C_{\mu\nu\rho\sigma}$ and $K_{\mu\nu\rho\sigma\lambda\eta}$, respectively, i.e., $E_{\mu\nu\rho\sigma}$ have the same symmetry properties as $A_{\mu\nu\rho\sigma}$, $F_{\mu\nu\rho\sigma}$ have the same symmetry properties as $B_{\mu\nu\rho\sigma}$, $U_{\mu\nu\rho\sigma\lambda\eta}$ have the same symmetry properties as $P_{\mu\nu\rho\sigma\lambda\eta}$, and $V_{\mu\nu\rho\sigma\lambda\eta}$ have the same symmetry properties as $Q_{\mu\nu\rho\sigma\lambda\eta}$.

Likewise, the asymmetric tensor given by

$$L_{\mu\nu} = L_{(\mu\nu)} + L_{[\mu\nu]}$$

is comparable to the displacement gradient tensor.

Introducing a new infinitesimal spin potential via ϕ_μ, let the covariant dual form of the intrinsic spin tensor be given by

$$\bar{\omega}_{\mu\nu} = \frac{1}{2}\,\in_{\mu\nu\rho\sigma}\,\omega^{\rho\sigma} = \frac{1}{2}\left(\nabla_\nu\,\phi_\mu - \nabla_\mu\,\phi_\nu\right).$$

Let us now introduce a completely anti-symmetric third-rank spin tensor via

$$S^{\mu\nu\rho} = -\frac{1}{2}\left(\beta - \gamma\right)\in^{\mu\nu\rho\sigma}\,\phi_\sigma\,.$$

As a direct consequence, we see that

$$\nabla_\rho\,S^{\mu\nu\rho} = \left(\beta - \gamma\right)\omega^{\mu\nu}$$

In other words,

$$\nabla_\rho\,S^{\mu\nu\rho} = T^{[\mu\nu]} - N^{\mu\nu} = \frac{1}{\kappa}\left(R^{[\mu\nu]} - \Lambda^{\mu\nu}\right)$$

where

$$N^{\mu\nu} = 2\left(b_4 + b_6\right)\Phi\,\omega^{\mu\nu} + \left(b_8 + b_{10}\right)\times$$
$$\times\left(D^{\mu\rho}\Phi^\nu{}_\rho - D^{\nu\rho}\Phi^\mu{}_\rho\right) + \left(b_9 + b_9\right)\left(D^{\mu\rho}\omega^\nu{}_\rho - D^{\nu\rho}\omega^\mu{}_\rho\right),$$

$$\Lambda^{\mu\nu} = c_8\Phi\,\omega^{\mu\nu} + \frac{1}{2}\left(c_9 + c_{11}\right)\left(D^{\mu\rho}\Phi^\nu{}_\rho - D^{\nu\rho}\Phi^\mu{}_\rho\right) +$$
$$+\frac{1}{2}\left(c_{10} + c_{11}\right)\left(D^{\mu\rho}\omega^\nu{}_\rho - D^{\nu\rho}\omega^\mu{}_\rho\right).$$

We can now form the second Lagrangian density of our theory as

$$\bar{H} = \sqrt{\det(g)}\Big\{ M^{\mu\nu}\left(\nabla_\nu \phi_\mu - L_{\mu\nu}\right) + \frac{1}{2}\, J^{\mu\nu}{}_{\rho\sigma} L_{\mu\nu} L^{\rho\sigma} + \\ + \frac{1}{3}\, H^{\mu\nu}{}_{\rho\sigma\lambda\eta} L_{\mu\nu} L^{\rho\sigma} L^{\lambda\eta} - \in^{\mu}{}_{\rho\sigma\lambda}\left(\nabla_\nu \phi_\mu\right) S^{\rho\sigma\nu} u^\lambda + \\ + u^\mu \left(\nabla_\mu \phi_\nu\right)\left(h\,\phi^\nu - I\rho\, s^\nu\right)\Big\}$$

where h is a scalar function, I is the moment of inertia, and s^ν are the components of the angular velocity vector.

Letting $L_{(\mu\nu)} = X_{\mu\nu}$ and $L_{[\mu\nu]} = Z_{\mu\nu}$, the corresponding action integral is

$$J = \int_{\mathbb{S}^4} \Big\{ M^{\mu\nu}\left(\nabla_{(\nu}\,\phi_{\mu)} - X_{\mu\nu}\right) + M^{\mu\nu}\left(\nabla_{[\nu}\,\phi_{\mu]} - Z_{\mu\nu}\right) + \\ + \frac{1}{2}\, E^{\mu\nu}{}_{\rho\sigma} X_{\mu\nu} X^{\rho\sigma} + \frac{1}{2}\, F^{\mu\nu}{}_{\rho\sigma} Z_{\mu\nu} Z^{\rho\sigma} + \\ + \frac{1}{3}\, U^{\mu\nu}{}_{\rho\sigma\lambda\eta} X_{\mu\nu} X^{\rho\sigma} X^{\lambda\eta} + \frac{1}{3}\, V^{\mu\nu}{}_{\rho\sigma\lambda\eta} Z_{\mu\nu} Z^{\rho\sigma} Z^{\lambda\eta} - \\ - \in^{\mu}{}_{\rho\sigma\lambda}\left(\nabla_\nu \phi_\mu\right) S^{\rho\sigma\nu} u^\lambda + u^\mu\left(\nabla_\mu \phi_\nu\right)\left(h\,\phi^\nu - I\rho\, s^\nu\right)\Big\}\, d\Sigma\,.$$

As before, writing $\bar{H} = \sqrt{\det(g)}H$ and performing the variation $\delta J = 0$, we have

$$\delta J = \int_{\mathbb{S}^4} \Big\{ \frac{\partial H}{\partial M^{\mu\nu}}\, \delta M^{\mu\nu} + \frac{\partial H}{\partial X^{\mu\nu}}\, \delta X^{\mu\nu} + \\ + \frac{\partial H}{\partial Z^{\mu\nu}}\, \delta Z^{\mu\nu} - \nabla_\mu \left(\frac{\partial H}{\partial\left(\nabla_\mu \phi_\nu\right)}\right)\delta\phi_\nu \Big\} d\Sigma = 0$$

with arbitrary variations $\delta M^{\mu\nu}$, $\delta X^{\mu\nu}$, $\delta Z^{\mu\nu}$, and $\delta\phi_\nu$.

From $\frac{\partial H}{\partial M^{\mu\nu}} = 0$, we obtain

$$X_{\mu\nu} = \nabla_{(\nu}\,\phi_{\mu)}\,,$$

$$Z_{\mu\nu} = \nabla_{[\nu}\,\phi_{\mu]}\,.$$

From $\frac{\partial H}{\partial X^{\mu\nu}} = 0$, we obtain

$$M^{(\mu\nu)} = E^{\mu\nu}{}_{\rho\sigma} X^{\rho\sigma} + U^{\mu\nu}{}_{\rho\sigma\lambda\eta} X^{\rho\sigma} X^{\lambda\eta}\,.$$

From $\frac{\partial H}{\partial Z^{\mu\nu}} = 0$, we obtain

$$M^{[\mu\nu]} = F^{\mu\nu}{}_{\rho\sigma} Z^{\rho\sigma} + V^{\mu\nu}{}_{\rho\sigma\lambda\eta} Z^{\rho\sigma} Z^{\lambda\eta}\,.$$

We therefore have the constitutive relation

$$M^{\mu\nu} = J^{\mu\nu}{}_{\rho\sigma} L^{\rho\sigma} + H^{\mu\nu}{}_{\rho\sigma\lambda\eta} L^{\rho\sigma} L^{\lambda\eta}\,.$$

Let us investigate the last variation

$$-\int_{\mathbb{S}^4} \nabla_\mu \left(\frac{\partial H}{\partial\left(\nabla_\mu \phi_\nu\right)}\right)\delta\phi_\nu\, d\Sigma = 0$$

in necessary detail.

Firstly,

$$\frac{\partial H}{\partial\left(\nabla_\mu \phi_\nu\right)} = M^{\mu\nu} - \in^{\nu}{}_{\lambda\rho\sigma}\, S^{\lambda\rho\mu} u^\sigma + u^\mu\left(h\,\phi^\nu - I\rho s^\nu\right).$$

Then we see that

$$\nabla_\mu \left(\frac{\partial H}{\partial\left(\nabla_\mu \phi_\nu\right)}\right) = \nabla_\mu M^{\mu\nu} - \\ - \in^{\nu}{}_{\mu\rho\sigma}\, T^{[\mu\rho]} u^\sigma - \in^{\nu}{}_{\lambda\rho\sigma}\, S^{\lambda\rho\mu} \nabla_\mu u^\sigma + \nabla_\mu\left(h u^\mu\right)\phi^\nu + \\ + h u^\mu \nabla_\mu \phi^\nu - I\,\nabla_\mu\left(\rho u^\mu\right) s^\nu - I\rho u^\mu \nabla_\mu s^\nu\,.$$

We now define the angular acceleration by

$$\alpha^\mu = u^\nu\, \nabla_\nu\, s^\mu = \frac{Ds^\mu}{Ds}$$

and the angular body force per unit mass by

$$\beta^\mu = \frac{1}{\rho}\left(\bar{l}\,\phi^\mu + h\,\frac{D\phi^\mu}{Ds} - I\left(\nabla_\nu J^\nu\right) s^\mu\right)$$

where $\bar{l} = \nabla_\mu\left(h u^\mu\right)$.

We have

$$\int_{\mathbb{S}^4} \Big\{ \nabla_\mu M^{\mu\nu} - \in^{\nu}_{\mu\rho\sigma}\left(T^{[\mu\rho]} u^\sigma + S^{\mu\rho\lambda} \nabla_\lambda u^\sigma\right) + \\ + \rho\beta^\nu - I\rho\,\alpha^\nu \Big\}\, \delta\phi_\nu\, d\Sigma = 0\,.$$

Hence we obtain the equation of motion

$$\nabla_\mu M^{\mu\nu} = \in^{\nu}{}_{\mu\rho\sigma} \times \\ \times \Big\{ \left(T^{[\mu\rho]} - N^{\mu\rho}\right) u^\sigma + S^{\mu\rho\lambda} \nabla_\lambda u^\sigma \Big\} + \rho\left(I\alpha^\nu - \beta^\nu\right)$$

i.e.,

$$\nabla_\mu M^{\mu\nu} = \in^{\nu}{}_{\mu\rho\sigma} \times \\ \times \Big\{ \frac{1}{\kappa}\left(R^{[\mu\rho]} - \Lambda^{\mu\rho}\right) u^\sigma + S^{\mu\rho\lambda} \nabla_\lambda u^\sigma \Big\} + \rho\left(I\alpha^\nu - \beta^\nu\right).$$

7 Final remarks

We have seen that the classical fields of physics can be unified in a simple manner by treating space-time itself as a four-dimensional finite (but unbounded) elastic medium capable of undergoing extensions (dilations) and internal point-rotations in the presence of material-energy fields. In the present framework, the classical physical fields indeed appear on an equal footing as they are of purely geometric character. In addition, we must note that this apparent simplicity still leaves the constitutive invariants undetermined. At the moment, we leave this aspect of the theory to more specialized attempts. However, it can be said, in general, that we expect the constitutive invariants of the theory to be functions of the known physical properties of matter such as material density, energy density, compressibility, material symmetry, etc. This way, we have successfully built a significant theoretical framework that holds in all classical physical situations.

We would also like to remark that once the constitutive invariants are determined and incorporated into the possible equations of state, the fully non-linear formulation of the present theory should be very satisfactory for describing the dynamics of astrophysical objects especially various fluids which exhibit the characteristics of non-degenerate relativistic and non-Newtonian fluids.

We have seen that the general dynamical behavior of a material body as determined by the equations of motion given in Section 5, is intrinsically related to the underlying geometry of the space-time continuum which in turn is largely determined by the constitutive relations given in Sections 3 and 4. In Section 6, we have also constructed a framework in which the motion of a point-like particle is always subject to the conservation law of matter and energy regardless of the particle's intrinsic spin.

We also note that a material body in our continuum representation of space-time can be regarded as the three-dimensional boundary of a so-called world-tube such that outside the world-tube the region is said to be free or empty. This three-dimensional boundary can be represented by a time-like hypersurface. Such hypersurfaces can be seen as disturbances in the space-time continuum. Furthermore, such disturbances are equivalent to three-dimensional representations of material waves (not necessarily gravitational waves). In this context, one may formulate the dynamic discontinuity conditions as purely geometric and kinematic compatibility conditions over the hypersurfaces.

In common with standard general relativity, a region of the space-time continuum is said to be statical if it can be covered by a space-time coordinate system relative to which the components of the metric tensor are independent of time. It may be that such a region can be covered by one or more such coordinate systems. As such, material waves are propagated into a fixed (three-dimensional) curved space along trajectories normal to the family of hypersurfaces given by the successive positions of a material body in the fixed space. In various cases, such trajectories can be represented as curves of zero length in the space-time continuum.

The microscopic substructure of the space-time continuum provides us room for additional degrees of freedom. In other words, there exist intrinsic length scales associated with these additional degrees of freedom. Correspondingly, one may define the so-called microrotational inertial field. In fact, the internal rotation of the points in the space-time continuum is seen as representing the intrinsic spin of elementary particles. On microscopic scales, the structure of the space-time continuum can indeed appear to be granular. Due to possible effects arising from this consideration, it is often not sufficient to model the space-time continuum itself as continuous, isotropic, and homogeneous. Furthermore, the rather predominant presence of twisting paths may give rise to particles exhibiting micropolar structure.

In geometrizing microspin phenomena, we emphasize that the initial microspin variables are not to be freely chosen to be included in the so-called elasticity scalar functional of the space-time continuum which is equivalent to a Lagrangian density. Rather, one must first identify them with the internal geometric properties of the space-time continuum. In other words, one must primarily unfold their underlying geometric existence in the space-time continuum itself. This is precisely what we have done in this work.

Finally, we note that geometric discontinuities can also be incorporated into the present theory. Such discontinuities can be seen as topological defects in the space-time continuum. Holographic four-dimensional continua with cellular, fibrous, or foamy structure may indeed represent admissible semi-classical models of the Universe which can be realized in the framework of the present theory. In such a case, the metric must therefore be quantized. It remains to be seen how this might correspond to any conventional quantum description of the space-time continuum.

Acknowledgements

I would like to sincerely thank D. Rabounski and S. J. Crothers for their kind assistance and the numerous discussions devoted to making this work appear in a somewhat more readable form.

Submitted on June 22, 2007
Accepted on July 04, 2007

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SPECIAL REPORT

A New Semi-Symmetric Unified Field Theory of the Classical Fields of Gravity and Electromagnetism

Indranu Suhendro

Department of Physics, Karlstad University, Karlstad 651 88, Sweden
E-mail: spherical_symmetry@yahoo.com

We attempt to present a classical theoretical framework in which the gravitational and electromagnetic fields are unified as intrinsic geometric objects in the space-time manifold. For this purpose, we first present the preliminary geometric considerations dealing with the metric differential geometry of Cartan connections. The unified field theory is then developed as an extension of the general theory of relativity based on a semi-symmetric Cartan connection which is meant to be as close as possible structurally to the symmetric connection of the Einstein-Riemann space-time.

1 Introduction

It is now well-known that there are various paths available, provided by geometry alone, to a unified description of physical phenomena. The different possibilities for the interpretation of the underlying nature and fabric of the Universe in a purely geometric fashion imply that there is a deep underlying structural reason for singular harmony that lies in the depths of Nature's unity. It appears that the Universe is a self-descriptive continuum which connects what seem to be purely intrinsic mathematical objects to physical observables. It is the belief that analytical geometry alone is able to provide the profoundest description of the complexity and harmony of our structured Universe that has led generations of mathematicians and physicists to undertake the task of geometrizing the apparently systematic laws of Nature. Indeed this is, as Einstein once described, the effect of the sense of universal causation on the inquisitive mind.

The above-mentioned wealth of the inherent mathematical possibility for the geometrization of physics has resulted in the myriad forms of unified field theory which have been proposed from time to time, roughly since 1918 when H. Weyl's applied his so-called purely infinitesimal geometry which was a relaxation of the geometry of Riemann spaces to the task of geometrizing the electromagnetic field in the hope to unify it with the already geometrized gravitational field of general relativity [6]. However, often for want of simplicity, this fact which basically gives us a vision of a solid, reified reality may also lead us to think that the Universe of phenomena must be ultimately describable in the somewhat simplest and yet perhaps most elegant mathematical (i.e., geometric) formalism. Furthermore, when one is exposed to the different forms of unified field theory, especially for the first time, I believe it is better for one to see a less complicated version, otherwise one might get overloaded mentally and it follows that there is a chance that such a thing will just prevent one from absorbing the essence of our desired simplicity which is intuitively expected to be present in any objective task of unification.

Given the freedom of choice, we do not attempt, in this work, to speak about which version of unified field theory out of many is true, rather we shall present what I believe should qualify among the logically simplest geometric descriptions of the classical fields of gravity and electromagnetism. Indeed, for the reason that we may not still be fully aware of the many hidden aspects of the Universe on the microscopic (quantum) scales, at present we shall restrict our attention to the unification and geometrization of the classical fields alone.

As we know, there are many types of differential geometry, from affine geometry to non-affine geometry, from metric (i.e., metric-compatible) geometry to non-metric geometry. However, the different systems of differential geometry that have been developed over hundreds of years can be most elegantly cast in the language of Cartan geometry. The geometric system I will use throughout this physical part of our work is a metric-compatible geometry endowed with a semi-symmetric Cartan connection. It therefore is a variant of the so-called Riemann-Cartan geometry presented in Sections 1.1-1.6. As we know, the standard form of general relativity adopts the symmetric, twist-free, metric-compatible Christoffel connection. We are also aware that the various extensions of standard general relativity [7] tend to employ more general connections that are often asymmetric (e.g., the Sciama-Kibble theory [8, 9]) and even non-metric in general (e.g., the Weyl theory [6]). However, in the present work, we shall insist on logical simplicity and on having meaningful physical consequences. Once again, we are in no way interested in pointing out which geometric system is most relevant to physics, rather we are simply concerned with describing in detail what appears to be among the most consistent and accurate views of the physical world. We only wish to construct a unified field theory on the common foundation of beauty, simplicity, and observational accuracy without having to deal with unnecessarily complex physical implications that might dull our perspective on the workings of Nature. I myself have always been fond of employing the most general type of connection for the purpose of unification. However, after years of poring over the almost universally held and (supposedly) objectively existing physical evidence, I have come to the conclusion that there is more reason to impose a simpler geometric formulation than a more general type of geometry such as non-metric geometry. In this work, it is my hope to dovetail the classical fields of gravity and electromagnetism with the conventional Riemann-Cartan geometry in general and with a newly constructed semi-symmetric Cartan connection in particular. Our resulting field equations are then just the distillation of this motive, which will eventually give us a penetrating and unified perspective on the nature of the classical fields of gravity and electromagnetism as *intrinsic* geometric fields, as well as on the possible interaction between the translational and rotational symmetries of the space-time manifold.

I believe that the semi-symmetric nature of the present theory (which keeps us as close as possible to the profound, observable physical implications of standard general relativity) is of great generality such that it can be applied to a large class of problems, especially problems related to the more general laws of motion for objects with structure.

2 A comprehensive evaluation of the differential geometry of Cartan connections with metric structure

The splendid, profound, and highly intuitive interpretation of differential geometry by E. Cartan, which was first applied to Riemann spaces, has resulted in a highly systematic description of a vast range of geometric and topological properties of differentiable manifolds. Although it possesses a somewhat abstract analytical foundation, to my knowledge there is no instance where Riemann-Cartan geometry, cast in the language of differential forms (i.e., exterior calculus), gives a description that is in conflict with the classical tensor analysis as formalized, e.g., by T. Levi-Civita. Given all its successes, one might expect that any physical theory, which relies on the concept of a field, can be elegantly built on its rigorous foundation. Therefore, as long as the reality of metric structure (i.e., metric compatibility) is assumed, it appears that a substantial modified geometry is not needed to supersede Riemann-Cartan geometry.

A common overriding theme in both mathematics and theoretical physics is that of unification. And as long as physics can be thought of as geometry, the geometric objects within Riemann-Cartan geometry (such as curvature for gravity and twist for intrinsic spin) certainly help us visualize and conceptualize the essence of unity in physics. Because of its intrinsic unity and its breadth of numerous successful applications, it might be possible for nearly all the laws governing physical phenomena to be combined and written down in compact form via the structural equations. By the intrinsic unity of Riemann-Cartan geometry, I simply refer to its tight interlock between algebra, analysis, group representation theory, and geometry. At least in mathematics alone, this is just as close as one can get to a "final" unified description of things. I believe that the unifying power of this beautiful piece of mathematics extends further still.

I'm afraid the title I have given to this first part of our work (which deals with the essential mathematics) has a somewhat narrow meaning, unlike the way it sounds. In writing this article, my primary goal has been to present Riemann-Cartan geometry in a somewhat simpler, more concise, and therefore more efficient form than others dealing with the same subject have done before [1, 4]. I have therefore had to drop whatever mathematical elements or representations that might seem somewhat highly counterintuitive at first. After all, not everyone, unless perhaps he or she is a mathematician, is familiar with abstract concepts from algebra, analysis, and topology, just to name a few. Nor is he or she expected to understand these things. But one thing remains essential, namely, one's ability to catch at least a glimpse of the beauty of the presented subject via deep, often simple, real understanding of its basics. As a non-mathematician (or simply a "dabbler" in pure mathematics), I do think that pure mathematics as a whole has grown extraordinarily "strange", if not complex (the weight of any complexity is really relative of course), with a myriad of seemingly separate branches, each of which might only be understood at a certain level of depth by the pure mathematicians specializing in that particular branch themselves. As such, a comparable complexity may also have occurred in the case of theoretical physics itself as it necessarily feeds on the latest formalism of the relevant mathematics each time. Whatever may be the case, the real catch is in the *essential understanding* of the basics. I believe simplicity alone will reveal it without necessarily having to diminish one's perspectives at the same time.

2.1 A brief elementary introduction to the Cartan (-Hausdorff) manifold $\mathbb{C}^\infty$

Let $\omega_a = \frac{\partial X^i}{\partial x^a} E_i = \partial_a X^i E_i$ (summation convention employed throughout this article) be the covariant (*frame*) basis spanning the n-dimensional base manifold $\mathbb{C}^\infty$ with local coordinates $x^a = x^a\left(X^k\right)$. The contravariant (*coframe*) basis θ^b is then given via the orthogonal projection $\left\langle\theta^b, \omega_a\right\rangle = \delta^b_a$, where δ^b_a are the components of the Kronecker delta (whose value is unity if the indices coincide or null otherwise). Now the set of linearly independent local directional derivatives $E_i = \frac{\partial}{\partial X^i} = \partial_i$ gives the coordinate basis of the locally flat tangent space $\mathbb{T}_x(M)$ at a point $x \in \mathbb{C}^\infty$. Here $\mathbb{M}$ denotes the topological space of the so-called n-tuples $h(x) = h\left(x^1, \ldots, x^n\right)$ such that relative to a given chart $\left(U, h(x)\right)$ on a neighborhood U of a local coordinate point, our $\mathbb{C}^\infty$-differentiable manifold itself is a topological space. The dual basis to E_i spanning the locally flat cotangent space $\mathbb{T}^*_x(\mathbb{M})$ will then be given by the differential elements dX^k via the relation $\langle dX^k, \partial_i \rangle = \delta^k_i$. In fact and in general, the *one-forms* dX^k indeed act as a linear map $\mathbb{T}_x(\mathbb{M}) \rightarrow \mathbb{R}$ when applied to an arbitrary vector field $F \in \mathbb{T}_x(\mathbb{M})$ of the explicit form $F = F^i \frac{\partial}{\partial X^i} = f^a \frac{\partial}{\partial x^a}$. Then it is easy to see that $F^i = F X^i$ and $f^a = F x^a$, from which we obtain the usual transformation laws for the contravariant components of a vector field, i.e., $F^i = \partial_a X^i f^a$ and $f^i = \partial_i x^a F^i$, relating the localized components of F to the general ones and vice versa. In addition, we also see that $\langle dX^k, F \rangle = F X^k = F^k$.

The components of the metric tensor $g = g_{ab}\theta^a \otimes \theta^b$ of the base manifold $\mathbb{C}^\infty$ are readily given by

$$g_{ab} = \langle \omega_a, \omega_b \rangle .$$

The components of the metric tensor $g(x_N) = \eta_{ik} dX^i \otimes dX^k$ describing the locally flat tangent space $\mathbb{T}_x(\mathbb{M})$ of rigid frames at a point $x_N = x_N(x^a)$ are given by

$$\eta_{ik} = \langle E_i, E_k \rangle = \text{diag}\,(\pm 1, \pm 1, \ldots, \pm 1) .$$

In four dimensions, the above may be taken to be the components of the Minkowski metric tensor, i.e., $\eta_{ik} = \langle E_i, E_k \rangle = \text{diag}\,(1, -1, -1, -1)$.

Then we have the expression

$$g_{ab} = \eta_{ik}\, \partial_a X^i\, \partial_b X^k$$

satisfying

$$g_{ac}\, g^{bc} = \delta^b_a$$

where $g^{ab} = \langle \theta^a, \theta^b \rangle$.

The manifold $\mathbb{C}^\infty$ is a metric space whose line-element in this formalism of a differentiable manifold is directly given by the metric tensor itself, i.e.,

$$ds^2 = g = g_{ab} \left(\partial_i x^a\, \partial_k x^b \right) dX^i \otimes dX^k ,$$

where the coframe basis is given by the one-forms $\theta^a = \partial_i x^a dX^i$.

2.2 Exterior calculus in n dimensions

As we know, an arbitrary tensor field $T \in \mathbb{C}^\infty$ of rank (p, q) is the object

$$T = T^{i_1 i_2 \ldots i_q}_{j_1 j_2 \ldots j_p} \omega_{i_1} \otimes \omega_{i_2} \otimes \ldots \otimes \omega_{i_q} \otimes \theta^{j_1} \otimes \theta^{j_2} \otimes \ldots \otimes \theta^{j_p} .$$

Given the existence of a local coordinate transformation via $x^i = x^i(\bar{x}^\alpha)$ in $\mathbb{C}^\infty$, the components of $T \in \mathbb{C}^\infty$ transform according to

$$T^{ij\ldots s}_{kl\ldots r} = T^{\alpha\beta\ldots\lambda}_{\mu\nu\ldots\eta} \partial_\alpha x^i \partial_\beta x^j \ldots \partial_\lambda x^s \partial_k \bar{x}^\mu \partial_l \bar{x}^\nu \ldots \partial_r \bar{x}^\eta .$$

Taking a local coordinate basis $\theta^i = dx^i$, a Pfaffian p-form ω is the completely anti-symmetric tensor field

$$\omega = \omega_{i_1 i_2 \ldots i_p} dx^{i_1} \wedge dx^{i_2} \wedge \ldots \wedge dx^{i_p} ,$$

where

$$dx^{i_1} \wedge dx^{i_2} \wedge \ldots \wedge dx^{i_p} \equiv \frac{1}{p!}\, \delta^{i_1 i_2 \ldots i_p}_{j_1 j_2 \ldots j_p} dx^{j_1} \otimes dx^{j_2} \otimes \ldots \otimes dx^{j_p} .$$

In the above, the $\delta^{i_1 i_2 \ldots i_p}_{j_1 j_2 \ldots j_p}$ are the components of the generalized Kronecker delta. They are given by

$$\delta^{i_1 i_2 \ldots i_p}_{j_1 j_2 \ldots j_p} = \in_{j_1 j_2 \ldots j_p} \in^{i_1 \ldots i_p} = \det \begin{pmatrix} \delta^{i_1}_{j_1} & \delta^{i_2}_{j_1} & \ldots & \delta^{i_p}_{j_1} \\ \delta^{i_1}_{j_2} & \delta^{i_2}_{j_2} & \ldots & \delta^{i_p}_{j_2} \\ \ldots & \ldots & \ldots & \ldots \\ \delta^{i_1}_{j_p} & \delta^{i_2}_{j_p} & \ldots & \delta^{i_p}_{j_p} \end{pmatrix}$$

where $\in_{j_1 j_2 \ldots j_p} = \sqrt{\det(g)}\, \epsilon_{j_1 j_2 \ldots j_p}$ and $\in^{i_1 i_2 \ldots i_p} = \frac{\epsilon^{i_1 i_2 \ldots i_p}}{\sqrt{\det(g)}}$ are the covariant and contravariant components of the completely anti-symmetric Levi-Civita permutation tensor, respectively, with the ordinary permutation symbols being given as usual by $\epsilon_{j_1 j_2 \ldots j_q}$ and $\epsilon^{i_1 i_2 \ldots i_p}$.

We can now write

$$\omega = \frac{1}{p!}\, \delta^{i_1 i_2 \ldots i_p}_{j_1 j_2 \ldots j_p} \omega_{i_1 i_2 \ldots i_p} dx^{j_1} \wedge dx^{j_2} \wedge \ldots \wedge dx^{j_p} .$$

such that for a null p-form $\omega = 0$ its components satisfy the relation $\delta^{i_1 i_2 \ldots i_p}_{j_1 j_2 \ldots j_p} \omega_{i_1 i_2 \ldots i_p} = 0$.

By meticulously moving the dx^i from one position to another, we see that

$$dx^{i_1} \wedge dx^{i_2} \wedge \ldots \wedge dx^{i_{p-1}} \wedge dx^{i_p} \wedge dx^{j_1} \wedge dx^{j_2} \wedge \ldots \wedge dx^{j_q} = (-1)^p dx^{i_1} \wedge dx^{i_2} \wedge \ldots \wedge dx^{i_{p-1}} \wedge dx^{j_1} \wedge dx^{j_2} \wedge \ldots \wedge dx^{j_q} \wedge dx^{i_p}$$

and

$$dx^{i_1} \wedge dx^{i_2} \wedge \ldots \wedge dx^{i_p} \wedge dx^{j_1} \wedge dx^{j_2} \wedge \ldots \wedge dx^{j_q} = (-1)^{pq} dx^{j_1} \wedge dx^{j_2} \wedge \ldots \wedge dx^{j_q} \wedge dx^{i_1} \wedge dx^{i_2} \wedge \ldots \wedge dx^{i_p} .$$

Let ω and π be a p-form and a q-form, respectively. Then, in general, we have the following relations:

$$\omega \wedge \pi = (-1)^{pq} \pi \wedge \omega = \omega_{i_1 i_2 \ldots i_p} \pi_{j_1 j_2 \ldots j_q} dx^{i_1} \wedge dx^{i_2} \wedge \ldots \wedge dx^p \wedge dx^{j_1} \wedge dx^{j_2} \wedge \ldots \wedge dx^{j_q}$$

$$d(\omega + \pi) = d\omega + d\pi$$

$$d(\omega \wedge \pi) = d\omega \wedge \pi + (-1)^p\, \omega \wedge d\pi$$

Note that the mapping $d : \omega = d\omega$ is a $(p+1)$-form. Explicitly, we have

$$d\omega = \frac{(-1)^p}{(p+1)!}\, \delta^{i_1 i_2 \ldots i_p}_{j_1 j_2 \ldots j_p} \frac{\partial \omega_{i_1 i_2 \ldots i_p}}{\partial x^{i_{p+1}}} dx^{j_1} \wedge dx^{j_2} \wedge \ldots \wedge dx^{j_p} \wedge dx^{i_{p+1}} .$$

For instance, given a (continuous) function f, the one-form $df = \partial_i f dx^i$ satisfies $d^2 f \equiv ddf = \partial_k \partial_i f dx^k \wedge dx^i = 0$. Likewise, for the one-form $A = A_i dx^i$, we have $dA = \partial_k A_i dx^k \wedge dx^i$ and therefore $d^2 A = \partial_l \partial_k A_i dx^l \wedge dx^k \wedge dx^i = 0$, i.e., $\delta^{ikl}_{rst} \partial_l \partial_k A_i = 0$ or $\partial_l \partial_k A_i + \partial_k \partial_i A_l + \partial_i \partial_l A_k = 0$. Obviously, the last result holds for arbitrary p-forms $\Pi^{ij\ldots s}_{kl\ldots r}$, i.e.,

$$d^2 \Pi^{ij\ldots s}_{kl\ldots r} = 0\,.$$

Let us now consider a simple two-dimensional case. From the transformation law $dx^i = \partial_\alpha x^i d\bar{x}^\alpha$, we have, upon employing a positive definite Jacobian, i.e., $\frac{\partial(x^i, x^j)}{\partial(\bar{x}^\alpha, \bar{x}^\beta)} > 0$, the following:

$$dx^i \wedge dx^j = \partial_\alpha x^i \partial_\beta x^j d\bar{x}^\alpha \wedge d\bar{x}^\beta = \frac{1}{2} \frac{\partial (x^i, x^j)}{\partial (\bar{x}^\alpha, \bar{x}^\beta)} d\bar{x}^\alpha \wedge d\bar{x}^\beta .$$

It is easy to see that

$$dx^1 \wedge dx^2 = \frac{\partial (x^1, x^2)}{\partial (\bar{x}^1, \bar{x}^2)} d\bar{x}^1 \wedge d\bar{x}^2 .$$

which gives the correct transformation law of a surface element.

We can now elaborate on the so-called *Stokes theorem*. Given an arbitrary function f, the integration in a domain D in the manifold $\mathbb{C}^\infty$ is such that

$$\iint\limits_D f(x^i)\, dx^1 \wedge dx^2 = \iint\limits_D f(x^i(\bar{x}^\alpha)) \frac{\partial (x^1, x^2)}{\partial (\bar{x}^1, \bar{x}^2)} d\bar{x}^1 d\bar{x}^2 .$$

Generalizing to n dimensions, for any $\psi^i = \psi^i(x^k)$ we have

$$d\psi^1 \wedge d\psi^2 \wedge \ldots \wedge d\psi^n = \frac{\partial (\psi^1, \psi^2, \ldots, \psi^n)}{\partial (x^1, x^2, \ldots, \bar{x}^n)} dx^1 \wedge dx^2 \wedge \ldots \wedge dx^n .$$

Therefore (in a particular domain)

$$\iint \ldots \int f d\psi^1 \wedge d\psi^2 \wedge \ldots \wedge d\psi^n = \iint \ldots \int f(x^i) \frac{\partial (\psi^1, \psi^2, \ldots, \psi^n)}{\partial (x^1, x^2, \ldots, x^n)} dx^1 \wedge dx^2 \wedge \ldots \wedge dx^n .$$

Obviously, the value of this integral is independent of the choice of the coordinate system. Under the coordinate transformation given by $x^i = x^i(\bar{x}^\alpha)$, the Jacobian can be expressed as

$$\frac{\partial (\psi^1, \psi^2, \ldots, \psi^n)}{\partial (x^1, x^2, \ldots, x^n)} = \frac{\partial (\psi^1, \psi^2, \ldots, \psi^n)}{\partial (\bar{x}^1, \bar{x}^2, \ldots, \bar{x}^n)} \frac{\partial (\bar{x}^1, \bar{x}^2, \ldots, \bar{x}^n)}{\partial (x^1, x^2, \ldots, x^n)} .$$

If we consider a $(n-m)$-dimensional subspace (hypersurface) $\mathbb{S} \in \mathbb{C}^\infty$ whose local coordinates u^A parametrize the coordinates x^i, we have

$$\iint \ldots \int f d\psi^1 \wedge d\psi^2 \wedge \ldots \wedge d\psi^n = \iint \ldots \int f(x^i(u^A)) \times \frac{\partial (\psi^1(x^i(u^A)), \psi^2(x^i(u^A)), \ldots, \psi^n(x^i(u^A)))}{\partial (u^1, u^2, \ldots, u^{n-m})} \times du^1 du^2 \ldots du^{n-m} .$$

2.3 Geometric properties of a curved manifold

Let us recall a few concepts from conventional tensor analysis for a while. Introducing a generally asymmetric connection Γ via the covariant derivative

$$\partial_b \omega_a = \Gamma^c_{ab} \omega_c$$

i.e.,

$$\Gamma^c_{ab} = \langle \theta^c, \partial_b \omega_a \rangle = \Gamma^c_{(ab)} + \Gamma^c_{[ab]}$$

where the round index brackets indicate symmetrization and the square ones indicate anti-symmetrization, we have, by means of the local coordinate transformation given by $x^a = x^a(\bar{x}^\alpha)$ in $\mathbb{C}^\infty$

$$\partial_b e^\alpha_a = \Gamma^c_{ab} e^\alpha_c - \bar{\Gamma}^\alpha_{\beta\lambda} e^\beta_a e^\lambda_b ,$$

where the tetrads of the *moving frames* are given by $e^\alpha_a = \partial_a \bar{x}^\alpha$ and $e^a_\alpha = \partial_\alpha x^a$. They satisfy $e^a_\alpha e^\alpha_b = \delta^a_b$ and $e^\alpha_a e^a_\beta = \delta^\alpha_\beta$. In addition, it can also be verified that

$$\partial_\beta e^a_\alpha = \bar{\Gamma}^\lambda_{\alpha\beta} e^a_\lambda - \Gamma^a_{bc} e^b_\alpha e^c_\beta \partial_b e^a_\alpha = e^a_\lambda \bar{\Gamma}^\lambda_{\alpha\beta} e^\beta_b - \Gamma^a_{cb} e^c_\alpha .$$

From conventional tensor analysis, we know that Γ is a non-tensorial object, since its components transform as

$$\Gamma^c_{ab} = e^c_\alpha \partial_b e^\alpha_a + e^c_\alpha \bar{\Gamma}^\alpha_{\beta\lambda} e^\beta_a e^\lambda_b .$$

However, it can be described as a kind of displacement field since it is what makes possible a comparison of vectors from point to point in $\mathbb{C}^\infty$. In fact the relation $\partial_b \omega_a = \Gamma^c_{ab} \omega_c$ defines the so-called metricity condition, i.e., the change (during a displacement) in the basis can be measured by the basis itself. This immediately translates into

$$\nabla_c g_{ab} = 0\,,$$

where we have just applied the notion of a covariant derivative to an arbitrary tensor field T:

$$\nabla_k T^{ij\ldots s}_{lm\ldots r} = \partial_k T^{ij\ldots s}_{lm\ldots r} + \Gamma^i_{pk} T^{pj\ldots s}_{lm\ldots r} + \Gamma^j_{pk} T^{ip\ldots s}_{lm\ldots r} + \ldots + \Gamma^s_{pk} T^{ij\ldots p}_{lm\ldots r} - \Gamma^p_{lk} T^{ij\ldots s}_{pm\ldots r} - \Gamma^p_{mk} T^{ij\ldots s}_{lp\ldots r} - \ldots - \Gamma^p_{rk} T^{ij\ldots s}_{lm\ldots p}$$

such that $(\partial_k T)^{ij\ldots s}_{lm\ldots r} = \nabla_k T^{ij\ldots s}_{lm\ldots r}$.

The condition $\nabla_c\, g_{ab} = 0$ can be solved to give

$$\Gamma^c_{ab} = \frac{1}{2}\, g^{cd} \left(\partial_b g_{da} - \partial_d g_{ab} + \partial_a g_{bd}\right) + \Gamma^c_{[ab]} - \\ - g^{cd} \left(g_{ae}\Gamma^e_{[db]} + g_{be}\Gamma^e_{[da]}\right)$$

from which it is customary to define

$$\Delta^c_{ab} = \frac{1}{2}\, g^{cd} \left(\partial_b g_{da} - \partial_d g_{ab} + \partial_a g_{bd}\right)$$

as the Christoffel symbols (symmetric in their two lower indices) and

$$K^c_{ab} = \Gamma^c_{[ab]} - g^{cd} \left(g_{ae}\Gamma^e_{[db]} + g_{be}\Gamma^e_{[da]}\right)$$

as the components of the so-called contwist tensor (anti-symmetric in the first two mixed indices).

Note that the components of the twist tensor are given by

$$\Gamma^a_{[bc]} = \frac{1}{2}\, e^a_\alpha \left(\partial_c\, e^\alpha_b - \partial_b\, e^\alpha_c + e^\beta_b\, \bar{\Gamma}^\alpha_{\beta c} - e^\beta_c\, \bar{\Gamma}^\alpha_{\beta b}\right)$$

where we have set $\bar{\Gamma}^\alpha_{\beta c} = \bar{\Gamma}^\alpha_{\beta\lambda}\, e^\lambda_c$, such that for an arbitrary scalar field Φ we have

$$\left(\nabla_a \nabla_b - \nabla_b \nabla_a\right)\Phi = 2\Gamma^c_{[ab]}\nabla_c \Phi\,.$$

The components of the curvature tensor R of $\mathbb{C}^\infty$ are then given via the relation

$$\left(\nabla_q \nabla_p - \nabla_p \nabla_q\right) T^{ab\ldots s}_{cd\ldots r} = T^{ab\ldots s}_{wd\ldots r} R^w{}_{cpq} + T^{ab\ldots s}_{cw\ldots r} R^w{}_{dpq} + \\ \ldots + T^{ab\ldots s}_{cd\ldots w} R^w{}_{rpq} - T^{wb\ldots s}_{cd\ldots r} R^a{}_{wpq} - T^{aw\ldots s}_{cd\ldots r} R^b{}_{wpq} - \ldots \\ - T^{ab\ldots w}_{cd\ldots r} R^s{}_{wpq} - 2\Gamma^w_{[pq]} \nabla_w T^{ab\ldots s}_{cd\ldots r}$$

where

$$R^d{}_{abc} = \partial_b \Gamma^d_{ac} - \partial_c \Gamma^d_{ab} + \Gamma^e_{ac}\Gamma^d_{eb} - \Gamma^e_{ab}\Gamma^d_{ec} = \\ = B^d{}_{abc}\left(\Delta\right) + \hat{\nabla}_b K^d{}_{ac} - \hat{\nabla}_c K^d{}_{ab} + K^e{}_{ac} K^d{}_{eb} - K^e{}_{ab} K^d{}_{ec}$$

where $\hat{\nabla}$ denotes covariant differentiation with respect to the Christoffel symbols alone, and where

$$B^d{}_{abc}\left(\Delta\right) = \partial_b \Delta^d_{ac} - \partial_c \Delta^d_{ab} + \Delta^e_{ac}\Delta^d_{eb} - \Delta^e_{ab}\Delta^d_{ec}$$

are the components of the Riemann-Christoffel curvature tensor of $\mathbb{C}^\infty$.

From the components of the curvature tensor, namely, $R^d{}_{abc}$, we have (using the metric tensor to raise and lower indices)

$$R_{ab} \equiv R^c{}_{acb} = B_{ab}\left(\Delta\right) + \hat{\nabla}_c K^c_{ab} - K^c_{ad} K^d_{cb} - \\ - 2\hat{\nabla}_b \Gamma^c_{[ac]} + 2K^c_{ab}\Gamma^d_{[cd]}$$

$$R \equiv R^a{}_a = B\left(\Delta\right) - 4g^{ab}\hat{\nabla}_a \Gamma^c_{[bc]} - \\ - 2g^{ac}\Gamma^b_{[ab]}\Gamma^d_{[cd]} - K_{abc}K^{acb}$$

where $B_{ab}\left(\Delta\right) \equiv B^c{}_{acb}\left(\Delta\right)$ are the components of the symmetric Ricci tensor and $B\left(\Delta\right) \equiv B^a{}_a\left(\Delta\right)$ is the Ricci scalar. Note that $K_{abc} \equiv g_{ad} K^d_{bc}$ and $K^{acb} \equiv g^{cd} g^{be} K^a_{de}$.

Now since

$$\Gamma^b_{ba} = \Delta^b_{ba} = \Delta^b_{ab} = \partial_a \left(\ln \sqrt{\det\left(g\right)}\right)$$

$$\Gamma^b_{ab} = \partial_a \left(\ln \sqrt{\det\left(g\right)}\right) + 2\Gamma^b_{[ab]}$$

we see that for a continuous metric determinant, the so-called homothetic curvature vanishes:

$$H_{ab} \equiv R^c{}_{cab} = \partial_a \Gamma^c_{cb} - \partial_b \Gamma^c_{ca} = 0$$

Introducing the traceless Weyl tensor C, we have the following decomposition theorem:

$$R^d{}_{abc} = C^d{}_{abc} + \frac{1}{n-2}\left(\delta^d_b R_{ac} + g_{ac} R^d{}_b - \delta^d_c R_{ab} - g_{ab} R^d{}_c\right) + \\ + \frac{1}{\left(n-1\right)\left(n-2\right)}\left(\delta^d_c g_{ab} - \delta^d_b g_{ac}\right) R$$

which is valid for $n > 2$. For $n = 2$, we have

$$R^d{}_{abc} = K_G\left(\delta^d_b g_{ac} - \delta^d_c g_{ab}\right)$$

where

$$K_G = \frac{1}{2}\, R$$

is the Gaussian curvature of the surface. Note that (in this case) the Weyl tensor vanishes.

Any n-dimensional manifold (for which $n > 1$) with constant sectional curvature R and vanishing twist is called an Einstein space. It is described by the following simple relations:

$$R^d{}_{abc} = \frac{1}{n(n-1)}\left(\delta^d_b g_{ac} - \delta^d_c g_{ab}\right) R$$

$$R_{ab} = \frac{1}{n}\, g_{ab} R\,.$$

In the above, we note especially that

$$R^d{}_{abc} = B^d{}_{abc}\left(\Delta\right),$$

$$R_{ab} = B_{ab}\left(\Delta\right),$$

$$R = B\left(\Delta\right).$$

Furthermore, after some elaborate (if not tedious) algebra, we obtain, in general, the following *generalized* Bianchi identities:

$$R^a{}_{bcd} + R^a{}_{cdb} + R^a{}_{dbc} = -2\big(\partial_d \Gamma^a_{[bc]} + \\ + \partial_b \Gamma^a_{[cd]} + \partial_c \Gamma^a_{[db]} + \Gamma^a_{eb}\Gamma^e_{[cd]} + \Gamma^a_{ec}\Gamma^e_{[db]} + \Gamma^a_{ed}\Gamma^e_{[bc]}\big)$$

$$\nabla_e R^a{}_{bcd} + \nabla_c R^a{}_{bde} + \nabla_d R^a{}_{bec} = \\ = 2\left(\Gamma^f_{[cd]} R^a{}_{bfe} + \Gamma^f_{[de]} R^a{}_{bfc} + \Gamma^f_{[ec]} R^a{}_{bfd}\right)$$

$$\nabla_a \left(R^{ab} - \frac{1}{2}\, g^{ab} R\right) = 2g^{ab}\Gamma^c_{[da]} R^d{}_c + \Gamma^a_{[cd]} R^{cdb}{}_a$$

for any metric-compatible manifold endowed with both curvature and twist.

In the last of the above set of equations, we have introduced the generalized Einstein tensor, i.e.,

$$G_{ab} \equiv R_{ab} - \frac{1}{2}\, g_{ab} R\,.$$

In particular, we also have the following specialized identities, i.e., the *regular* Bianchi identities:

$$B^a{}_{bcd} + B^a{}_{cdb} + B^a{}_{dbc} = 0\,,$$

$$\hat{\nabla}_e B^a{}_{bcd} + \hat{\nabla}_c B^a{}_{bde} + \hat{\nabla}_d B^a{}_{bec} = 0$$

$$\hat{\nabla}_a \left(B^{ab} - \frac{1}{2}\, g^{ab} B \right) = 0\,.$$

In general, these hold in the case of a symmetric, metric-compatible connection. Non-metric differential geometry is beyond the scope of our present consideration. We will need the identities presented in this section in the development of our semi-symmetric, metric-compatible unified field theory.

2.4 The structural equations

The results of the preceding section can be expressed in the language of exterior calculus in a somewhat more compact form.

In general, we can construct arbitrary p-forms $\omega^{ab\ldots e}_{cd\ldots f}$ through arbitrary $(p-1)$ forms $\alpha^{ab\ldots e}_{cd\ldots f}$, i.e.,

$$\omega^{ab\ldots e}_{cd\ldots f} = d\alpha^{ab\ldots e}_{cd\ldots f} = \frac{\partial \alpha^{ab\ldots e}_{cd\ldots f}}{\partial x^h} \wedge dx^h.$$

The covariant exterior derivative is then given by

$$D\omega^{ab\ldots e}_{cd\ldots f} = \nabla_h \omega^{ab\ldots e}_{cd\ldots f} \wedge dx^h$$

i.e.,

$$D\omega^{ab\ldots e}_{cd\ldots.f} = d\omega^{ab\ldots e}_{cd\ldots f} + (-1)^p \left(\omega^{hb\ldots e}_{cd\ldots f} \wedge \Gamma^a_h + \right.$$
$$+ \, \omega^{ah\ldots e}_{cd\ldots f} \wedge \Gamma^b_h + \ldots + \omega^{ab\ldots h}_{cd\ldots f} \wedge \Gamma^e_h - \omega^{ab\ldots e}_{hd\ldots f} \wedge \Gamma^h_c -$$
$$\left. - \, \omega^{ab\ldots e}_{ch\ldots f} \wedge \Gamma^h_d - \ldots - \omega^{ab\ldots e}_{cd\ldots h} \wedge \Gamma^h_f \right)$$

where we have defined the connection one-forms by

$$\Gamma^a_b \equiv \Gamma^a_{bc}\, \theta^c$$

with respect to the coframe basis θ^a.

Now we write the twist two-forms τ^a as

$$\tau^a = D\theta^a = d\theta^a + \Gamma^a_b \wedge \theta^b.$$

This gives the first structural equation. With respect to another local coordinate system (with coordinates $\bar{x}^\alpha$) in $\mathbb{C}^\infty$ spanned by the basis $\epsilon^\alpha = e^\alpha_a \theta^a$, we see that

$$\tau^a = -e^a_\alpha \bar{\Gamma}^\alpha_{[\beta\lambda]} \epsilon^\beta \wedge \epsilon^\lambda.$$

We shall again proceed to define the curvature tensor. For a triad of arbitrary vectors u, v, w, we may define the following relations with respect to the frame basis ω_a:

$$\nabla_u \nabla_v w \equiv u^c \nabla_c \left(v^b \nabla_b w^a \right) \omega_a$$

$$\nabla_{[u,v]} w \equiv \nabla_b w^a \left(u^c \nabla_c v^b - v^c \nabla_c u^b \right)$$

where ∇_u and ∇_v denote covariant differentiation in the direction of u and of v, respectively.

Then we have

$$\left(\nabla_u \nabla_v - \nabla_v \nabla_u \right) w = {}^*R^a{}_{bcd}\, w^b u^c v^d \omega_a\,.$$

Note that

$${}^*R^a{}_{bcd} = \partial_c \Gamma^a_{bd} - \partial_d \Gamma^a_{bc} + \Gamma^e_{bd} \Gamma^a_{ec} - \Gamma^e_{bc} \Gamma^a_{ed} +$$
$$+ \, 2\Gamma^e_{[cd]} \Gamma^a_{be} = R^a{}_{bcd} + 2\Gamma^e_{[cd]} \Gamma^a_{be}$$

are the components of the extended curvature tensor ${}^*R$.

Define the curvature two-forms by

$${}^*R^a{}_b \equiv \frac{1}{2}\, {}^*R^a{}_{bcd}\, \theta^c \wedge \theta^d.$$

The second structural equation is then

$${}^*R^a{}_b = d\Gamma^a_b + \Gamma^a_c \wedge \Gamma^c_b\,.$$

The third structural equation is given by

$$d^2 \Gamma^a_b = d\,{}^*R^a{}_b - {}^*R^a{}_c \wedge \Gamma^c_b + \Gamma^a_c \wedge {}^*R^c{}_b = D\,{}^*R^a{}_b$$

which is equivalent to the generalized Bianchi identities given in the preceding section.

In fact the second and third structural equations above can be directly verified using the properties of exterior differentiation given in Section 1.2.

Now, as we have seen, the covariant exterior derivative of arbitrary one-forms ϕ^a is given by $D\phi^a = d\phi^a + \Gamma^a_b \wedge \phi^b$. Then

$$DD\phi^a = d\left(D\phi^a \right) + \Gamma^a_b \wedge D\phi^b \; =$$
$$= d\left(d\phi^a + \Gamma^a_b \wedge \phi^b \right) + \Gamma^a_c \wedge \left(d\phi^c + \Gamma^c_d \wedge \phi^d \right) =$$
$$= \; d\Gamma^a_b \wedge \phi^b - \Gamma^a_b \wedge \Gamma^b_c \wedge \phi^c \; =$$
$$= \left(d\Gamma^a_b + \Gamma^a_c \wedge \Gamma^c_b \right) \wedge \phi^b$$

where we have used the fact that the $D\phi^a$ are two-forms. Therefore, from the second structural equation, we have

$$DD\phi^a = {}^*R^a{}_b \wedge \phi^b.$$

Finally, taking $\phi^a = \theta^a$, we give the fourth structural equation as

$$DD\theta^a = D\tau^a = {}^*R^a{}_b \wedge \theta^b$$

or,

$$d\tau^a = {}^*R^a{}_b \wedge \theta^b - \Gamma^a_b \wedge \tau^b.$$

Remarkably, this is equivalent to the first generalized Bianchi identity given in the preceding section.

2.5 The geometry of distant parallelism

Let us now consider a special situation in which our n-dimensional manifold $\mathbb{C}^\infty$ is embedded *isometrically* in a flat n-dimensional (pseudo-)Euclidean space $\mathbb{E}^n$ (with coordinates $v^{\bar m}$) spanned by the constant basis $e_{\bar m}$ whose dual is denoted by $s^{\bar n}$. This embedding allows us to globally cover the manifold $\mathbb{C}^\infty$ in the sense that its geometric structure can be parametrized by the Euclidean basis $e_{\bar m}$ satisfying

$$\eta_{\bar m \bar n} = \langle e_{\bar m}, e_{\bar n} \rangle = \mathrm{diag}\,(\pm 1, \pm 1, \ldots, \pm 1)\,.$$

It is important to note that this situation is different from the one presented in Section 1.1, in which case we may refer the structural equations of $\mathbb{C}^\infty$ to the locally flat tangent space $\mathbb{T}_x(\mathbb{M})$. The results of the latter situation (i.e., the *localized* structural equations) should not always be regarded as globally valid since the tangent space $\mathbb{T}_x(\mathbb{M})$, though ubiquitous in the sense that it can be defined everywhere (at any point) in $\mathbb{C}^\infty$, cannot cover the whole structure of the curved manifold $\mathbb{C}^\infty$ without changing orientation from point to point.

One can construct geometries with special connections that will give rise to what we call *geometries with parallelism*. Among others, the geometry of *distant parallelism* is a famous case. Indeed, A. Einstein adopted this geometry in one of his attempts to geometrize physics, and especially to unify gravity and electromagnetism [5]. In its application to physical situations, the resulting field equations of a unified field theory based on distant parallelism, for instance, are quite remarkable in that the so-called energy-momentum tensor appears to be geometrized via the twist tensor. We will therefore dedicate this section to a brief presentation of the geometry of distant parallelism in the language of Riemann-Cartan geometry.

In this geometry, it is possible to orient vectors such that their directions remain invariant after being displaced from a point to some distant point in the manifold. This situation is made possible by the vanishing of the curvature tensor, which is given by the integrability condition

$$R^d{}_{abc} = e^d_{\bar m} \left(\partial_b \partial_c - \partial_c \partial_b\right) e^{\bar m}_a = 0$$

where the connection is now given by

$$\Gamma^c_{ab} = e^c_{\bar m}\, \partial_b e^{\bar m}_a$$

where $e^{\bar m}_a = \partial_a \xi^{\bar m}$ and $e^a_{\bar m} = \partial_{\bar m} x^a$.

However, while the curvature tensor vanishes, one still has the twist tensor given by

$$\Gamma^a_{[bc]} = \frac{1}{2}\, e^a_{\bar m} \left(\partial_c e^{\bar m}_b - \partial_b e^{\bar m}_c\right)$$

with the $e^{\bar m}_a$ acting as the components of a spin "potential". Thus the twist can now be considered as the primary geometric object in the manifold $\mathbb{C}^\infty_p$ endowed with distant parallelism.

Also, in general, the Riemann-Christoffel curvature tensor is non-vanishing as

$$B^d{}_{abc} = \hat\nabla_c K^d_{ab} - \hat\nabla_b K^d_{ac} + K^e_{ab} K^d_{ec} - K^e_{ac} K^d_{eb}\,.$$

Let us now consider some facts. Taking the covariant derivative of the tetrad $e^{\bar m}_a$ with respect to the Christoffel symbols alone, we have

$$\hat\nabla_b\, e^{\bar m}_a = \partial_b\, e^{\bar m}_a - e^{\bar m}_d \Delta^d_{ab} = e^{\bar m}_c K^c_{ab}$$

i.e.,

$$K^c_{ab} = e^c_{\bar m} \hat\nabla_b\, e^{\bar m}_a = -e^{\bar m}_a \hat\nabla_b\, e^c_{\bar m}\,.$$

In the above sense, the components of the contwist tensor give the so-called *Ricci rotation coefficients*. Then from

$$\hat\nabla_c \hat\nabla_b\, e^{\bar m}_a = e^{\bar m}_d \left(\hat\nabla_c K^d_{ab} + K^e_{ab} K^d_{ec}\right)$$

it is elementary to show that

$$\left(\hat\nabla_c \hat\nabla_b - \hat\nabla_b \hat\nabla_c\right) e^{\bar m}_a = e^{\bar m}_d B^d{}_{abc}\,.$$

Likewise, we have

$$\tilde\nabla_b\, e^{\bar m}_a = \partial_b\, e^{\bar m}_a - e^{\bar m}_d K^d_{ab} = e^{\bar m}_c \Delta^c_{ab}$$

$$\Delta^c_{ab} = e^c_{\bar m} \tilde\nabla_b\, e^{\bar m}_a = -e^{\bar m}_a \tilde\nabla_b\, e^c_{\bar m}$$

where now $\tilde\nabla$ denotes covariant differentiation with respect to the Ricci rotation coefficients alone. Then from

$$\tilde\nabla_c \tilde\nabla_b e^{\bar m}_a = e^{\bar m}_d \left(\tilde\nabla_c \Delta^d_{ab} + \Delta^e_{ab} \Delta^d_{ec}\right)$$

we get

$$\left(\tilde\nabla_c \tilde\nabla_b - \tilde\nabla_b \tilde\nabla_c\right) e^{\bar m}_a = -e^{\bar m}_d \big(B^d{}_{abc} - 2\Delta^d_{ae} \Gamma^e_{[bc]} - \\ - \Delta^e_{ab} K^d_{ec} + \Delta^e_{ac} K^d_{eb} - K^e_{ab} \Delta^d_{ec} + K^e_{ac} \Delta^d_{eb}\big)\,.$$

In this situation, one sees, with respect to the coframe basis $\theta^a = e^a_{\bar m} s^{\bar m}$, that

$$d\theta^a = -\Gamma^a_b \wedge \theta^b \equiv T^a$$

i.e.,

$$T^a = \Gamma^a_{[bc]} \theta^b \wedge \theta^c.$$

Thus the twist two-forms of this geometry are now given by T^a (instead of τ^a of the preceding section). We then realize that

$$D\theta^a = 0\,.$$

Next, we see that

$$d^2\theta^a = dT^a = -d\Gamma^a_b \wedge \theta^b + \Gamma^a_b \wedge d\theta^b = \\ = -\left(d\Gamma^a_b + \Gamma^a_c \wedge \Gamma^c_b\right) \wedge \theta^b = \\ = -{}^*R^a{}_b \wedge \theta^b.$$

But, as always, $d^2\theta^a = 0$, and therefore we have

$$ {}^*R^a{}_b \wedge \theta^b = 0 $$

Note that in this case, ${}^*R^a{}_b \neq 0$ (in general) as

$$ {}^*R^a{}_{bcd} = 2\Gamma^e_{[cd]}\Gamma^a_{be} $$

will not vanish in general. We therefore see immediately that

$$ {}^*R^a{}_{bcd} + {}^*R^a{}_{cdb} + {}^*R^a{}_{dbc} = 0 $$

giving the integrability condition

$$ \Gamma^e_{[cd]}\Gamma^a_{be} + \Gamma^e_{[db]}\Gamma^a_{ce} + \Gamma^e_{[bc]}\Gamma^a_{de} = 0\,. $$

Meanwhile, the condition

$$ dT^a = 0 $$

gives the integrability condition

$$ \partial_d\Gamma^a_{[bc]} + \partial_b\Gamma^a_{[cd]} + \partial_c\Gamma^a_{[db]} = 0\,. $$

Contracting, we find

$$ \partial_c\Gamma^c_{[ab]} = 0\,. $$

It is a curious fact that the last two relations somehow remind us of the algebraic structure of the components of the electromagnetic field tensor in physics.

Finally, from the contraction of the components $B^d{}_{abc}$ of the Riemann-Christoffel curvature tensor (the Ricci tensor), one defines the regular Einstein tensor by

$$ \hat{G}_{ab} \equiv B_{ab} - \frac{1}{2}\,g_{ab}B \equiv kE_{ab} $$

where k is a physical coupling constant and E_{ab} are the components of the so-called energy-momentum tensor. We therefore see that

$$ E_{ab} = \frac{1}{k}\left(K^c_{ad}K^d_{cb} - \hat{\nabla}_c K^c_{ab} + 2\hat{\nabla}_b\Gamma^c_{[ac]} - 2K^c_{ab}\Gamma^d_{[cd]}\right) - $$
$$ -\frac{1}{2k}\,g_{ab}\left(4g^{cd}\hat{\nabla}_c\Gamma^e_{[de]} + 2g^{ce}\Gamma^d_{[cd]}\Gamma^f_{[ef]} + K_{cde}K^{ced}\right). $$

In addition, the following two conditions are satisfied:

$$ E_{[ab]} = 0\,, $$
$$ \hat{\nabla}_a E^{ab} = 0\,. $$

We have now seen that, in this approach we have applied here, the energy-momentum tensor (matter field) is fully geometrized. This way, gravity arises from twistal (spin) interaction (possibly, on the microscopic scales) and becomes an emergent phenomenon rather than a fundamental one. This seems rather speculative. However, it may have profound consequences.

2.6 Spin frames

A spin frame is described by the anti-symmetric tensor product

$$ \Omega^{ik} = \frac{1}{2}\left(\theta^i \otimes \theta^k - \theta^k \otimes \theta^i\right) = \theta^i \wedge \theta^k \equiv \frac{1}{2}\left[\theta^i, \theta^k\right]\,. $$

In general, then, for arbitrary vector field fields A and B, we can form the commutator

$$ [A, B] = A \otimes B - B \otimes A\,. $$

Introducing another vector field C, we have the so-called Jacobi identity

$$ [A, [B, C]] + [B, [C, A]] + [C, [A, B]] = 0\,. $$

With respect to the local coordinate basis elements $E_i = \partial_i$ of the tangent space $\mathbb{T}_x(\mathbb{M})$, we see that, astonishingly enough, the anti-symmetric product $[A, B]$ is what defines the Lie (exterior) derivative of B with respect to A:

$$ L_A B \equiv [A, B] = \left(A^i\partial_i B^k - B^i\partial_i A^k\right)\frac{\partial}{\partial X^k}\,. $$

(Note that $L_A A = [A, A] = 0$.) The terms in the round brackets are just the components of our Lie derivative which can be used to define a diffeomorphism invariant (i.e., by taking $A^i = \xi^i$ where ξ represents the displacement field in a neighborhood of coordinate points).

Furthermore, for a vector field U and a tensor field T, both arbitrary, we have (in component notation) the following:

$$ L_U T^{ij\ldots s}_{kl\ldots r} = \partial_m T^{ij\ldots s}_{kl\ldots r}U^m + T^{ij\ldots s}_{ml\ldots r}\partial_k U^m + $$
$$ + T^{ij\ldots s}_{km\ldots r}\partial_l U^m + \ldots + T^{ij\ldots s}_{kl\ldots m}\partial_r U^m - T^{mj\ldots s}_{kl\ldots r}\partial_m U^i - $$
$$ - T^{im\ldots s}_{kl\ldots r}\partial_m U^j - \ldots - T^{ij\ldots m}_{kl\ldots r}\partial_m U^s $$

It is not immediately apparent whether these transform as components of a tensor field or not. However, with the help of the twist tensor and the relation

$$ \partial_k U^i = \nabla_k U^i - \Gamma^i_{mk}U^m = \nabla_k U^i - \left(\Gamma^i_{km} - 2\Gamma^i_{[km]}\right)U^m $$

we can write

$$ L_U T^{ij\ldots s}_{kl\ldots r} = \nabla_m T^{ij\ldots s}_{kl\ldots r}U^m + T^{ij\ldots s}_{ml\ldots r}\nabla_k U^m + $$
$$ + T^{ij\ldots s}_{km\ldots r}\nabla_l U^m + \ldots + T^{ij\ldots s}_{kl\ldots m}\nabla_r U^m - T^{mj\ldots s}_{kl\ldots r}\nabla_m U^i - $$
$$ - T^{im\ldots s}_{kl\ldots r}\nabla_m U^j - \ldots - T^{ij\ldots m}_{kl\ldots r}\nabla_m U^s + $$
$$ + 2\Gamma^i_{[mp]}T^{mj\ldots s}_{kl\ldots r}U^p + 2\Gamma^j_{[mp]}T^{im\ldots s}_{kl\ldots r}U^p + \ldots $$
$$ + 2\Gamma^s_{[mp]}T^{ij\ldots m}_{kl\ldots r}U^p - 2\Gamma^m_{[kp]}T^{ij..s}_{ml\ldots r}U^p - $$
$$ - 2\Gamma^m_{[lp]}T^{ij\ldots s}_{km\ldots r}U^p - \ldots - 2\Gamma^m_{[rp]}T^{ij\ldots s}_{kl\ldots m}U^p. $$

Hence, noting that the components of the twist tensor, namely, $\Gamma^i_{[kl]}$, indeed transform as components of a tensor field, it is seen that the $L_U T^{ij\dots s}_{kl\dots r}$ do transform as components of a tensor field. Apparently, the beautiful property of the Lie derivative (applied to an arbitrary tensor field) is that it is connection-independent even in a curved manifold.

If we now apply the commutator to the frame basis of the base manifold $\mathbb{C}^\infty$ itself, we see that (for simplicity, we again refer to the coordinate basis of the tangent space $\mathbb{T}_x(\mathbb{M})$)

$$[\omega_a, \omega_b] = \left(\partial_a X^i\, \partial_i \partial_b X^k - \partial_b X^i\, \partial_i \partial_a X^k\right) \frac{\partial}{\partial X^k}\,.$$

Again, writing the tetrads simply as $e^i_a = \partial_a X^i$, $e^a_i = \partial_i x^a$, we have

$$[\omega_a, \omega_b] = \left(\partial_a e^k_b - \partial_b e^k_a\right) \frac{\partial}{\partial X^k}$$

i.e.,

$$[\omega_a, \omega_b] = -2\Gamma^c_{[ab]}\omega_c\,.$$

Therefore, in the present formalism, the components of the twist tensor are by themselves proportional to the so-called *structure constants* Ψ^c_{ab} of our rotation group:

$$\Psi^c_{ab} = -2\Gamma^c_{[ab]} = -e^c_i\left(\partial_a\, e^i_b - \partial_b\, e^i_a\right).$$

As before, here the tetrad represents a spin potential.

Also note that

$$\Psi^d_{ab}\Psi^e_{dc} + \Psi^d_{bc}\Psi^e_{da} + \Psi^d_{ca}\Psi^e_{db} = 0\,.$$

We therefore observe that, as a consequence of the present formalism of differential geometry, spin fields (*objects of anholonomicity*) in the manifold $\mathbb{C}^\infty$ are generated *directly* by the twist tensor.

3 The new semi-symmetric unified field theory of the classical fields of gravity and electromagnetism

In this part, we develop our semi-symmetric unified field theory on the foundations of Riemann-Cartan geometry presented in Sections 1.1–1.6. We shall concentrate on physical events in the four-dimensional space-time manifold $\mathbb{S}^4$ with the usual Lorentzian signature. As we will see, the choice of a semi-symmetric Cartan twist will lead to a set of physically meaningful field equations from which we will obtain not only the generally covariant Lorentz equation of motion of a charged particle, but also its generalizations.

We are mainly concerned with the dynamical equations governing a cluster of individual particles and their multiple field interactions and also the possibility of defining geometrically and phenomenologically conserved currents in the theory. We will therefore not assume dimensional (i.e., structural) homogeneity with regard to the particles. Classically, a point-like (i.e., structureless) particle which characterizes a particular physical field is only a mere idealization which is not subject, e.g., to any possible dilation when interacting with other particles or fields. Still within the classical context, we relax this condition by assigning a structural configuration to each individual particle. Therefore, the characteristic properties of the individual particles allow us to describe a particle as a field in a physically meaningful sense. In this sense, the particle-field duality is abolished on the phenomenological level as well. In particular, this condition automatically takes into account both the rotational and reflectional symmetries of individual particles which have been developed separately. As such, without having to necessarily resort to particle isotropy, the symmetry group in our theory is a general one, i.e., it includes all rotations about all possible axes and reflections in any plane in the space-time manifold $\mathbb{S}^4$.

The presence of the semi-symmetric twist causes any local (hyper)surface in the space-time manifold $\mathbb{S}^4$ to be *non-orientable* in general. As a result, the trajectories of individual particles generally depend on the twisted path they trace in $\mathbb{S}^4$. It is important to note that this twist is the generator of the so-called microspin, e.g., in the simplest case, a spinning particle is simply a point-rotation in the sense of the so-called Cosserat continuum theory [10]. As usual, the semi-symmetric twist tensor enters the curvature tensor as an integral part via the general (semi-symmetric) connection. This way, all classical physical fields, not just the gravitational field, are intrinsic to the space-time geometry.

3.1 A semi-symmetric connection based on a semi-simple (transitive) rotation group

Let us now work in *four* space-time dimensions (since this number of dimensions is most relevant to physics). For a semi-simple (transitive) rotation group, we can show that

$$[\omega_a, \omega_b] = -\gamma\, \in_{abcd}\, \varphi^c\, \theta^d$$

where $\in_{abcd} = \sqrt{\det(g)}\, \epsilon_{abcd}$ are the components of the completely anti-symmetric four-dimensional Levi-Civita permutation tensor and φ is a vector field normal to a three-dimensional space (hypersurface) $\sum(t)$ defined as the time section $ct = x^0 = \text{const.}$ (where c denotes the speed of light in vacuum) of $\mathbb{S}^4$ with local coordinates z^A. It satisfies $\varphi_a\varphi^a = \gamma = \pm 1$ and is given by

$$\varphi_a = \frac{1}{6}\,\gamma\, \in_{abcd} \in^{ABC}\, \lambda^b_A \lambda^c_B \lambda^d_C$$

where

$$\lambda^a_A \equiv \partial_A x^a, \qquad \lambda^A_a \equiv \partial_a z^A,$$

$$\lambda^b_A \lambda^A_a = \delta^b_a - \gamma\varphi_a\varphi^b,$$

$$\lambda^a_A \lambda^B_a = \delta^B_A\,.$$

More specifically,

$$\in_{ABC}\, \varphi_d = \in_{abcd}\, \lambda^a_A \lambda^b_B \lambda^c_C$$

from which we find

$$\in_{abcd} = \in_{ABC}\, \lambda^A_a \lambda^B_b \lambda^C_c\, \varphi_d + \Lambda_{abcd}$$

where

$$\Lambda_{abcd} = \gamma \left(\in_{ebcd} \varphi_a + \in_{aecd} \varphi_b + \in_{abed} \varphi_c\right) \varphi^e.$$

Noting that $\Lambda_{abcd}\varphi^d = 0$, we can define a completely anti-symmetric, three-index, four-dimensional "permutation" tensor by

$$\Phi_{abc} \equiv \in_{abcd} \varphi^d = \gamma \in_{ABC} \lambda^A_a \lambda^B_b \lambda^C_c .$$

Obviously, the hypersurface $\sum(t)$ can be thought of as representing the position of a material body at any time t. As such, it acts as a boundary of the so-called world-tube of a family of world-lines covering an arbitrary four-dimensional region in $\mathbb{S}^4$.

Meanwhile, in the most general four-dimensional case, the twist tensor can be decomposed according to

$$\Gamma^c_{[ab]} = \frac{1}{3}\left(\delta^c_b \Gamma^d_{[ad]} - \delta^c_a \Gamma^d_{[bd]}\right) + \\ + \frac{1}{6} \in^c{}_{abd} \in^d{}_{pqr} g^{qs} g^{rt} \Gamma^p_{[st]} + g^{cd} Q_{dab} ,$$

$$Q_{abc} + Q_{bca} + Q_{cab} = 0 ,$$

$$Q^a_{ab} = Q^a_{ba} = 0 .$$

In our special case, the twist tensor becomes completely anti-symmetric (in its three indices) as

$$\Gamma^c_{[ab]} = -\frac{1}{2}\gamma g^{ce} \in_{abed} \varphi^d$$

from which we can write

$$\varphi^a = -\frac{1}{3} \in^{abcd} \Gamma_{b[cd]}$$

where, as usual, $\Gamma_{b[cd]} = g_{be}\Gamma^e_{[cd]}$. Therefore, at this point, the full connection is given by (with the Christoffel symbols written explicitly)

$$\Gamma^c_{ab} = \frac{1}{2} g^{cd}\left(\partial_b g_{da} - \partial_d g_{ab} + \partial_a g_{bd}\right) - \frac{1}{2}\gamma \in^c{}_{abd} \varphi^d.$$

We shall call this special connection "*semi-symmetric*". This gives the following simple conditions:

$$\Gamma^c_{(ab)} = \Delta^c_{ab} = \frac{1}{2} g^{cd}\left(\partial_b g_{da} - \partial_d g_{ab} + \partial_a g_{bd}\right),$$

$$K^c_{ab} = \Gamma^c_{[ab]} = -\frac{1}{2}\gamma \in^c_{abd} \varphi^d,$$

$$\Gamma^b_{[ab]} = 0 ,$$

$$\Gamma^b_{ab} = \Gamma^b_{ba} = \partial_a\left(\ln\sqrt{\det(g)}\right).$$

Furthermore, we can extract a projective metric tensor ϖ from the twist (via the structure constants) as follows:

$$\varpi_{ab} = g_{ab} - \gamma\varphi_a\varphi_b = 2\Gamma^c_{[ad]}\Gamma^d_{[cb]}.$$

In three dimensions, the above relation gives the so-called Cartan metric.

Finally, we are especially interested in how the existence of twist affects a coordinate frame spanned by the basis ω_a and its dual θ^b in a geometry endowed with distant parallelism. Taking the *four-dimensional curl* of the coframe basis θ^b, we see that

$$[\nabla, \theta^a] = 2d\theta^a = 2T^a \\ = -\gamma \in^{\bar m\bar n\bar p\bar q}\left(\partial_{\bar m} e^a_{\bar n}\right)\varphi_{\bar p} e_{\bar q}$$

where $\nabla = \theta^b\nabla_b = s^{\bar m}\partial_{\bar m}$ and $\in^{abcd} = \frac{1}{\sqrt{\det(g)}}\,\epsilon^{abcd}$. From the metricity condition of the tetrad (with respect to the basis of E^n), namely, $\nabla_b e^{\bar m}_a = 0$, we have

$$\partial_b e^{\bar m}_a = \Gamma^c_{ab} e^{\bar m}_c ,$$

$$\partial^{\bar n} e^{\bar m}_a = \eta^{\bar n\bar p} e^b_{\bar p}\partial_b e^{\bar m}_a = e^{\bar m}_c \Gamma^c_{ab} e^{\bar n b}.$$

It is also worthwhile to note that from an equivalent metricity condition, namely, $\nabla_a e^b_{\bar m} = 0$, one finds

$$\partial_{\bar n} e^a_{\bar m} = -\Gamma^a_{bc} e^b_{\bar m} e^c_{\bar n} .$$

Thus we find

$$[\nabla, \theta^a] = -\gamma \in^{bcde} \Gamma^a_{[bc]}\varphi_d\,\omega_e .$$

In other words,

$$T^a = d\theta^a = -\frac{1}{2}\gamma \in^{bcde} \Gamma^a_{[bc]}\varphi_d\,\omega_e .$$

For the frame basis, we have

$$[\nabla, \omega_a] = -\gamma \in^{bcde} \Gamma_{a[bc]}\varphi_d\,\omega_e .$$

At this point it becomes clear that the presence of twist in $\mathbb{S}^4$ rotates the frame and coframe bases themselves. The basics presented here constitute the reality of the so-called *spinning frames*.

3.2 Construction of the semi-symmetric field equations

In the preceding section, we have introduced the semi-symmetric connection

$$\Gamma^c_{ab} = \frac{1}{2} g^{cd}\left(\partial_b g_{da} - \partial_d g_{ab} + \partial_a g_{bd}\right) - \frac{1}{2}\gamma \in^c{}_{abd} \varphi^d$$

based on the semi-simple rotation group

$$[\omega_a, \omega_b] = -\gamma \in_{abcd} \varphi^c\theta^d.$$

Now we are in a position to construct a classical unified field theory of gravity and electromagnetism based on this connection. We shall then call the resulting field equations semi-symmetric, hence the name semi-symmetric unified field theory. (Often the terms "symmetric" and "asymmetric" refer to the metric rather than the connection.)

Using the results we have given in Section 1.3, we see that the curvature tensor built from our semi-symmetric connection is given by

$$R^d{}_{abc} = B^d{}_{abc} - \frac{1}{2}\,\gamma\left(\in^d{}_{ace}\,\hat{\nabla}_b\,\varphi^e - \in^d{}_{abe}\,\hat{\nabla}_c\,\varphi^e\right) + \\ + \frac{3}{2}\,\gamma\left(g_{eb}\,\delta^{def}_{acg} - g_{ec}\,\delta^{def}_{abg}\right)\varphi_f\,\varphi^g.$$

As before, the generalized Ricci tensor is then given by the contraction $R_{ab} = R^c{}_{acb}$, i.e.,

$$R_{ab} = B_{ab} - \frac{1}{2}\,(g_{ab} - \gamma\,\varphi_a\varphi_b) - \frac{1}{2}\,\gamma \in^{cd}{}_{ab}\,\hat{\nabla}_c\varphi_d\,.$$

Then we see that its symmetric and anti-symmetric parts are given by

$$R_{(ab)} = B_{ab} - \frac{1}{2}\,(g_{ab} - \gamma\,\varphi_a\,\varphi_b)$$

$$R_{[ab]} = -\frac{1}{2}\,\gamma \in^{cd}{}_{ab}\,F_{cd}$$

where

$$F_{ab} = \frac{1}{2}\,(\partial_a\,\varphi_b - \partial_b\,\varphi_a)$$

are the components of the intrinsic spin tensor of the first kind in our unified field theory. Note that we have used the fact that $\hat{\nabla}_a\,\varphi_b - \hat{\nabla}_b\,\varphi_a = \partial_a\,\varphi_b - \partial_b\,\varphi_a$.

Note that if

$$\varphi^a = \gamma\,\delta^a_0$$

then the twist tensor becomes covariantly constant throughout the space-time manifold, i.e.,

$$\nabla_d\,\Gamma^c_{[ab]} = \hat{\nabla}_d\,\Gamma^c_{[ab]} = 0\,.$$

This special case may indeed be anticipated as in the present theory, the two fundamental geometric objects are the metric and twist tensors.

Otherwise, in general let us define a vector-valued gravo-electromagnetic potential A via

$$\varphi^a = \lambda A^a$$

where

$$\lambda = \left(\frac{\gamma}{A_a A^a}\right)^{1/2}.$$

Letting $\in\, = \lambda^2\gamma$, we then have

$$R_{ab} = B_{ab} - \frac{1}{2}\,(g_{ab} - \in A_a A_b) - \frac{1}{2}\,\gamma \in^{cd}{}_{ab}\,\left(\lambda\bar{F} + H_{cd}\right)$$

where

$$\bar{F}_{ab} = \frac{1}{2}\,(\partial_a\,A_b - \partial_b\,A_a)\,,$$

$$H_{ab} = -\frac{1}{2}\,(A_a\,\partial_b\,\lambda - A_b\,\partial_a\,\lambda)\,.$$

We may call $\bar{F}_{ab}$ the components of the intrinsic spin tensor of the second kind. The components of the anti-symmetric field equation then take the form

$$R_{[ab]} = -\frac{1}{2}\,\gamma \in^{cd}{}_{ab}\,\left(\lambda\bar{F}_{cd} + H_{cd}\right).$$

Using the fact that

$$\partial_a\,F_{bc} + \partial_b\,F_{ca} + \partial_c\,F_{ab} = 0$$

we obtain

$$\nabla_a\,R^{[ab]} = 0\,.$$

The dual of the anti-symmetric part of the generalized Ricci tensor is then given by

$$\tilde{R}_{[ab]} = \frac{1}{2}\,\in_{abcd}\,R^{[cd]} = -\frac{1}{2}\,(\partial_a\,\varphi_b - \partial_b\,\varphi_a)$$

i.e.,

$$\tilde{R}_{[ab]} = -\left(\lambda\bar{F}_{ab} + H_{ab}\right).$$

We therefore see that

$$\partial_a\,\tilde{R}_{[bc]} + \partial_b\,\tilde{R}_{[ca]} + \partial_c\,\tilde{R}_{[ab]} = 0\,.$$

At this point, the components of the intrinsic spin tensor take the following form:

$$\bar{F}_{ab} = -\frac{1}{2\lambda}\left(\in_{abcd}\,R^{[cd]} + 2H_{ab}\right).$$

The generalized Einstein field equation is then given by

$$G_{ab} = R_{ab} - \frac{1}{2}\,g_{ab}\,R = k\,T_{ab}$$

where k is a coupling constant, $R = R^a{}_a = B - \frac{3}{2}$ (in our geometrized units) is the generalized Ricci scalar, and T_{ab} are the components of the energy-momentum tensor of the coupled matter and spin fields. Taking the covariant divergence of the generalized Einstein tensor with the help of the relations

$$\nabla_a\,R^{ab} = \hat{\nabla}_a\,R^{ab} - \Gamma^b_{[ac]}\,R^{[ac]}\,,$$

$$\nabla_a\,R = \partial_a\,R = \partial_a\,B\,,$$

$$F_{ab}\,\varphi^b = -\frac{1}{2}\,\varphi^b\,\hat{\nabla}_b\,\varphi_a\,,$$

we obtain

$$\nabla_a\,G^{ab} = \hat{\nabla}_a\,G^{ab} - \gamma\,F^b{}_a\,\varphi^a.$$

On the other hand, using the integrability condition

$$\in^{abcd}\,\hat{\nabla}_b\hat{\nabla}_c\varphi_d = \in^{abcd}\,\partial_b\partial_c\,\varphi_d = 0$$

we have

$$\hat{\nabla}_a\,R^{ab} = \hat{\nabla}_a\,B^{ab} - \frac{1}{2}\,\gamma\,\hat{\nabla}_a\left(\varphi^a\,\varphi^b\right).$$

Therefore

$$\nabla_a G^{ab} = \hat{\nabla}_a \hat{G}^{ab} + \frac{1}{2}\gamma\left(\varphi^b \hat{\nabla}_a \varphi^a + \varphi^a \hat{\nabla}_a \varphi^b\right) - \gamma F^b{}_a \varphi^a$$

where, as before, $\hat{G}_{ab} = B_{ab} - \frac{1}{2} g_{ab} B$. But as $\hat{\nabla}_a \hat{G}^{ab} = 0$, we are left with

$$\nabla_a G^{ab} = \frac{1}{2}\gamma\left(\varphi^b \hat{\nabla}_a \varphi^a + \varphi^a \hat{\nabla}_a \varphi^b\right) - \gamma F^b{}_a \varphi^a .$$

We may notice that in general the above divergence does not vanish.

We shall now seek a possible formal correspondence between our present theory and both general relativistic gravitomagnetism and Maxwellian electrodynamics. We shall first assume that particles do not necessarily have point-like structure. Now let the rest (inertial) mass of a particle and the speed of light in vacuum (again) be denoted by m and c, respectively. Also, let ϕ represent the scalar gravoelectromagnetic potential and let g_a and B_a denote the components of the gravitational spin potential and the electromagnetic four-potential, respectively. We now make the following ansatz:

$$\lambda = \text{const} = -\frac{\bar{g}}{2mc^2} ,$$

$$A_a = \partial_a \phi + v\, g_{0a} = \partial_a \phi + g_a + B_a ,$$

where v is a constant and

$$\bar{g} = (1+m)\, n + 2\,(1+s_\pi)\, e$$

is the *generalized gravoelectromagnetic charge*. Here n is the structure constant (i.e., a volumetric number) which is different from zero for structured particles, s_π is the spin constant, and e is the electric charge (or, more generally, the electromagnetic charge).

Now let the gravitational vorticity tensor be given by

$$\omega_{ab} = \frac{1}{2}\left(\partial_a g_b - \partial_b g_a\right)$$

which vanishes in spherically symmetric (i.e., centrally symmetric) situations. Next, the electromagnetic field tensor is given as usual by

$$f_{ab} = \partial_b B_a - \partial_a B_b .$$

The components of the intrinsic spin tensor can now be written as

$$\bar{F}_{ab} = \omega_{ab} - \frac{1}{2} f_{ab} .$$

As a further consequence, we have $H_{ab} = 0$ and therefore

$$\bar{F}_{ab} = -\frac{1}{2\lambda} \in_{abcd} R^{[cd]} = \frac{mc^2}{\bar{g}} \in^{cd}{}_{ab} R_{[cd]} .$$

The electromagnetic field tensor in our unified field theory is therefore given by

$$f_{ab} = -2\left(\frac{mc^2}{\bar{g}} \in^{cd}{}_{ab} R_{[cd]} - \omega_{ab}\right) .$$

Here we see that when the gravitational spin is present, the electromagnetic field does interact with the gravitational field. Otherwise, in the presence of a centrally symmetric gravitational field we have

$$f_{ab} = -\frac{2mc^2}{\bar{g}} \in^{cd}{}_{ab} R_{[cd]}$$

and there is no physical interaction between gravity and electromagnetism.

3.3 Equations of motion

Now let us take the unit vector field φ to represent the unit velocity vector field, i.e.,

$$\varphi^a = u^a = \frac{dx^a}{ds}$$

where ds is the (infinitesimal) world-line satisfying

$$1 = g_{ab} \frac{dx^a}{ds} \frac{dx^b}{ds} .$$

This selection defines a general material object in our unified field theory as a hypersurface $\sum(t)$ whose world-velocity u is normal to it. Indeed, we will soon see some profound physical consequences.

Invoking this condition, we immediately obtain the following equation of motion:

$$\nabla_a G^{ab} = \frac{1}{2}\gamma\left(u^b \nabla_a u^a + \frac{Du^b}{Ds}\right) - \gamma F^b{}_a \varphi^a$$

where we have used the following relations:

$$\Gamma^c_{(ab)} = \Delta^c_{ab}$$

$$\Gamma^c_{[ab]} = -\frac{1}{2}\gamma \in^c{}_{abd} u^d$$

$$\frac{Du^a}{Ds} = u^b \nabla_b u^a = \frac{du^a}{ds} + \Gamma^a_{(bc)} u^b u^c = \\ = \frac{du^a}{ds} + \Delta^a_{bc} u^b u^c = u^b \hat{\nabla}_a u^a .$$

What happens now if we insist on guaranteeing the conservation of matter and spin? Letting

$$\nabla_a G^{ab} = 0$$

and inserting the value of λ, we obtain the equation of motion

$$\frac{Du^a}{Ds} = -\frac{\bar{g}}{mc^2} \bar{F}^a_b u^b - u^a \nabla_b u^b$$

i.e., the *generalized* Lorentz equation of motion

$$\frac{Du^a}{Ds} = \frac{\bar{g}}{2mc^2} \left(f^a_b - 2\omega^a{}_b\right) u^b - u^a \nabla_b u^b .$$

From the above equation of motion we may derive special equations of motion such as those in the following cases:

1. For an electrically charged, non-spinning, incompressible, structureless (point-like) particle moving in a static, centrally symmetric gravitational field, we have $m \neq 0$, $e \neq 0$, $s_\pi = 0$, $n = 0$, $\nabla_a u^a = 0$, $f_{ab} \neq 0$, $\omega_{ab} = 0$. Therefore its equation of motion is given by
$$\frac{Du^a}{Ds} = \frac{e}{mc^2}\, f^a{}_b\, u^b$$
which is just the standard, relativistically covariant Lorentz equation of motion.

2. For an electrically charged, spinning, incompressible, structureless particle moving in a non-static, spinning gravitational field, we have $m \neq 0$, $e \neq 0$, $s_\pi \neq 0$, $n = 0$, $\nabla_a u^a = 0$, $f_{ab} \neq 0$, $\omega_{ab} \neq 0$. Therefore its equation of motion is given by
$$\frac{Du^a}{Ds} = \frac{(1+s_\pi)}{mc^2}\, e \left(f^a{}_b - 2\omega^a{}_b\right) u^b.$$

3. For a neutral, non-spinning, incompressible, structureless particle moving in a static, centrally symmetric gravitational field, we have $m \neq 0$, $e = 0$, $s_\pi = 0$, $n = 0$, $\nabla_a u^a = 0$, $f_{ab} = 0$, $\omega_{ab} = 0$. Therefore its equation of motion is given by the usual geodesic equation of motion
$$\frac{Du^a}{Ds} = 0\,.$$
In general, this result does not hold for arbitrary incompressible bodies with structure.

4. For a neutral, static, non-spinning, compressible body moving in a static, non-spinning, centrally symmetric gravitational field, we have $m \neq 0$, $e = 0$, $s_\pi = 0$, $n \neq 0$, $\nabla_a u^a \neq 0$, $f_{ab} = 0$, $\omega_{ab} = 0$. Therefore its equation of motion is given by
$$\frac{Du^a}{Ds} = -u^a\, \nabla_b\, u^b$$
which holds for non-Newtonian fluids in classical hydrodynamics.

5. For an electrically charged, non-spinning, compressible body moving in a static, non-spinning, centrally symmetric gravitational field, we have $m \neq 0$, $e \neq 0$, $s_\pi = 0$, $n \neq 0$, $\nabla_a u^a \neq 0$, $f_{ab} \neq 0$, $\omega_{ab} = 0$. Therefore its equation of motion is given by
$$\frac{Du^a}{Ds} = \frac{n\,(1+m)}{mc^2}\, e f^a{}_b\, u^b - u^a\, \nabla_b\, u^b$$
which holds for a variety of classical Maxwellian fluids.

6. For a neutral, spinning, compressible body moving in a non-static, spinning gravitational field, the parametric (structural) condition is given by $m \neq 0$, $e = 0$, $s_\pi, \neq 0$, $n \neq 0$, $\nabla_a u^a \neq 0$, $f_{ab} = 0$, $\omega_{ab} \neq 0$. Therefore its equation of motion is given by
$$\frac{Du^a}{Ds} = -\frac{n\,(1+m)}{mc^2}\, \omega^a{}_b\, u^b - u^a\, \nabla_b\, u^b.$$

Note that the exact equation of motion for massless, neutral particles cannot be directly extracted from the general form of our equation of motion.

We now proceed to give the most general form of the equation of motion in our unified field theory. Using the general identity (see Section 1.3)
$$\nabla_a G^{ab} = 2g^{ab}\Gamma^c_{[da]} R^d{}_c + \Gamma^a_{[cd]} R^{cdb}{}_a$$
we see that
$$\nabla_a G^{ab} = \gamma \left(\in^b{}_{cda}\, R^{[cd]} + \frac{1}{2} \in_{cdea}\, R^{cdeb} \right) u^a.$$

After some algebra, we can show that the above relation can also be written in the form
$$\frac{Du^a}{Ds} = -\in^a{}_{bcd}\, R^{[bc]} u^d.$$

Note that the above general equation of motion is true whether the covariant divergence of the generalized Einstein tensor vanishes or not. Otherwise, let $\Phi^a = \nabla_b G^{ba}$ represent the components of the non-conservative vector of the coupled matter and spin fields. Our equation of motion can then be written alternatively as
$$\frac{Du^a}{Ds} = \frac{1}{2} \in_{bcde}\, R^{bcda} u^e - \gamma\Phi^a.$$

Let us once again consider the conservative case, in which $\Phi^a = 0$. We now have the relation
$$\frac{1}{2} \in_{bcde}\, R^{bcda} u^e = -\frac{2\bar{g}}{mc^2}\, \bar{F}^a{}_b\, u^b - u^a\, \nabla_b\, u^b$$
i.e.,
$$\frac{1}{2} \left(\in_{cdhb}\, R^{cdha} + \frac{4\bar{g}}{mc^2}\, \bar{F}^a{}_b \right) u^b = -u^a\, \nabla_b\, u^b.$$

For a structureless spinning particle, we are left with
$$\left(\in_{cdhb}\, R^{cdha} + \frac{4\,(1+s_\pi)}{mc^2}\, e\bar{F}^a{}_b \right) u^b = 0$$
for which the general solution may read
$$\bar{F}_{ab} = e\, \frac{mc^2}{4\,(1+s_\pi)} \left(\in_{acde}\, R_b{}^{cde} - \in_{bcde}\, R_a{}^{cde} \right) + S_{ab}$$
where $S_{ab} \neq 0$ are the components of a generally asymmetric tensor satisfying
$$S_{ab}\, u^b = -\, e\, \frac{mc^2}{4\,(1+s_\pi)} \in_{acde}\, R_b{}^{cde}\, u^b.$$

In the case of a centrally symmetric gravitational field, this condition should again allow us to determine the electromagnetic field tensor from the curvature tensor alone.

Now, with the help of the decomposition
$$R^d{}_{abc} = C^d{}_{abc} + \frac{1}{2}\left(\delta^d_b R_{ac} + g_{ac} R^d{}_b - \delta^d_c R_{ab} - g_{ab} R^d{}_c\right) + \\ + \frac{1}{6}\left(\delta^d_c\, g_{ab} - \delta^d_b\, g_{ac}\right) R$$

we obtain the relation

$$\in_{bcde} R^{bcda} = \in_{bcde} \left(C^{bcda} + \frac{1}{2} \left(g^{ac} R^{[bd]} - g^{ab} R^{[cd]} \right) \right).$$

However, it can be shown that the last two terms in the above relation cancel each other, since

$$\in_{bcde} g^{ac} R^{[bd]} = \in_{bcde} g^{ab} R^{[cd]} = -\gamma\, g^{ac} \left(\partial_e u_c - \partial_c u_e \right)$$

therefore we are left with the simple relation

$$\in_{bcde} R^{bcda} = \in_{bcde} C^{bcda}.$$

If the space-time under consideration is conformally flat (i.e., ${C^d}_{abc} = 0$), we obtain the following integrability condition for the curvature tensor:

$$\in_{bcde} R^{bcda} = 0\,.$$

It is easy to show that this is generally true if the components of the curvature tensor are of the form

$$R_{abcd} = \frac{1}{12} \left(g_{ac}\, g_{bd} - g_{ad}\, g_{bc} \right) B + P_{abcd}$$

where

$$P_{abcd} = \varepsilon \left(g_{ac}\, g_{bd} - g_{ad}\, g_{bc} \right) \bar{F}_{rs} \bar{F}^{rs}$$

with ϵ being a constant of proportionality. In this case, the generalized Ricci tensor is completely symmetric, i.e.,

$$R_{(ab)} = \frac{1}{4}\, g_{ab} \left(B + 12\,\varepsilon\, \bar{F}_{rs} \bar{F}^{rs} \right) \oplus$$

$$R_{[ab]} = 0\,.$$

We also have

$$R = B + 12\,\varepsilon\, \bar{F}_{ab} \bar{F}^{ab}$$

such that the variation $\delta S = 0$ of the action integral

$$S = \iiiint \sqrt{\det(g)}\, R\, d^4x \; = \; = \iiiint \sqrt{\det(g)} \left(B + 12\,\varepsilon\, \bar{F}_{ab} \bar{F}^{ab} \right) d^4x$$

where $dV = \sqrt{\det(g)}\, dx^0 dx^1 dx^2 dx^3 = \sqrt{\det(g)}\, d^4x$ defines the elementary four-dimensional volume, gives us a set of generalized Einstein-Maxwell equations. Note that in this special situation, the expression for the curvature scalar is true irrespective of whether the Ricci scalar B is constant or not. Furthermore, this gives a generalized Einstein space endowed with a generally non-vanishing spin density. Electromagnetism, in this case, appears to be inseparable from the gravitational vorticity and therefore becomes an emergent phenomenon. Also, the motion then becomes purely geodesic:

$$\frac{du^a}{ds} + \Delta^a_{bc}\, u^b u^c = 0\,,$$

$$\bar{F}_{ab}\, u^b = 0\,.$$

3.4 The conserved gravoelectromagnetic currents of the theory

Interestingly, we can obtain more than one type of conserved gravoelectromagnetic current from the intrinsic spin tensor of the present theory.

We have seen in Section 2.2 that the intrinsic spin tensor in the present theory is given by

$$\bar{F}_{ab} = \frac{mc^2}{\bar{g}} \in^{cd}{}_{ab}\, R_{[cd]}\,.$$

We may note that

$$\hat{j}^a \equiv \hat{\nabla}_b \bar{F}^{ba} = 0$$

which is a covariant "source-free condition" in its own right.

Now, we shall be particularly interested in obtaining the conservation law for the gravoelectromagnetic current in the most general sense.

Define the absolute (i.e., global) gravoelectromagnetic current via the total covariant derivative as follows:

$$j^a \equiv \nabla_b \bar{F}^{ba} = \frac{mc^2}{\bar{g}} \in^{abcd} \nabla_d R_{bc}\,.$$

Now, with the help of the relation

$$\nabla_c \bar{F}_{ab} + \nabla_a \bar{F}_{bc} + \nabla_b \bar{F}_{ca} = -2 \left(\Gamma^d_{[ab]} \bar{F}_{cd} + \Gamma^d_{[bc]} \bar{F}_{ad} + \Gamma^d_{[ca]} \bar{F}_{bd} \right)$$

we see that

$$j^a = -\frac{6mc^2}{\bar{g}}\, g^{ce} \delta^{ab}_{cd} R_{[be]} u^d.$$

Simplifying, we have

$$j^a = \frac{6mc^2}{\bar{g}}\, R^{[ab]} u_b\,.$$

At this moment, we have nothing definitive to say about gravoelectromagnetic charge confinement. We cannot therefore speak of a globally admissible gravoelectromagnetic current density yet. However, we can show that our current is indeed conserved. As a start, it is straightforward to see that we have the relative conservation law

$$\hat{\nabla}_a j^a = 0\,.$$

Again, this is not the most desired conservation law as we are looking for the most generally covariant one.

Now, with the help of the relations

$$\in^{abcd} \nabla_c F_{ab} = - \in^{abcd} \left(\Gamma^e_{[ac]} F_{eb} + \Gamma^e_{[bc]} F_{ae} \right)$$

$$\Gamma^a_{[bc]} = -\frac{1}{2}\, \gamma \in^a{}_{bcd}\, u^d$$

we obtain

$$\nabla_a R^{[ab]} = -2 F^{ab} u_a\,.$$

Therefore
$$u_b \nabla_a R^{[ab]} = 0\,.$$

Using this result together with the fact that
$$R^{[ab]} \nabla_a u_b = -\frac{1}{2}\,\gamma \in^{abcd} F_{ab} F_{cd} = 0$$
we see that
$$\nabla_a j^a = \frac{6mc^2}{\bar{g}} \left(u_b \nabla_a R^{[ab]} + R^{[ab]} \nabla_a u_b \right) = 0$$
i.e., our gravoelectromagnetic current is conserved in a fully covariant manner.

Let us now consider a region in our space-time manifold in which the gravoelectromagnetic current vanishes. We have, from the boundary condition $j^a = 0$, the governing equation
$$R_{[ab]} u^b = 0$$
which is equivalent to the following integrability condition:
$$\in^{abcd} u_a \left(\partial_c u_d - \partial_d u_c \right) = 0\,.$$

In three dimensions, if in general $\mathrm{curl} u \neq 0$, this gives the familiar integrability condition
$$u \cdot \mathrm{curl}\, u = 0$$
where the dot represents three-dimensional scalar product.

We are now in a position to define the phenomenological gravoelectromagnetic current density which shall finally allow us to define gravoelectromagnetic charge confinement. However, in order to avoid having extraneous sources, we do not in general expect such confinement to hold globally. From our present perspective, what we need is a relative (i.e., local) charge confinement which can be expressed solely in geometric terms.

Therefore we first define the spin tensor density (of weight +2) as
$$\bar{f}^{ab} \equiv \det(g)\, \bar{F}^{ab} = \frac{mc^2}{\bar{g}} \sqrt{\det(g)}\, \varepsilon^{abcd} R_{[cd]}\,.$$

The phenomenological (i.e., relative) gravoelectromagnetic current density is given here by
$$\bar{j}^a = \partial_b \bar{f}^{ab} = \frac{mc^2}{\bar{g}} \left(\partial_b \sqrt{\det(g)} \right) \varepsilon^{abcd} R_{[cd]}$$
i.e.,
$$\bar{j}^a = \frac{mc^2}{2\bar{g}}\, \varepsilon^{abcd} g^{rs} \left(\partial_b g_{rs} \right) R_{[cd]}\,.$$

Meanwhile, using the identity
$$\partial_a g^{bc} = -g^{br} g^{cs} \partial_a g_{rs}$$
we see that
$$\left(\partial_a g^{rs} \right) \left(\partial_b g_{rs} \right) = \left(\partial_a g_{rs} \right) \left(\partial_b g^{rs} \right).$$

Using this result and imposing continuity on the metric tensor, we finally see that
$$\partial_a \bar{j}^a = \frac{mc^2}{2\bar{g}}\, \varepsilon^{abcd} \times$$
$$\times \left(\frac{1}{2} g^{rs} g^{pq} \left(\partial_a g_{rs} \right) \left(\partial_b g_{pq} \right) - \left(\partial_a g^{rs} \right) \left(\partial_b g_{rs} \right) \right) R_{[cd]} = 0$$
which is the desired local conservation law. In addition, it is easy to show that
$$\hat{\nabla}_a \bar{j}^a = 0\,.$$

Unlike the geometric current represented by j^a, the phenomenological current density given by $\bar{j}^a$ corresponds directly to the hydrodynamical analogue of a gravoelectromagnetic current density if we set
$$\bar{j}^a = \det(g)\, \rho\, u^a$$
which defines charge confinement in our gravoelectrodynamics. Combining this relation with the previously given equivalent expression for j^a, we obtain
$$\rho = \frac{mc^2}{2\bar{g}} \in^{abcd} u_a\, g^{rs} \left(\partial_b g_{rs} \right) R_{[cd]}$$
i.e.,
$$\rho = \frac{mc^2}{\bar{g}} \in^{abcd} u_a \Gamma^h_{hb} R_{[cd]}$$
for the gravoelectromagnetic charge density. Note that this is a pseudo-scalar.

At this point, it becomes clear that the gravoelectromagnetic charge density is generated by the properties of the curved space-time itself, i.e., the non-unimodular character of the space-time geometry, for which $\sqrt{\det(g)} = 1$ and $\Gamma^h_{hb} \neq 0$, and the twist (intrinsic spin) of space-time which in general causes material points (whose characteristics are given by $\bar{g}$) to rotate on their own axes such that in a finite region in the space-time manifold, an "individual" energy density emerges. Therefore, in general, a material body is simply a collection of individual material points confined to interact gravoelectrodynamically with each other in a finite region in our curved space-time. More particularly, this can happen in the absence of either the electromagnetic field or the gravitational vorticity, but not in the absence of both fields. To put it more simply, it requires both local curvature and twist to generate a material body out of an energy field.

4 Final remarks

At this point, we may note that we have not considered the conditions for the balance of spin (intrinsic angular momentum) in detail. This may be done, in a straightforward manner, by simply expressing the anti-symmetric part of the generalized Ricci tensor in terms of the so-called spin density tensor as well as the couple stress tensor. This can then be used to develop a system of equations governing the balance of energy-momentum in our theory. Therefore, we also need to obtain a formal representation for the energy-momentum tensor in terms of the four-momentum vector. This way, we obtain a set of constitutive equations which characterize the theory.

This work has simply been founded on the feeling that it *could* be physically correct as a unified description of physical phenomena due to its manifest simplicity. Perhaps there remains nothing more beyond the simple appreciation of that possibility. It is valid for a large class of particles and (space-time) continua in which the coordinate points themselves are allowed to rotate and translate. Since the particles are directly related to the coordinate points, they are but intrinsic objects in the space-time manifold, just as the fields are.

It remains, therefore, to consider a few physically meaningful circumstances in greater detail for the purpose of finding particular solutions to the semi-symmetric field equations of our theory.

Acknowledgements

I am indebted to my dear friends S. J. Crothers and D. Rabounski and also Prof. B. Suhendro, Prof. S. Hwang, and Prof. S. Antoci for the enlightening discussions in a large number of seemingly separate theoretical and experimental disciplines. I would also like to sincerely express my gratitude for their continuous moral support.

Submitted on July 02, 2007
Accepted on July 16, 2007

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Optical-Fiber Gravitational Wave Detector: Dynamical 3-Space Turbulence Detected

Reginald T. Cahill

School of Chemistry, Physics and Earth Sciences, Flinders University, Adelaide 5001, Australia

E-mail: Reg.Cahill@flinders.edu.au

Preliminary results from an optical-fiber gravitational wave interferometric detector are reported. The detector is very small, cheap and simple to build and operate. It is assembled from readily available opto-electronic components. A parts list is given. The detector can operate in two modes: one in which only instrument noise is detected, and data from a 24 hour period is reported for this mode, and in a 2nd mode in which the gravitational waves are detected as well, and data from a 24 hour period is analysed. Comparison shows that the instrument has a high S/N ratio. The frequency spectrum of the gravitational waves shows a pink noise spectrum, from 0 to 0.1 Hz.

1 Introduction

Preliminary results from an optical-fiber gravitational wave interferometric detector are reported. The detector is very small, cheap and simple to build and operate, and is shown in Fig. 1. It is assembled from readily available opto-electronic components, and is suitable for amateur and physics undergraduate laboratories. A parts list is given. The detector can operate in two modes: one in which only instrumental noise is detected, and the 2nd in which the gravitational waves are detected as well. Comparison shows that the instrument has a high S/N ratio. The frequency spectrum of the gravitational waves shows a pink noise spectrum, from essentially 0 to 0.1 Hz. The interferometer is 2nd order in v/c and is analogous to a Michelson interferometer. Michelson interferometers in vacuum mode cannot detect the light-speed anisotropy effect or the gravitational waves manifesting as light-speed anisotropy fluctuations. The design and operation as well as preliminary data analysis are reported here so that duplicate detectors may be constructed to study correlations over various distances. The source of the gravitational waves is unknown, but a 3D multi-interferometer detector will soon be able to detect directional characteristics of the waves.

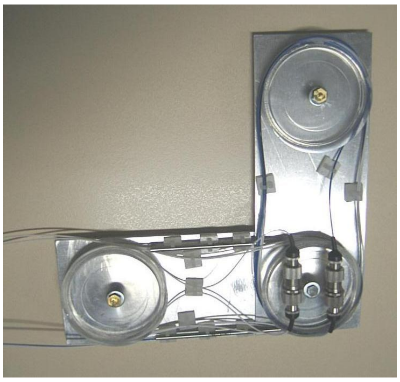

Fig. 1: Photograph of the detector showing the fibers forming the two orthogonal arms. See Fig. 2 for the schematic layout. The beam splitter and joiner are the two small stainless steel cylindrical tubes. The two FC to FC mating sleeves are physically adjacent, and the fibers can be re-connected to change from Mode A (Active detector — gravitational wave and device noise detection) to Mode B (Background — device noise measurements only). The overall dimensions are 160mm×160mm. The 2×2 splitter and joiner each have two input and two output fibers, with one not used.

2 Light speed anisotropy

In 2002 it was reported [1, 2] that light-speed anisotropy had been detected repeatedly since the Michelson-Morley experiment of 1887 [3]. Contrary to popular orthodoxy they reported a light-speed anisotropy up to 8 km/s based on their analysis of their observed fringe shifts. The Michelson-Morley experiment was everything except *null*. The deduced speed was based on Michelson's Newtonian-physics calibration for the interferometer. In 2002 the necessary special relativity effects and the effects of the air present in the light paths were first taken into account in calibrating the interferometer. This reanalysis showed that the actual observed fringe shifts corresponded to a very large light-speed anisotropy, being in excess of 1 part in 1000 of $c = 300{,}000$ km/s. The existence of this light-speed anisotropy is not in conflict with the successes of Special Relativity, although it is in conflict with Einstein's postulate that the speed of light is invariant. This large light-speed anisotropy had gone unnoticed throughout the twentieth century, although we now know that it was detected in seven experiments, ranging from five 2nd order in v/c gas-mode Michelson-interferometer experiments [3–7] to two 1st order in v/c one-way RF coaxial cable travel-speed measurements using atomic clocks [8, 9]. In 2006 another RF travel time coaxial cable experiment was performed [10]. All eight light-speed anisotropy experiments agree [11, 12]. Remarkably five of these experiments [3, 4, 8, 9, 10] reveal pronounced *gravitational wave* effects, where the meaning of this term is explained below. In particular detailed analysis of

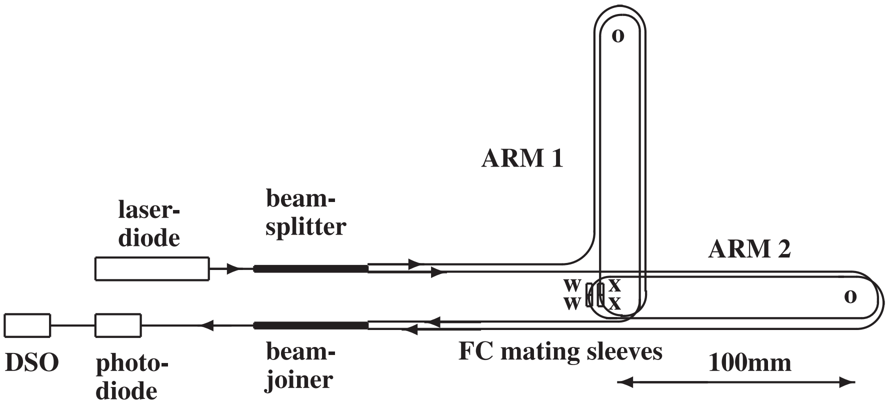

Fig. 2: Schematic layout of the interferometric optical-fiber light-speed anisotropy/gravitational wave detector (in Mode **A**). Actual detector is shown in Fig.1, with ARM2 located to the left, so as to reduce lengths of fiber feeds and overall size. Coherent 650 nm light from the laser diode is split into two 1m length single-mode polarisation preserving fibers by the beam splitter. The two fibers take different directions, ARM1 and ARM2, after which the light is recombined in the beam joiner, which also has 1m length fibers, in which the phase differences lead to interference effects that are indicated by the outgoing light intensity, which is measured in the photodiode, and then recorded in the Digital Storage Oscilloscope (DSO). In Mode **A** the optical fibers are joined $x - x$ and $w - w$ at the FC to FC mating sleeves, as shown. In the actual layout the fibers make four loops in each arm, and the length of one straight section is 100 mm, which is the center to center spacing of the plastic turners, having diameter = 52 mm, see Fig. 1. The two FC to FC mating sleeves are physically adjacent and by re-connecting the fibers as $x - w$ and $w - x$ the light paths can be made symmetrical wrt the arms, giving Mode **B**, which only responds to device noise — the **B**ackground mode. In Mode **A** the detector is **A**ctive, and responds to both flowing 3-space and device noise. The relative travel times, and hence the output light intensity, are affected by the fluctuating speed and direction of the flowing 3-space, by affecting differentially the speed of the light, and hence the net phase difference between the two arms.

the Michelson-Morley fringe shift data shows that they not only detected a large light-speed anisotropy, but that their data also reveals large wave effects [12]. The reason why their interferometer could detect these phenomena was that the light paths passed through air; if a Michelson interferometer is operated in vacuum then changes in the geometric light-path lengths exactly cancel the Fitzgerald-Lorentz arm-length contraction effects. This cancellation is incomplete when a gas is present in the light paths. So modern vacuum Michelson interferometers are incapable of detecting the large light-speed anisotropy or the large gravitational waves. Here we detail the construction of a simple optical-fiber light-speed anisotropy detector, with the main aim being to record and characterise the gravitational waves. These waves reveal a fundamental aspect to reality that is absent in the prevailing models of reality.

3 Dynamical 3-Space and gravitational waves

The light-speed anisotropy experiments reveal that a dynamical 3-space exists, with the speed of light being c only wrt to this space: observers in motion "through" this 3-space detect that the speed of light is in general different from c, and is different in different directions. The notion of a dynamical 3-space is reviewed in [11, 12]. The dynamical equations for this 3-space are now known and involve a velocity field $\mathbf{v}(\mathbf{r}, t)$, but where only relative velocities are observable. The coordinates $\mathbf{r}$ are relative to a non-physical mathematical embedding space. These dynamical equations involve Newton's gravitational constant G and the fine structure constant α. The discovery of this dynamical 3-space then required a generalisation of the Maxwell, Schrödinger and Dirac equations. In particular these equations showed that the phenomenon of gravity is a wave refraction effect, for both EM waves and quantum matter waves [12, 13]. This new physics has been confirmed by explaining the origin of gravity, including the Equivalence Principle, gravitational light bending and lensing, bore hole g anomalies, spiral galaxy rotation anomalies (so doing away with the need for dark matter), black hole mass systematics, and also giving an excellent parameter-free fit to the supernovae and gamma-ray burst Hubble expansion data [14] (so doing away with the need for dark energy). It also predicts a novel spin precession effect in the GPB satellite gyroscope experiment [15]. This physics gives an explanation for the successes of the Special Relativity formalism, and the geodesic formalism of General Relativity. The wave effects already detected correspond to fluctuations in the 3-space velocity field $\mathbf{v}(\mathbf{r}, t)$, so they are really 3-space turbulence or wave effects. However they are better known, if somewhat inappropriately as "gravitational waves" or "ripples" in "spacetime". Because the 3-space dynamics gives a deeper understanding of the spacetime formalism, we now know that the metric of the induced spacetime, merely a mathematical construct having no ontological significance, is related to $\mathbf{v}(\mathbf{r}, t)$ according to [11, 12]

$$ds^2 = dt^2 - \frac{\left(d\mathbf{r} - \mathbf{v}(\mathbf{r}, t)dt\right)^2}{c^2} = g_{\mu\nu}\, dx^\mu dx^\nu. \qquad (1)$$

The gravitational acceleration of matter, and of the structural patterns characterising the 3-space, is given by [12, 13]

$$\mathbf{g} = \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\,\mathbf{v} \tag{2}$$

and so fluctuations in $\mathbf{v}(\mathbf{r}, t)$ may or may not manifest as a gravitational force. The general characteristics of $\mathbf{v}(\mathbf{r}, t)$ are now known following the detailed analysis of the eight experiments noted above, namely its average speed, over an hour or so, of some 420±30 km/s, and direction RA = 5.5±2$^{\rm hr}$, Dec = 70±10° S, together with large wave/turbulence effects. The magnitude of this turbulence depends on the timing resolution of each particular experiment, and here we report that the speed fluctuations are very large, as also seen in [10] Here we employ a new detector design that enables a detailed study of $\mathbf{v}(\mathbf{r}, t)$, and with small timing resolutions. A key experimental test of the various detections of $\mathbf{v}(\mathbf{r}, t)$ is that the data shows that the time-averaged $\mathbf{v}(\mathbf{r}, t)$ has a direction that has a specific Right Ascension and Declination as given above, i.e. the time for say a maximum averaged speed depends on the local sidereal time, and so varies considerably throughout the year, as do the directions to all astronomical processes/objects. This sidereal effect constitutes an absolute proof that the direction of $\mathbf{v}(\mathbf{r}, t)$ and the accompanying wave effects are real "astronomical" phenomena, as there is no known earth-based effect that can emulate the sidereal effect.

4 Gravitational wave detector

To measure $\mathbf{v}(\mathbf{r}, t)$ has been difficult until now. The early experiments used gas-mode Michelson interferometers, which involved the visual observation of small fringe shifts as the relatively large devices were rotated. The RF coaxial cable experiments had the advantage of permitting electronic recording of the RF travel times, over 500 m [8] and 1.5km [9], by means of two or more atomic clocks, although the experiment reported in [10] used a novel technique that enable the coaxial cable length to be reduced to laboratory size. The new detector design herein has the advantage of electronic recording as well as high precision because the travel time differences in the two orthogonal fibers employ light interference effects, with the interference effects taking place in an optical beam-joiner, and so no optical projection problems arise. The device is very small, very cheap and easily assembled from readily available opto-electronic components. The schematic layout of the detector is given in Fig. 2, with a detailed description in the figure caption. The detector relies on the phenomenon where the 3-space velocity $\mathbf{v}(\mathbf{r}, t)$ affects differently the light travel times in the optical fibers, depending on the projection of $\mathbf{v}(\mathbf{r}, t)$ along the fiber directions. The differences in the light travel times are measured by means of the interference effects in the beam joiner. However at present the calibration constant k of the device is not yet known, so it is not yet known what speed corresponds to the measured time difference Δt, although comparison with the earlier experiments gives a guide. In general we expect

$$\Delta t = k^2 \frac{L v_P^2}{c^3} \cos\big(2(\theta - \psi)\big) \tag{3}$$

where k is the instrument calibration constant. For gas-mode Michelson interferometers k is known to be given by $k^2 \approx \approx n^2 - 1$, where n is the refractive index of the gas. Here $L = 4 \times 100$ mm is the effective arm length, achieved by having four loops of the fibers in each arm, and v_P is the projection of $\mathbf{v}(\mathbf{r}, t)$ onto the plane of the detector. The angle θ is that of the arm relative to the local meridian, while ψ is the angle of the projected velocity, also relative to the local meridian. A photograph of the prototype detector is shown in Figure 1.

A key component is the light source, which can be the laser diode listed in the Table of parts. This has a particularly long coherence length, unlike most cheap laser diodes, although the data reported herein used a more expensive He-Ne laser. The other key components are the fiber beam splitter/joiner, which split the light into the fibers for each arm, and recombine the light for phase difference measurements by means of the fiber-joiner and photodiode detector and amplifier. A key feature of this design is that the detector can oper-

Parts	**Thorlabs** http://www.thorlabs.com/
1x Si Photodiode Detector/ Amplifier/Power Supply	PDA36A or PDA36A-EC select for local AC voltage
1xFiber Adaptor for above	SM1FC
1xFC Fiber Collimation Pkg	F230FC-B
1xLens Mounting Adaptor	AD1109F
2xFC to FC Mating Sleeves	ADAFC1
2x 2x2 Beam Splitters	FC632-50B-FC
Fiber Supports	PFS02
	Midwest Laser Products http://www.midwest-laser.com/
650nm Laser Diode Module	VM65003
LDM Power Supply/3VDC	**Local Supplier or Batteries**
BNC 50Ω coaxial cable	**Local Supplier**
	PoLabs
PoScope USB DSO	http://www.poscope.com/

Table 1: List of parts and possible suppliers for the detector. The FC Collimation Package and Lens Mounting Adaptor together permit the coupling of the Laser Diode Module to the optical fiber connector. This requires unscrewing the lens from the Laser Diode Module and screwing the diode into above and making judicious adjustment to maximise light coupling. The coaxial cable is required to connect the photodiode output to the DSO. Availability of a PC to host the USB DSO is assumed. The complete detector will cost ≈ \$1100 US dollars.

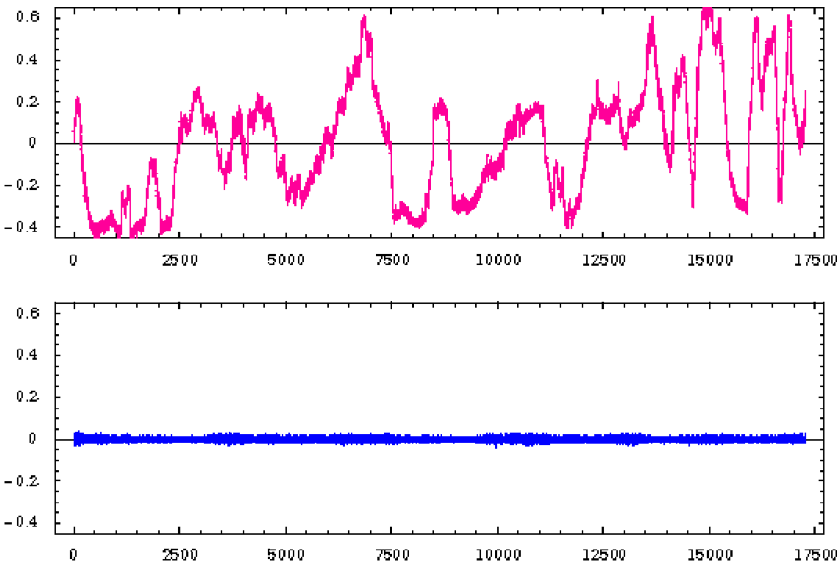

Fig. 3: Photodiode voltages over a 24 hour period with data recording every 5 s, with the detector arms orientated in a NS-EW direction and horizontal. Upper plot (red) is for detector in Mode A, i.e responding to 3-space dynamics and instrument noise, while lower plot (blue) is for Mode B in which detector only responds to instrumental noise, and demonstrates the high S/N ratio of the detector. The lower plot is dominated by higher frequency noise, as seen in the frequency spectrum in Fig. 5. A selection of the above data over a 1 hour time interval, from time steps 4900 to 5620, is shown in Fig. 4 indicating details of the 3-space wave forms.

ate in two different modes. In Mode **A** the detector is **A**ctive, and responds to both flowing 3-space and device noise. Because the two fiber coupler (FC) mating sleeves are physically adjacent a re-connection of the fibers at the two mating sleeves makes the light paths symmetrical wrt the arms, and then the detector only responds to device noise; this is the **B**ackground mode. The data stream may be mostly cheaply recorded by a PoScope USB Digital Storage Oscilloscope (DSO) that runs on a PC.

The interferometer operates by detecting the travel time difference between the two arms as given by (3). The cycle-averaged light intensity emerging from the beam joiner is given by

$$I(t) \propto \left|\mathbf{E}_1 e^{i\omega t} + \mathbf{E}_2 e^{i\omega(t+\tau+\Delta t)}\right|^2 = |\mathbf{E}|^2 \cos\left(\frac{\omega(\tau+\Delta t)}{2}\right)^2 \approx a + b\Delta t\,. \qquad (4)$$

Here $\mathbf{E}_i$ are the electric field amplitudes and have the same value as the fiber splitter/joiner are 50%-50% types, and having the same direction because polarisation preserving fibers are used, ω is the light angular frequency and τ is a travel time difference caused by the light travel times not being identical, even when $\Delta t = 0$, mainly because the various splitter/joiner fibers will not be identical in length. The last expression follows because Δt is small, and so the detector operates in a linear regime, in general, unless τ has a value equal to modulo(T), where T is the light period. The main temperature effect in the detector is that τ will be temperature dependent. The photodiode detector output voltage $V(t)$ is proportional to $I(t)$, and so finally linearly related to Δt. The detector calibration constants a and b depend on k and τ, and are unknown at present, and indeed τ will be instrument dependent. The results reported herein show that the value of the calibration constant b is not given by using the effective refractive index of the optical fiber in (3), with b being much smaller than that calculation would suggest. This is in fact very fortunate as otherwise the data would be affected by the need to use the cosine form in (4), and thus would suffer from modulo effects. It is possible to determine the voltages for which (4) is in the non-linear regime by spot heating a segment of one fiber by touching with a finger, as this produces many full fringe shifts.

By having three mutually orthogonal optical-fiber interferometers it is possible to deduce the vectorial direction of $\mathbf{v}(\mathbf{r},t)$, and so determine, in particular, if the pulses have any particular direction, and so a particular source. The simplicity of this device means that an international network of detectors may be easily set up, primarily to test for correlations in the waveforms.

5 Data analysis

Photodiode voltage readings from the detector in Mode **A** on July 11, 2007, from approximately 12:30pm local time for 24 hours, and in Mode **B** June 24 from 4pm local time for 24 hours, are shown in Fig. 3, with an arbitrary zero. The photodiode output voltages were recorded every 5 s. Most importantly the data are very different, showing that only in Mode **A** are gravitational waves detected, and with a high S/N

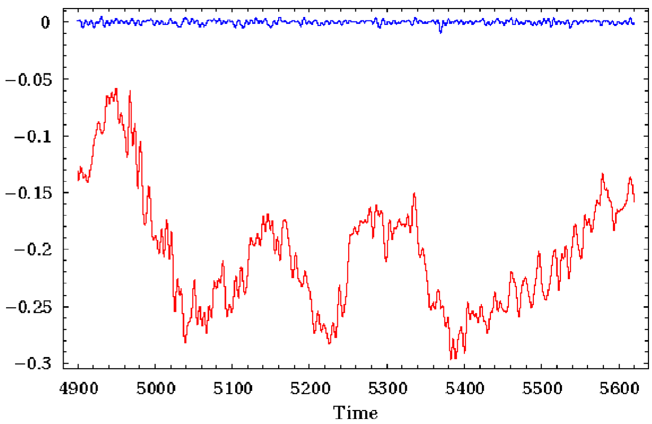

Fig. 4: Lower plot (red) shows the time series data over a 1 hour period, from time steps 4900 to 5620 in Fig. 3, showing the wave forms present in Fig. 3 in greater detail. Similar complex wave forms were seen in [10]. These plots were reconstructed from the FT after band passing the frequencies (1–3000)$\times$1.16$\times 10^{-5}$Hz=(0.000116–0.034) Hz to reduce the instrument noise component, which is very small as shown in upper plot (blue).

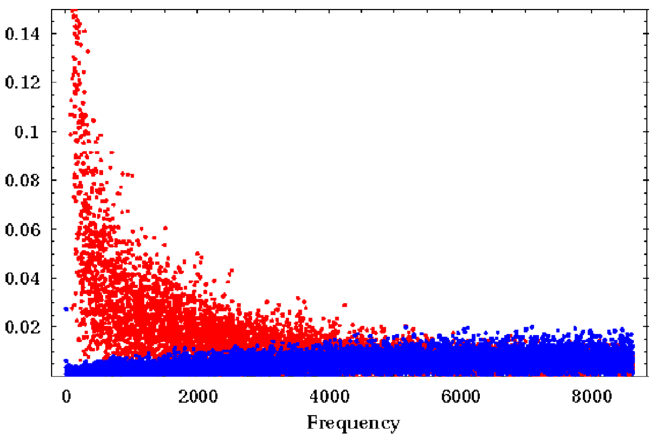

Fig. 5: Two plots of $|\widetilde{V}_s|$ from Fast Fourier Transforms of the photodiode detector voltage V_r at 5 second intervals for 24 fours. Frequency step corresponds to 1.157×10^{-5}Hz. Upper frequency spectrum (red) is for detector in Mode A, i.e responding to 3-space dynamics and instrument noise, while lower spectrum (blue) is for Mode B in which detector only responds to instrumental noise. We see that the signal in Mode A is very different from that Mode B operation, showing that the S/N ratio for the detector is very high. The instrumental noise has a mild "blue" noise spectrum, with a small increase at higher frequencies, while the 3-space turbulence has a distinctive "pink" noise spectrum.

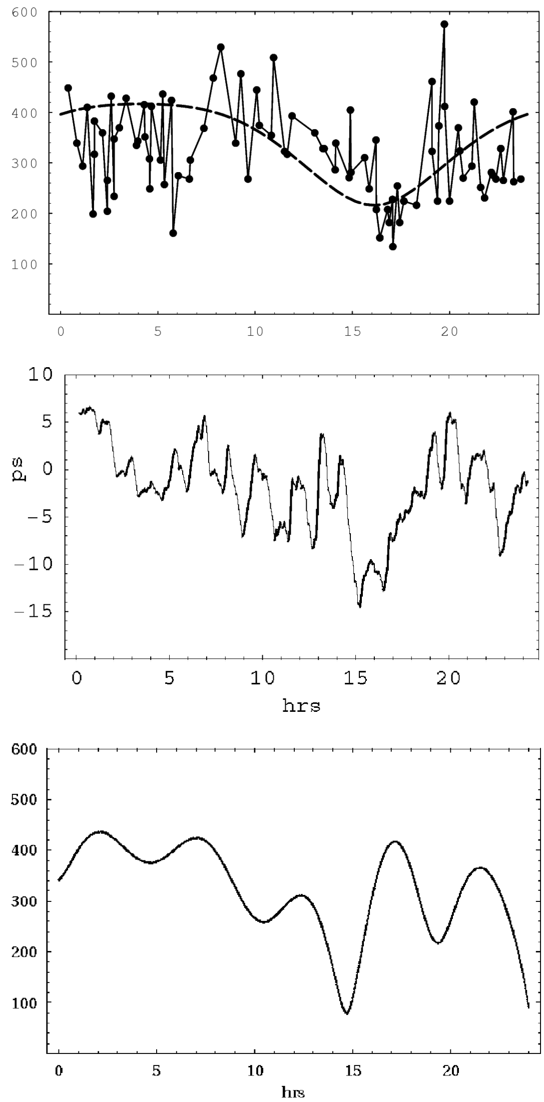

Fig. 6: *Top*: Absolute projected speeds v_P in the Miller experiment plotted against sidereal time in hours for a composite day collected over a number of days in September 1925. Maximum projected speed is 417 km/s. The data shows considerable fluctuations. The dashed curve shows the non-fluctuating variation expected over one day as the Earth rotates, causing the projection onto the plane of the interferometer of the velocity of the average direction of the 3-space flow to change. *Middle*: Data from the Cahill experiment [10] for one sidereal day on approximately August 23, 2006. We see similar variation with sidereal time, and also similar wave structure. This data has been averaged over a running 1hr time interval to more closely match the time resolution of the Miller experiment. These fluctuations are real wave phenomena of the 3-space. *Bottom*: Data from the optical-fiber experiment herein with only low frequencies included to simulate the time averaging in the other two experiments. Comparison permits an approximate calibration for the optical fiber detector, as indicated by the speed in km/s.

ratio. A 1 hour time segment of that data is shown in Fig. 4. In that plot the higher frequencies have been filtered out from both data time series, showing the exceptional S/N ratio that can be achieved.

In Fig. 5 the Fourier Transfoms of the two data time series are shown, again revealing the very different characteristics of the data from the two operating modes. The instrumental noise has a mild "blue" noise spectrum, with a small increase at higher frequencies, while the 3-space turbulence has a distinctive "pink" noise spectrum, and ranging essentially from 0 to 0.1 Hz. The FT is defined by

$$\widetilde{V}_s = \frac{1}{\sqrt{n}} \sum_{r=1}^{n} V_r e^{2\pi i(r-1)(s-1)/n} \tag{5}$$

where $n = 17280$ corresponds to a 5 s timing interval over 24 hours.

By removing all but the FT amplitudes 1–10, and then inverse Fourier Transforming we obtain the slow changes occurring over 24 hours. The resulting data has been presented in terms of possible values for the projected speed v_P in (3), and is shown in Fig. 6 and plotted against sidereal time, after adjusting the unknown calibration constants to give a form resembling the Miller and coaxial cable experimental results so as to give some indication of the calibration of the detector. The experiment was run in an unoccupied office in which temperatures varied by some 10°C over the 24 hour periods. In future temperature control will be introduced.

6 Conclusions

As reviewed in [11, 12] gravitational waves, that is, fluctuations or turbulence in the dynamical 3-space, have been detected since the 1887 Michelson-Morley experiment, although this all went unrealised until recently. As the timing resolution improved over the century, from initially one hour to seconds now, the characteristics of the turbulence of the dynamical 3-space have become more apparent, and that at smaller timing resolutions the turbulence is seen to be very large. As shown herein this wave/pulse phenomenon is very easy to detect, and opens up a whole new window on the universe. The detector reported here took measurements every 5 s, but can be run at millisecond acquisition rates. A 3D version of the detector with three orthogonal optical-fiber interferometers will soon become operational. This will permit the determination of the directional characteristics of the 3-space pulses.

That the average 3-space flow will affect the gyroscope precessions in the GP-B satellite experiment through vorticity effects was reported in [15]. The fluctuations are also predicted to be detectable in that experiment as noted in [16]. However the much larger fluctuations detected in [10] and herein imply that these effects will be much large than reported in [16] where the time averaged waves from the DeWitte experiment [9] were used; essentially the gyro precessions will appear to have a large stochasticity.

Special thanks to Peter Morris, Thomas Goodey, Tim Eastman, Finn Stokes and Dmitri Rabounski.

Submitted on July 09, 2007
Accepted on July 16, 2007

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On the “Size” of Einstein’s Spherically Symmetric Universe

Stephen J. Crothers

Queensland, Australia

E-mail: thenarmis@yahoo.com

It is alleged by the Standard Cosmological Model that Einstein’s Universe is finite but unbounded. Although this is a longstanding and widespread allegation, it is nonetheless incorrect. It is also alleged by this Model that the Universe is expanding and that it began with a Big Bang. These are also longstanding and widespread claims that are demonstrably false. The FRW models for an expanding, finite, unbounded Universe are inconsistent with General Relativity and are therefore invalid.

1 Historical basis

Non-static homogeneous models were first investigated theoretically by Friedmann in 1922. The concept of the Big Bang began with Lemaître, in 1927, who subsequently asserted that the Universe, according to General Relativity, came into existence from a “primaeval atom”.

Following Friedmann, the work of Robertson and Walker resulted in the FRW line-element,

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right],$$

from which is obtained the so-called “Friedmann equation”,

$$\dot{R}^2 + k = \frac{8\pi G}{3}\,\rho R^2,$$

where ρ is the macroscopic proper density of the Universe and k a constant. Applying the continuity condition $T^{\mu\nu}{}_{;\mu} = 0$, to the stress tensor $T_{\mu\nu}$ of a perfect fluid

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - p\,g_{\mu\nu},$$

where p is the pressure and u_μ the covariant world velocity of the fluid particles, the equation of continuity becomes

$$R\dot{\rho} + 3\dot{R}(\rho + p) = 0\,.$$

With the *ad hoc* assumption that $R(0) = 0$, the Friedmann equation is routinely written as

$$\dot{R}^2 + k = \frac{A^2}{R}\,,$$

where A is a constant. The so-called “Friedmann models” are:

(1) $k = 0$ — the flat model,

(2) $k = 1$ — the closed model,

(3) $k = -1$ — the open model,

wherein $t = 0$ is claimed to mark the beginning of the Universe and $R(0) = 0$ the cosmological singularity.

Big Bang and expansion now dominate thinking in contemporary cosmology. However, it is nonetheless easily proved that such cosmological models, insofar as they relate to the FRW line-element, with or without embellishments such as “inflation”, are in fact inconsistent with the mathematical structure of the line-elements from which they are alleged, and are therefore false.

2 Spherically symmetric metric manifolds

A 3-D spherically symmetric metric manifold has, in the spherical-polar coordinates, the following form ([1, 2]),

$$ds^2 = B(R_c)dR_c^2 + R_c^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\,, \qquad (1)$$

where $B(R_c)$ and $R_c = R_c(r)$ are *a priori* unknown analytic functions of the variable r of the simple line element

$$ds^2 = dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\,, \qquad (2)$$

$$0 \leqslant r \leqslant \infty\,.$$

Line elements (1) and (2) have precisely the same fundamental geometric form and so the geometric relations between the components of the metric tensor are exactly the same in each line element. The quantity R_c appearing in (1) is not the geodesic radial distance associated with the manifold it describes. It is in fact the *radius of curvature*, in that it determines the Gaussian curvature $G = 1/R_c^2$ (see [1, 2]). The geodesic radial distance distance, R_p, from an arbitrary point in the manifold described by (1) is an intrinsic geometric property of the line element, and is given by

$$R_p = \int \sqrt{B(R_c)}\, dR_c + C = \int \sqrt{B(R_c)}\, \frac{dR_c}{dr} dr + C\,,$$

where C is a constant of integration to be determined ([2]). Therefore, (1) can be written as

$$ds^2 = dR_p^2 + R_c^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\,,$$

where

$$dR_p = \sqrt{B(R_c)}\, dR_c\,,$$

and

$$0 \leqslant R_p < \infty\,,$$

with the possibility of the line element being singular (undefined) at $R_p = 0$, since $B(R_c)$ and $R_c = R_c(r)$ are *a priori* unknown analytic functions of the variable r. In the case of (2),

$$R_c(r) \equiv r, \quad dR_p \equiv dr, \quad B(R_c(r)) \equiv 1\,,$$

from which it follows that $R_c \equiv R_p \equiv r$ in the case of (2). Thus $R_c \equiv R_p$ is not general, and only occurs in the special case of (2), which describes an Efcleethean* space.

The volume V of (1), and therefore of (2), is

$$V = \int_0^{R_p} R_c^2 \, dR_p \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\varphi =$$
$$= 4\pi \int_{R_c(0)}^{R_c(r)} R_c^2(r) \sqrt{B(R_c(r))}\, dR_c(r) =$$
$$= 4\pi \int_0^{r} R_c^2(r) \sqrt{B(R_c(r))}\, \frac{dR_c(r)}{dr}\, dr\,,$$

although, in the general case (1), owing to the *a priori* unknown functions $B(R_c(r))$ and $R_c(r)$, the line element (1) may be undefined at $R_p(R_c(0)) = R_p(r = 0) = 0$, which is the location of the centre of spherical symmetry of the manifold of (1) at an arbitrary point in the manifold. Also, since $R_c(r)$ is *a priori* unknown, the value of $R_c(0)$ is unknown and so it cannot be assumed that $R_c(0) = 0$. In the special case of (2), both $B(R_c(r))$ and $R_c(r)$ *are known*.

Similarly, the surface area S of (1), and hence of (2), is given by the general expression,

$$S = R_c^2(r) \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\varphi = 4\pi R_c^2(r)\,.$$

This might not ever be zero, since, once again, $R_c(r)$ is an *a priori* unknown function and so $R_c(0)$ might not be zero. It all depends an the explicit form for $R_c(r)$, if it can be determined in a given situation, and on associated boundary conditions. References [1, 2] herein describe the mathematics in more detail.

3 The "radius" of Einstein's universe

Since a geometry is entirely determined by the *form* of its line element [3], everything must be determined from it. One cannot, as is usually done, merely foist assumptions upon it. The *intrinsic* geometry of the line element and the consequent geometrical relations between the components of the metric tensor determine all.

Consider the usual non-static cosmological line element

$$ds^2 = dt^2 - \frac{e^{g(t)}}{\left(1+\frac{k}{4}\,\bar{r}^2\right)^2}\left[d\bar{r}^2 + \bar{r}^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\right], \quad (3)$$

wherein it is usually simply *assumed* that $0 \leqslant \bar{r} < \infty$ [3–6]. However, the range on $\bar{r}$ must be determined, not assumed. It is easily proved that the foregoing usual assumption is patently false.

Once again note that in (3) the quantity $\bar{r}$ is not a radial geodesic distance. In fact, it is not even a radius of curvature on (3). It is merely a parameter for the radius of curvature and the proper radius, both of which are well-defined by the *form* of the line element (describing a spherically symmetric metric manifold). The radius of curvature, R_c, for (3), is

$$R_c = e^{\frac{1}{2}g(t)} \frac{\bar{r}}{1 + \frac{k}{4}\,\bar{r}^2}\,. \quad (4)$$

The proper radius for (3) is given by

$$R_p = e^{\frac{1}{2}g(t)} \int \frac{d\bar{r}}{1 + \frac{k}{4}\,\bar{r}^2} =$$
$$= \frac{2e^{\frac{1}{2}g(t)}}{\sqrt{k}} \left(\arctan \frac{\sqrt{k}}{2}\,\bar{r} + n\pi\right), \quad n = 0, 1, 2, \ldots \quad (5)$$

Since $R_p \geqslant 0$ by definition, $R_p = 0$ is satisfied when $\bar{r} = 0 = n$. So $\bar{r} = 0$ is the lower bound on $\bar{r}$. The upper bound on $\bar{r}$ must now be ascertained from the line element and boundary conditions.

It is noted that the spatial component of (4) has a maximum of $\frac{1}{\sqrt{k}}$ for any time t, when $\bar{r} = \frac{2}{\sqrt{k}}$. Thus, as $\bar{r} \to \infty$, the spatial component of R_c runs from 0 (at $\bar{r} = 0$) to the maximum $\frac{1}{\sqrt{k}}$ (at $\bar{r} = \frac{2}{\sqrt{k}}$), then back to zero, since

$$\lim_{\bar{r}\to\infty} \frac{\bar{r}}{1 + \frac{k}{4}\,\bar{r}^2} = 0. \quad (6)$$

Transform (3) by setting

$$R = R(\bar{r}) = \frac{\bar{r}}{1 + \frac{k}{4}\,\bar{r}^2}, \quad (7)$$

which carries (3) into

$$ds^2 = dt^2 - e^{g(t)}\left[\frac{dR^2}{1 - kR^2} + R^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\right]. \quad (8)$$

The quantity R appearing in (8) is *not* a radial geodesic distance. It is only a factor in a radius of curvature in that it determines the Gaussian curvature $G = \frac{1}{e^{g(t)}R^2}$. The radius of curvature of (8) is

$$R_c = e^{\frac{1}{2}g(t)} R, \quad (9)$$

and the proper radius of Einstein's universe is, by (8),

$$R_p = e^{\frac{1}{2}g(t)} \int \frac{dR}{\sqrt{1 - kR^2}} =$$
$$= \frac{e^{\frac{1}{2}g(t)}}{\sqrt{k}} \left(\arcsin \sqrt{k}R + 2m\pi\right), \quad m = 0, 1, 2, \ldots \quad (10)$$

Now according to (7), the minimum value of R is $R\,(\bar{r} = 0) = 0$. Also, according to (7), the maximum value

*For the geometry due to Efcleethees, usually and abominably rendered as Euclid.

of R is $R(\bar{r} = \frac{2}{\sqrt{k}}) = \frac{1}{\sqrt{k}}$. $R = \frac{1}{\sqrt{k}}$ makes (8) singular, although (3) is not singular at $\bar{r} = \frac{2}{\sqrt{k}}$. Since by (7), $\bar{r} \to \infty \Rightarrow R(\bar{r}) \to 0$, then if $0 \leqslant \bar{r} < \infty$ on (3) it follows that the proper radius of Einstein's universe is, according to (8),

$$R_p = e^{\frac{1}{2}g(t)} \int_0^0 \frac{dR}{\sqrt{1-kR^2}} \equiv 0\,. \tag{11}$$

Therefore, $0 \leqslant \bar{r} < \infty$ on (3) is false. Furthermore, since the proper radius of Einstein's universe cannot be zero and cannot depend upon a set of coordinates (it must be an invariant), expressions (5) and (10) must agree. Similarly, the radius of curvature of Einstein's universe must be an invariant (independent of a set of coordinates), so expressions (4) and (9) must also agree, in which case $0 \leqslant R < \frac{1}{\sqrt{k}}$ and $0 \leqslant \bar{r} < \frac{2}{\sqrt{k}}$. Then by (5), the proper radius of Einstein's universe is

$$R_p = \lim_{\alpha \to \frac{2}{\sqrt{k}}} e^{\frac{1}{2}g(t)} \int_0^{\alpha} \frac{d\bar{r}}{1+\frac{k}{4}\bar{r}^2} = \\ = \frac{2e^{\frac{1}{2}g(t)}}{\sqrt{k}} \left[\left(\frac{\pi}{4} + n\pi\right) - m\pi\right], \quad n, m = 0, 1, 2, \ldots \\ n \geqslant m\,.$$

Setting $p = n - m$ gives for the proper radius of Einstein's universe,

$$R_p = \frac{2e^{\frac{1}{2}g(t)}}{\sqrt{k}} \left(\frac{\pi}{4} + p\pi\right), \quad p = 0, 1, 2, \ldots \tag{12}$$

Now by (10), the proper radius of Einstein's universe is

$$R_p = \lim_{\alpha \to \frac{1}{\sqrt{k}}} e^{\frac{1}{2}g(t)} \int_0^{\alpha} \frac{dR}{\sqrt{1-kR^2}} = \\ = \frac{e^{\frac{1}{2}g(t)}}{\sqrt{k}} \left[\left(\frac{\pi}{2} + 2n\pi\right) - m\pi\right], \quad n, m = 0, 1, 2, \ldots \\ 2n \geqslant m\,.$$

Setting $q = 2n - m$ gives the proper radius of Einstein's universe as,

$$R_p = \frac{e^{\frac{1}{2}g(t)}}{\sqrt{k}} \left(\frac{\pi}{2} + q\pi\right), \quad q = 0, 1, 2, \ldots \tag{13}$$

Expressions (12) and (13) must be equal for all values of p and q. This can only occur if $g(t)$ is infinite for all values of t. Thus, the proper radius of Einstein's universe is infinite.

By (4), (7) and (9), the invariant radius of curvature of Einstein's universe is,

$$R_c\left(\frac{2}{\sqrt{k}}\right) = \frac{e^{\frac{1}{2}g(t)}}{\sqrt{k}}, \tag{14}$$

which is infinite by virtue of $g(t) = \infty \;\forall\, t$.

4 The "volume" of Einstein's universe

The volume of Einstein's universe is, according to (3),

$$V = e^{\frac{3}{2}g(t)} \int_0^{\frac{2}{\sqrt{k}}} \frac{\bar{r}^2 d\bar{r}}{\left(1+\frac{k}{4}\bar{r}^2\right)^3} \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\varphi = \\ = \frac{4\pi e^{\frac{3}{2}g(t)}}{k^{\frac{3}{2}}} \left(\frac{\pi}{4} + p\pi\right), \quad p = 0, 1, 2, \ldots \tag{15}$$

The volume of Einstein's universe is, according to (8),

$$V = e^{\frac{3}{2}g(t)} \int_0^{\frac{1}{\sqrt{k}}} \frac{R^2 dR}{\sqrt{1-kR^2}} \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\varphi = \\ = e^{\frac{3}{2}g(t)} \frac{2\pi}{k^{\frac{3}{2}}} \left[\frac{\pi}{2} + (2n-m)\,\pi\right], \quad n, m = 0, 1, 2, \ldots \\ 2n \geqslant m\,,$$

and setting $q = 2n - m$ this becomes,

$$V = \frac{2\pi e^{\frac{3}{2}g(t)}}{k^{\frac{3}{2}}} \left(\frac{\pi}{2} + q\pi\right), \quad q = 0, 1, 2, \ldots \tag{16}$$

Since the volume of Einstein's universe must be an invariant, expressions (15) and (16) must be equal for all values of p and q. Equality can only occur if $g(t)$ is infinite for all values of the time t. Thus the volume of Einstein's universe is infinite.

In the usual treatment (8) is transformed by setting

$$R = \frac{1}{\sqrt{k}} \sin\chi, \tag{17}$$

to get

$$ds^2 = dt^2 - \frac{e^{g(t)}}{k} \left[d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta\, d\varphi^2)\right], \tag{18}$$

where it is usually asserted, without any proof (see e.g. [3, 4, 5, 6]), that

$$0 \leqslant \chi \leqslant \pi \quad (\text{or } 0 \leqslant \chi \leqslant 2\pi), \tag{19}$$

and whereby (18) is not singular. However, according to (7), (11), (12), and (13), χ can only take the values

$$2n\pi \leqslant \chi < \frac{\pi}{2} + 2n\pi, \quad n = 0, 1, 2, \ldots$$

so that the radius of curvature of Einstein's universe is, by (18),

$$R_c = \frac{e^{\frac{1}{2}g(t)} \sin\chi}{\sqrt{k}}$$

which must be evaluated for $\chi = \frac{\pi}{2} + 2n\pi$, $n = 0, 1, 2, \ldots$, giving

$$R_c = \frac{e^{\frac{1}{2}g(t)}}{\sqrt{k}}$$

as the radius of curvature of Einstein's universe, in concordance with (4), (7), and (9). The proper radius of Einstein's universe is given by

$$R_p = \frac{e^{\frac{1}{2}g(t)}}{\sqrt{k}} \int_{2n\pi}^{\frac{\pi}{2}+2n\pi} d\chi = \frac{e^{\frac{1}{2}g(t)}}{\sqrt{k}} \frac{\pi}{2}, \qquad (20)$$

and since the proper radius of Einstein's universe is an invariant, (20) must equal (12) and (13). Expression (20) is consistent with (12) and (13) only if $g(t)$ is infinite for all values of the time t, and so Einstein's universe is infinite.

According to (18), the volume of Einstein's universe is,

$$V = \frac{e^{\frac{3}{2}g(t)}}{k^{\frac{3}{2}}} \int_{2n\pi}^{\frac{\pi}{2}+2n\pi} \sin^2\chi\, d\chi \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\varphi = \\ = \frac{\pi^2 e^{\frac{3}{2}g(t)}}{k^{\frac{3}{2}}} \frac{\pi}{2}. \qquad (21)$$

Since this volume must be an invariant, expression (21) must give the same value as expressions (15) and (16). This can only occur for (21) if $g(t)$ is infinite for all values of the time t, and so Einstein's universe has an infinite volume.

5 The "area" of Einstein's universe

Using (3), the invariant surface area of Einstein's universe is

$$S = R_c^2 \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\varphi = 4\pi R_c^2$$

which must be evaluated for $R_c(\bar{r} = \frac{2}{\sqrt{k}})$, according to (4), and so

$$S = \frac{4\pi e^{g(t)}}{k}.$$

By (8) the invariant surface area is

$$S = e^{g(t)} R^2 \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\varphi = 4\pi R^2 e^{g(t)},$$

which must, according to (7), be evaluated for $R(\bar{r} = \frac{2}{\sqrt{k}}) = \frac{1}{\sqrt{k}}$, to give

$$S = \frac{4\pi e^{g(t)}}{k}.$$

By (18) the invariant surface area is

$$S = \frac{e^{g(t)}}{k} \sin^2\chi \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\varphi = \frac{4\pi e^{g(t)}}{k} \sin^2\chi,$$

and this, according to (17), must be evaluated for $\chi = \left(\frac{\pi}{2} + 2n\pi\right)$, $n = 0, 1, 2, \ldots$, which gives

$$S = \frac{4\pi e^{g(t)}}{k}.$$

Thus the invariant surface area of Einstein's universe is infinite for all values of the time t, since $g(t)$ is infinite for all values of t.

In similar fashion the invariant great "circumference", $C = 2\pi R_c$, of Einstein's universe is infinite at any particular time, given by

$$C = \frac{2\pi e^{\frac{1}{2}g(t)}}{\sqrt{k}}.$$

6 Generalisation of the line element

Line elements (3), (8) and (18) can be generalised in the following way. In (3), replace $\bar{r}$ by $|\bar{r} - \bar{r}_0|$ to get

$$ds^2 = dt^2 - \frac{e^{g(t)}}{\left(1 + \frac{k}{4}|\bar{r} - \bar{r}_0|^2\right)^2} \times \\ \times \left[d\bar{r}^2 + |\bar{r} - \bar{r}_0|^2 (d\theta^2 + \sin^2\theta\, d\varphi^2)\right], \qquad (22)$$

where $\bar{r}_0 \in \Re$ is entirely arbitrary. Line element (22) is defined on

$$0 \leqslant |\bar{r} - \bar{r}_0| < \frac{2}{\sqrt{k}} \;\; \forall \;\; \bar{r}_0,$$

i.e. on

$$\bar{r}_0 - \frac{2}{\sqrt{k}} < \bar{r} < \frac{2}{\sqrt{k}} + \bar{r}_0 \;\; \forall \;\; \bar{r}_0. \qquad (23)$$

This corresponds to $0 \leqslant R_c < \frac{1}{\sqrt{k}}$ irrespective of the value of $\bar{r}_0$, and amplifies the fact that $\bar{r}$ is merely a parameter. Indeed, (4) is generalised to

$$R_c = R_c(\bar{r}) = \frac{|\bar{r} - \bar{r}_0|}{1 + \frac{k}{4}|\bar{r} - \bar{r}_0|^2},$$

where (23) applies. Note that $\bar{r}$ can approach $\bar{r}_0$ from above or below. Thus, there is nothing special about $\bar{r}_0 = 0$. If $\bar{r}_0 = 0$ and $\bar{r} \geqslant 0$, then (3) is recovered as a special case, still subject of course to the range $0 \leqslant \bar{r} < \frac{2}{\sqrt{k}}$.

Expression (7) is generalised thus,

$$|R - R_0| = \frac{|\bar{r} - \bar{r}_0|}{1 + \frac{k}{4}|\bar{r} - \bar{r}_0|^2},$$

where R_0 is an entirely arbitrary real number, and so (8) becomes

$$ds^2 = dt^2 - e^{g(t)} \times \\ \times \left[\frac{dR^2}{1 - k|R - R_0|^2} + |R - R_0|^2 (d\theta^2 + \sin^2\theta\, d\varphi^2)\right], \qquad (24)$$

where

$$R_0 - \frac{1}{\sqrt{k}} < R < \frac{1}{\sqrt{k}} + R_0 \;\; \forall \;\; R_0. \qquad (25)$$

Note that R can approach R_0 from above or below. There is nothing special about $R_0 = 0$. If $R_0 = 0$ and $R \geqslant 0$, then (8) is recovered as a special case, subject of course to the range $0 \leqslant R < \frac{1}{\sqrt{k}}$.

Similarly, (18) is generalised, according to (24), by setting

$$|R - R_0| = \frac{1}{\sqrt{k}} \sin|\chi - \chi_0|,$$

where χ_0 is an entirely arbitrary real number, and

$$2n\pi \leqslant |\chi - \chi_0| < \frac{\pi}{2} + 2n\pi, \quad n = 0, 1, 2, \ldots$$

$$\forall\, \chi_0 \in \Re.$$

Note that χ can approach χ_0 from above or below. There is nothing special about $\chi_0 = 0$. If $\chi_0 = 0$ and $\chi \geqslant 0$, then (18) is recovered as a special case, subject of course to the range $2n\pi \leqslant \chi < \frac{\pi}{2} + 2n\pi$, $n = 0, 1, 2, \ldots$

The corresponding expressions for the great circumference, the surface area, and the volume are easily obtained in like fashion.

7 Conclusions

Einstein's universe has an infinite proper radius, an infinite radius of curvature, an infinite surface area and an infinite volume at any time. Thus, in relation to the Friedmann-Robertson-Walker line-element and its variations considered herein, the concept of the Big Bang cosmology is invalid.

Submitted on July 16, 2007
Accepted on July 20, 2007

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On the Nature of the Microwave Background at the Lagrange 2 Point. Part I

Pierre-Marie Robitaille

Dept. of Radiology, The Ohio State University, 130 Means Hall, 1654 Upham Drive, Columbus, Ohio 43210, USA

E-mail: robitaille.1@osu.edu

In this work, the nature of the microwave background is discussed. It is advanced that the 2.725 K monopole signal, first detected by Penzias and Wilson, originates from the Earth and therefore cannot be detected at the Lagrange 2 point (L2). Results obtained by the COBE, Relikt-1, and WMAP satellites are briefly reviewed. Attention is also placed on the upcoming PLANCK mission, with particular emphasis on the low frequency instrument (LFI). Since the LFI on PLANCK can operate both in absolute mode and in difference mode, this instrument should be able to unequivocally resolve any question relative to the origin of the 2.725 K monopole signal. The monopole will be discovered to originate from the Earth and not from the Cosmos. This will have implications relative to the overall performance of the PLANCK satellite, in particular, and for the future of astrophysics, in general.

1 Introduction

In 1965, a thermal signal of unknown origin, which appeared to completely engulf the Earth, irrespective of angle of observation, was first reported to exist at microwave frequencies [1]. Immediately considered of great importance, the strange finding was rapidly attributed to the universe by Dicke et al. [2] in a communication which preceded the disclosure of the actual measurements by A. A. Penzias and R. W. Wilson [1]. The observation became known as the "Cosmic Microwave Background (CMB)" nearly from the instant of discovery [1, 2]. For years, it had been predicted that such a signal must exist, if the universe evolved from a Big Bang scenario. With the advent of the Penzias and Wilson measurement [1], the long sought signature of creation seemed discovered, and cosmology entered the realm of modern science.

Since that time, the "CMB" has become a cornerstone of astrophysics [3–6]. The background and its characteristic 2.725 K monopole temperature [7, 8], the "relic of the Big Bang", is believed to span the entire known universe. While the "CMB" was initially considered weak, it is now clear that the signal was in fact quite powerful, at least when viewed from Earth orbit (8). Indeed, few experimental signals of natural origin have surpassed the microwave background in absolute signal to noise [8]. For cosmology, the "CMB" is the most important "astrophysical" finding. Experimental confirmations of its existence and characterization have consumed vast amounts of both financial and human capital. As a result, a more detailed understanding of the microwave background has emerged.

In addition to its characteristic monopole temperature at 2.725 K [8], the background has associated with it a strong (3.5 mK) dipole which is ascribed to the motion of the Earth and the Sun through the local group [9]. This powerful dipole has been observed not only on Earth, and in Earth orbit [9], but also by instruments located well beyond the Earth, like the Soviet Relikt-1 [10] and the NASA WMAP [11] satellites. Consequently, there can be little question that the dipole is real, and truly associated with motion through the local group.

Beyond the dipole, cosmology has also placed significant emphasis on the multipoles visible at microwave frequencies [12]. Accordingly, the universe has now been characterized by anisotropy maps, the most famous of which have been reported by the COBE [7] and WMAP [11] satellites. These maps reflect very slight differences in microwave power of the universe as a function of observational direction.

The recent array of scientific evidence, in support of a microwave background of cosmological origin, appears tremendous, and cosmology seems to have evolved into a precision science [13–19]. Should the 2.725 K microwave background truly belong to the universe, there can be little question that cosmology has joined the company of the established experimental disciplines. Yet, these claims remain directly linked to the validity of the assignment for the "Cosmic Microwave Background". Indeed, if the "CMB" is reassigned to a different source, astrophysics will undergo significant transformations.

2 The origin of the microwave background

Recently, the origin of the "CMB" has been brought into question, and the monopole of the microwave background has been formally reassigned to the Earth [20–29]. Such claims depend on several factors, as follows:

1. The assignment of a 2.725 K temperature to the Penzias and Wilson signal constitutes a violation of Kirchhoff's Law of Thermal Emission [30, 31]. The proper

assignment of thermal temperatures requires, according to Kirchhoff [31], equilibrium with an enclosure [30]. This is a condition which cannot be met by the universe. Therefore, the absolute magnitude of the temperature should be considered erroneous;

2. The cosmological community, in general, and the COBE [33] and WMAP [34] teams, in particular, have advanced that the Earth can be treated as a $\sim$300 K blackbody. In fact, since the Earth is 75% water covered, this assumption is not justified, based on the known behavior of sea emissions in the microwave region [26, 35]. The oceans exhibit thermal emission profiles, which depend on the Nadir angle, and are therefore not blackbody emitters at $\sim$300 K. Indeed, the oceans can produce signals very close to 0 K [26, 35]. It remains of concern that the signature of the microwave background is completely devoid of earthly interference. Not a single artifact has been reported over the entire frequency range [8] which could be attributed to an earthly signal of oceanic origin. At the same time, it is well established that water is a powerful absorber of microwave radiation. Consequently, it is reasonable to expect that the oceans cannot be microwave silent relative to this problem;

3. Powerful signals imply proximal sources. When measured from the Earth the monopole of the microwave background has a tremendous signal to noise [8]. To require that such extensive power fill the entire universe argues in favor of a nearly infinite power source well outside anything known to human science. Conversely, if the signal arises from the Earth, it would be expected to be strong when viewed from Earth [8]. The powerful nature of the microwave background in Earth orbit [8], and the lack of oceanic contaminating signal could very easily be solved, if the Penzias and Wilson signal [1] was generated by the Earth itself [20–29];

4. In the experimental setting, thermal photons, once released, report the temperature of the source which produced them in a manner which is independent of time elapsed and of subsequent source cooling. Once photons are emitted, they cannot shift their frequencies to account for changes at the source. Yet, the Big Bang scenario requires a constant and systematic shifting of photon frequencies towards lower temperatures in a manner wherein the cooling of the source is constantly monitored and reported. This is without experimental evidence in the laboratory. Experimental photons, once produced, can no longer monitor the cooling of the source. Arguments relative to photon shifting, based on an expanding universe, are theoretical and are not supported by laboratory measurements. In considering stellar red shifts, for instance, it is commonly held that the sources themselves are moving away from the observer. Thus, the photons are being shifted *as they are being produced*. In sharp contrast, a microwave background of cosmic origin requires *continuous shifting of photon frequencies long after emission*;

5. The monopole of the microwave background is characterized by a thermal profile [8]. It is a well recognized observation of physics, that a Lyman process is required to produce a group of Lyman lines. Likewise, a nuclear magnetic resonance process is required to obtain an NMR line. Similarly, a thermal process must occur to produce a thermal line. On Earth, thermal emission spectra are generated exclusively in the presence of matter in the condensed state [30]. The existence of a Planckian line in the microwave requires a process analogous to that which results in a thermal spectrum from a piece of graphite on Earth [30]. Physics has not provided a known mechanism for the creation of a photon by graphite [30]. As a result, Planck's equation, unlike all others in physics, remains detached from physical reality [30]. In this regard, it is maintained [30] that a thermal profile can only be obtained as the result of the vibration of atomic nuclei within the confines of a lattice field (or fleeting lattice field in the case of a liquid). Condensed matter, either in the solid or liquid state, is required. This condition cannot be met within the framework of Big Bang cosmology. Universality in blackbody radiation does not hold [30, 31];

6. Measurements performed by the COBE satellite reveal a systematic error relative to the measured value of the microwave background monopole temperature, derived either from the monopole or the dipole [26, 27]. These measurements can be interpreted as implying that still another field exists through which the Earth is moving [26, 27];

7. Currently, the "Cosmic Microwave Background" is thought to be continuously immersing the Earth in microwave photons from every conceivable direction in space. Under this steady state scenario, there can be no means for signal attenuation at high frequencies, as has been observed on Earth [28]. This strongly argues that the "CMB" cannot be of cosmic origin [28];

8. The "CMB" anisotropy maps reported by the WMAP satellite display instabilities which are unacceptable, given the need for reproducibility on a cosmological timescale. The results fail to meet accepted standards for image quality, based on a variety of criteria [23–25]. These findings demonstrate that the stability observed in the monopole at 2.725 K is not translated at the level of the anisotropy maps, as would be expected for a signal of cosmologic origin. This implies that the monopole arises from a stable source, while the anisotropies arise from separate unstable sources.

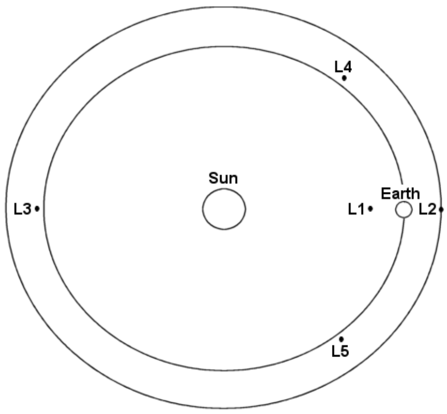

Fig. 1: Schematic representation of the Sun-Earth system depicting the position of the Lagrange 2 point, L2.

2.1 The CMB versus the EMB

Given this array of concerns relative to the assignment of the microwave background, it is clear that mankind must determine, without question, whether this signal is indeed of cosmic origin, or whether, as advanced herein and elsewhere [20–29], it is being generated by the Earth. Current satellite data make strong arguments relative to systematic errors [26, 27] and stability [25] that the monopole of the microwave background originates from the Earth. Conversely, the astrophysical community maintains that a cosmic origin remains the only valid explanation. This being said, it is perplexing that the thermal emission profile of the Earth itself, from space, has yet to be obtained. If the Earth's emission profile was obtained, over the infrared and microwave region, it would become evident that our planet is not a 300 K blackbody radiation source, as the COBE [33], WMAP [34], and PLANCK [36] teams assume. In this era of concern for global warming, it is critical to secure this data.

In the meantime, the PLANCK mission [36], planned by the European Space Agency, will provide the next opportunity to help resolve these questions. Because PLANCK [36] may well acquire the decisive evidence relative to an earthly origin for the monopole of the microwave background, it is important to understand this mission, relative to both COBE [7] and WMAP [11]. The area of greatest interest lies in the configuration of the PLANCK radiometers and the results which they should be able to deliver at the Lagrange 2 point (see Figure 1).

2.1.1 Scenario 1: a cosmic origin

The microwave background has always been viewed as a remnant of the Big Bang originating far beyond our own galaxy. The Earth, in this scenario, is being constantly bombarded by photons from every direction. The frequency distribution of these photons is represented by a 2.725 K blackbody [8]. Indeed, the "CMB" represents perhaps the most precise thermal radiation curve ever measured [8]. The Earth is traveling through the microwave background, as it continues to orbit the Sun and as the latter moves within the galaxy. This motion through the local group is associated with a strong dipole (3.346±0.017 mK) in the direction $l, b = 263.85^\circ \pm 0.1^\circ$, $48.25^\circ \pm 0.04^\circ$ [11], where l and b represent galactic longitude and latitude, respectively. In addition, the "CMB" is characterized by numerous multipoles derived from the analysis of the "CMB" anisotropy maps [11]. Under this scenario, the "CMB" field experienced at ground level, in Earth orbit, or at the Lagrange 2 point (see Figure 1), should be theoretically identical, neglecting atmospheric interference. If COBE [7] and Relikt-1 [10] were launched into Earth orbit, it was largely to avoid any interference from the Earth. The WMAP [11] and PLANCK [36] satellites seek a superior monitoring position, by traveling to the Lagrange 2 point. At this position, the Earth is able to shield the satellite, at least in part, from solar radiation.

2.1.2 Scenario 2: an earthly origin

Recently [20–29], it has been advanced that the microwave background is not of cosmic origin, but rather is simply being produced by the oceans of the Earth. Since the monopole can be visualized only on Earth, or in close Earth orbit [8], it will be referred to as the Earth Microwave Background or "EMB" [28]. In this scenario, the monopole of the Earth microwave background at 2.725 K (EMBM) reports an erroneous temperature, as a result of the liquid nature of the Earth's oceans. The oceans fail to meet the requirements set forth for setting a temperature using the laws of thermal emission [30–32]. For instance, Planck has warned that objects which sustain convection can never be treated as blackbodies [37]. A thermal signature may well appear, but the temperature which is extracted from it is not necessarily real. It may be only apparent. The fundamental oscillator responsible for this signature is thought to be the weak hydrogen bond between the water molecules of the oceans. The EMB has associated with it a dipole [9]. This dipole has been extensively measured from Earth and Earth orbit, and is directly reflecting the motion of the Earth through the local group, as above. Since the Earth is producing the monopole (EMBM), while in motion through the local group, the EMB dipole or "EMBD" would be expected to exist unrelated to the presence of any other fields.

At the Lagrange 2 point, the signal generated by the oceans (EMB) will be too weak to be easily observed [34, 38]. Nonetheless, L2 will not be devoid of all microwave signals. Indeed, at this position, a microwave field must exist. This field, much like noise, will not be characterized by a single temperature. Rather, it will be a weak field, best described through the summation of many apparent temperatures, not by a single monopole. In a sense, microwave noise will be found of significant intensity, but it will be devoid of the characteristics of typical signal. For the sake of clarity, this

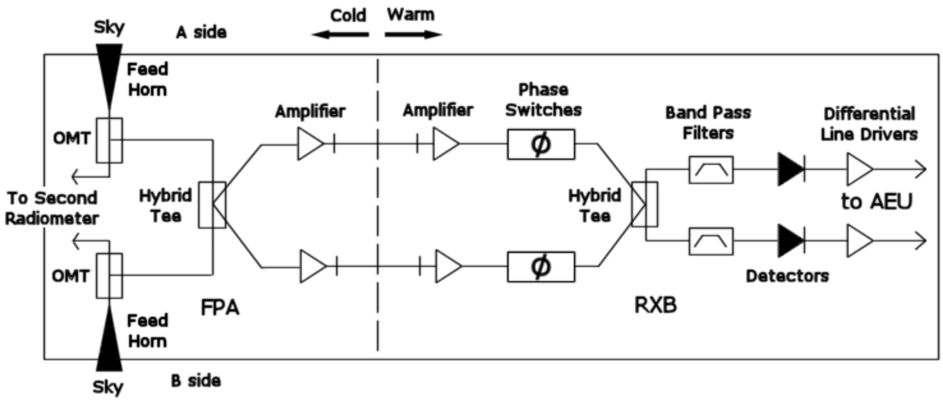

Fig. 2: Partial schematic representation of the WMAP pseudo-correlation differential radiometers [41]. Note that the signal from each horn first travels to an orthomode transducer (OMT) wherein two orthogonal outputs are produced, one for each radiometer. One output from the OMT then travels to the 180° hybrid tee before entering the phase-matched leg of the radiometer. Importantly, for the WMAP satellite, the signal from each horn is being compared directly to its paired counterpart. The satellite does not make use of internal reference loads and cannot operate in absolute mode. (Adapted from [34, 41].)

field will be referred to as the Weak Microwave Background (WMB). This weak background bathes, at least, our solar system, and perhaps much of the galaxy. However, it may or may not extend much power into intergalactic space. Interestingly, motion of the WMAP [11] or PLANCK [36] satellites through this WMB will be associated with the production of a dipole of exactly the same magnitude and direction as observed on Earth [9], since the nature of the motion through the local group has not changed at this point. As such, two dipoles can be considered. The first is associated with the EMB. It is referred to above by the acronym EMBD. The second is associated with the WMB and motion through the local group. It will be referred to henceforth as the WMBD. In actuality, even if the Earth did not produce the 2.725 K monopole, it would still sense the WMBD, as it is also traveling through the WMB. The fact, that both an EMBD and a WMBD are expected, has been used to reconcile the systematic error reported by the COBE satellite [26, 27].

In summary, under the second scenario, we now have a total of four fields to consider:

(1) the monopole of the Earth Microwave Background, the EMBM;

(2) the dipole associated directly with the Earth Microwave Background and motion through the local group, the EMBD;

(3) the Weak Microwave Background present at L2 and perhaps in much of the galaxy, the WMB, and finally

(4) the dipole associated when any object travels through the Weak Microwave Background, the WMBD.

2.1.3 The microwave anisotropies

Weak Microwave Background Anisotropies (MBA) are associated with either Scenario 1 or 2. The anisotropies form the basis of the microwave anisotropy maps now made famous by the WMAP satellite [11, 39, 40]. Under the first scenario, the MBA are tiny fluctuations in the fabric of space which represent relics of the Big Bang. However, careful analysis reveals that the anisotropy maps lack the stability required of cosmic signals [25], and are therefore devoid of cosmological significance. They represent the expected microwave variations, in the sky, associated with the fluctuating nature of microwave emissions originating from all galactic and extragalactic sources. These observations increase the probability that the second scenario is valid.

3 The WMAP versus PLANCK missions

3.1 WMAP

The WMAP satellite [11] is currently positioned at the Lagrange 2 point. WMAP operates in differential mode (see Figure 2), wherein the signal from two matched horns are constantly compared [34, 41]. In this sense, the WMAP satellite resembles the DMR instrument on COBE [33, 42]. Initially, WMAP was to rely exclusively on the magnitude of the dipole observable at L2, in order to execute the calibration of the radiometers (see Section 7.4.1 in [41]). Since the "CMB" and its 2.7 K signature are believed to be present at L2 by the WMAP team, then calibration involves the 1st derivative of the "CMB" and calculated temperature maps of the sky [41], describing the associated temperature variations based on the dipole [9]. Once WMAP reached L2, the initial approach to calibration appeared to be somewhat insufficient, and additional corrections were made for radiometer gains with the initial data release [45, 46].

WMAP is a pseudo-correlation differential spectrometer without absolute reference loads (see Figure 2). Correlation receivers are used extensively in radioastronomy, in part due to the inherent stability which they exhibit, when presented with two nearly identical signals [43, 44]. Since WMAP

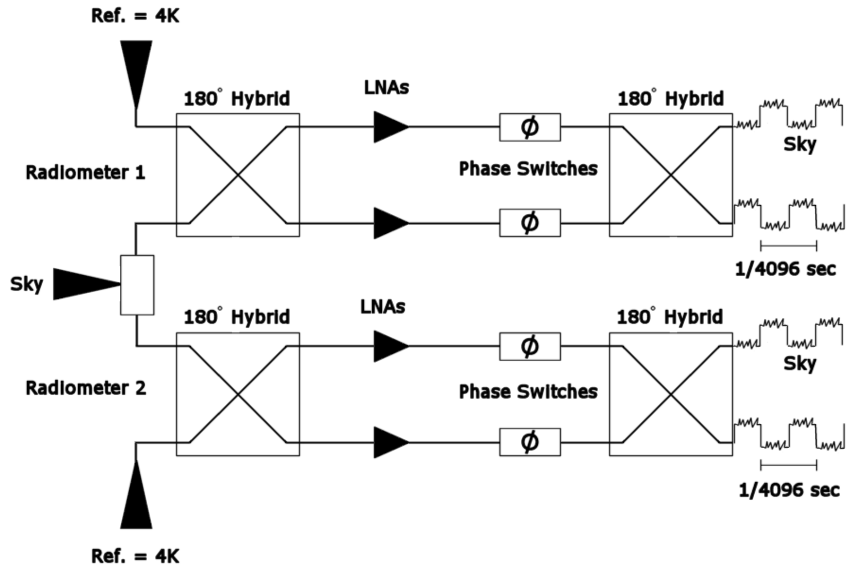

Fig. 3: Partial schematic representation of the PLANCK LFI pseudo-correlation differential radiometers [47, 48]. Prior to entering each radiometer, the signal from each sky horn travels to an orthomode transducer (OMT) where two orthogonal linearly polarized signals are produced. Each of these signals is then compared directly to a reference load maintained at 4 K. Unlike WMAP, PLANCK can operate both in absolute and differential mode. In absolute mode, PLANCK will be able to directly compare the amplitude signal observed from the sky with that produced by the reference loads. Importantly, in order to maintain a minimal knee frequency PLANCK assumes that the differences between the sky and reference signals will be small. (Adapted from [47–52].)

is devoid of reference loads, the satellite is unable to easily answer questions relative to the presence or absence of the 2.725 K "CMB" signal at the L2 point. Should only a WMB be present, WMAP could still be calibrated properly [41], because the magnitude and direction of the dipole itself ultimately governs the entire problem, independent of the underlying field. Because the dipole is being produced by motion through the local group, its magnitude and direction at L2 will be identical, irrespective of the scenario invoked above. This is true, of course, provided that the WMB exists. The WMAP team assumes the presence of a "CMB" monopole at L2 and uses its first derivative, in combination with an expected sky temperature difference map, based on the known dipole [41]. Alternatively, if only a WMB exists at L2, the dipole will still be present, and another set of theoretical constraints will also satisfy the requirements for calibration.

WMAP has been able to detect the dipole at the L2 point, but this is expected from both scenarios listed above. In any case, an objective analysis of the data products associated with this satellite reveals that, far from affirming the cosmic nature of anisotropy, WMAP refutes such conclusions [25]. The anisotropy maps derived from WMAP are much too unstable and unreliable to be fundamentally linked to signals of primordial origin [25]. WMAP has not been able to yield a definitive answer relative to the origin of the "CMB", and, to date, no signal has been measured which can be ascribed to the remnant of the Big Bang. Fortunately, it appears that the PLANCK satellite will be able to unambiguously resolve the issue.

3.2 PLANCK

Much like WMAP, the PLANCK satellite [36] is scheduled to be launched into an operational orbit at L2, the Lagrange 2 point of the Earth-Sun system. The satellite is equipped with two instruments, the low frequency instrument (LFI) and the high frequency instrument (HFI), scanning the sky at 30, 44, and 70 GHz [47–55] and 100, 143, 217, 353, 545, and 857 GHz [55–57], respectively. In contrast, the WMAP satellite scanned the 23, 33, 41, 61, and 94 GHz regions of the electromagnetic spectrum. Thus, PLANCK greatly extends the range of frequencies which will be sampled.

Still, more important differences exist between PLANCK and WMAP. The high frequency instrument on PLANCK is not differential, and frequencies from 100–857 GHz will be sampled in absolute mode, without subtraction. Moreover, while the low frequency instrument is designed to operate as a differential spectrometer, it can also function in absolute mode [47-54]. The low frequency instrument on PLANCK (see Figure 3) is also designed to function as a pseudo-correlation radiometer [47-53]. However, the signal from the sky, obtained by each horn, is being compared to a reference load maintained at 4 K (see Figure 3). These details constitute critical variations relative to the WMAP radiometer design.

Given that the LFI on PLANCK makes use of absolute reference loads, it resembles, in this important sense, the FIRAS Instrument on COBE [58]. Furthermore, since the LFI on PLANCK can operate either in absolute mode, or in difference mode [47–54], the spectrometer has a flexibility which appears to combine the best features possible for such an instrument. In absolute mode, the LFI on PLANCK will be able to quantify completely the signal originating from the sky relative to that produced by its 4 K references. Nonetheless, the LFI was designed to operate primarily in differential mode. This has implications for the quality of its data products based on whether or not the 2.725 K monopole signal is present at L2.

3.2.1 The PLANCK LFI

The PLANCK LFI is designed as a pseudo-correlation [52] receiver (see Figure 3). For this receiver, gain instabilities in the High Electron Mobility Transistor (HEMT) amplifier, within the receiver front end, result in $1/f$ noise. The $1/f$ noise, if not properly accounted for, can produce significant stripes in the final maps [47, 48]. These stripes are also dependent on scanning strategy. The behavior of the $1/f$ noise has been carefully analyzed for the PLANCK LFI [47, 48]. Since the LFI is designed to operate primarily in differential mode, it is important to minimize the difference between the reference load temperature, T_{ref}, and the sky temperature, T_{sky}.

Currently, the PLANCK team is making the assumption that $T_{sky} = 2.725$ K, as previously reported by the COBE group [7]. As such, they have chosen to use $T_{ref} = 4$ K. Any offset between T_{sky} and T_{ref} *"can be balanced before differencing either by a variable back-end gain stage with a feedback scheme to maintain the output power as close as possible to zero, or by multiplying in software one of the two signals by a so-called gain modulation factor"* [47].

If the differences between the sky temperature and the reference temperatures are large, then the idea of using back-end gain stage feedback, to balance the two channels, should introduce substantial noise directly into the system. The situation using software and a gain modulation factor would also introduce unexpected complications.

The gain modulation factor, r, is given by the following: $r = (T_{sky} + T_n)/(T_{ref} + T_n)$ where T_n corresponds to the radiometer noise temperature. The noise temperature of the radiometer, T_n, is a fundamental property of any receiver and is determined by the overall design and quality of the instrument. T_n is critical in establishing the sensitivity of the spectrometer. For instance, the radiometer sensitivity, ΔT_{rms}, over a given integration time, is directly dependent on both T_{sky} and T_n, as follows: $\Delta T_{rms} = 2\,(T_{sky} + T_n)/\sqrt{\beta}$, where β is the bandwidth of the radiometer (typically taken as 20%). Note that if T_n is large, then it will be easy to achieve gain modulation factors near 1. However, the radiometer sensitivity would be severely compromised. Low T_n values are central to the performance of any receiver. Under this constraint, the gain modulation factor will be strongly affected by any differences between the T_{sky} and T_{ref}.

PLANCK has the ability to calculate the gain modulation factor, r, directly from radiometer data acquired with the spectrometer operating in absolute mode [47]. Alternatively, r can be calculated from software, using up to three approaches including, for instance, minimizing the final differenced data knee frequency, f_k. The knee frequency is the frequency at which the value of $1/f$ noise and white noise contributions are equal.

In general, it is also true that for the PLANCK LFI *"the white noise sensitivity and the knee-frequency depend on the actual temperature in the sky"* [47]. Because excessive $1/f$ noise can degrade the final images and data products [47, 48], it is important to minimize its contribution. This can be achieved *"if the post detection knee frequency f_k (i.e. the frequency at which the $1/f$ noise contribution and the ideal white noise contribution are equal) is significantly lower than the spacecraft rotation frequency ($f_{spin} \sim 0.017$ Hz)"* [48]. If the f_k is greater than, or approximately equal to f_{spin}, a degradation in the final sensitivity of the satellite will occur [47]. As this inherently depends on the real sky temperature, there are some concerns relative to the performance of the PLANCK LFI instruments.

When the knee frequencies are too high, stripes will occur in the images generated by the satellite. It is true that algorithms do exist to help remove these artifacts, provided that they are not too strong [47]. Nonetheless, when the sky temperature and the reference temperatures are not balanced, the knee frequency will rise substantially. This could diminish the quality of the data products from this satellite.

The importance of maintaining a low knee frequency for the PLANCK LFI instruments cannot be overstated. *"If the knee frequency is sufficiently low (i.e. $f_k \leqslant 0.1$ Hz), with the application of such algorithms it is possible to maintain both the increase in rms noise within few % of the white noise, and the power increase at low multipole values (i.e. $l \leqslant 200$) at a very low level (two orders of magnitude less than the CMB power). If, on the other hand, the knee frequency is high (i.e. $\gg 0.1$ Hz) then even after destriping the degradation of the final sensitivity is of several tens of % and the excess power at low multipole values is significant (up to the same order of the CMB power for $f_k \sim 10$ Hz ...) Therefore, careful attention to instrument design, analysis, and testing is essential to achieve a low $1/f$ noise knee frequency"* [48]. The PLANCK team has emphasized this further, as follows: *"It is then of great importance to decrease as much as possible the impact of $1/f$ noise before destriping and $f_k = 0.01$ Hz is an important goal for instrument studies and prototypes."*

The manner in which the knee frequency is affected by both the gain modulation factor, r, and the absolute sky temperature [48], has been described algebraically:

$$f_k\,(T_n) = \beta \left[\frac{A\,(1-r)\,T_n}{2\,(T_{sky} + T_n)}\right]^2. \qquad (1)$$

In this equation, β corresponds to the bandwidth of the receiver, typically taken at 20%, T_n is the radiometer noise temperature, and A is a normalization factor for noise fluctuations [48]. Note that if the sky temperature, T_{sky}, is only some fraction of a Kelvin degree, this equation is moving towards:

$$f_k\,(T_n) = \beta \left[\frac{A\,(1-r)}{2}\right]^2 . \qquad (2)$$

Under test conditions, the PLANCK team estimated gain modulation factors ranging from 0.936 to 0.971 for the 30, 44, and 70 GHz radiometers [47]. In flight, T_n values of 7.5, 12, and 21.5 K are expected for the 30, 44, and 70 GHz radiometers [50]. This results in r values ranging from ∼0.89–0.95, if T_{sky} is taken as 2.725 K and $T_{ref} = 4$ K. Anticipated f_k values would therefore range from ∼0.0032 Hz to ∼0.0043 Hz, well below the 16 mHz requirement. This situation will not occur under Scenario 2, wherein T_{sky} at L2 is not 2.725 K, but rather only some fraction of a Kelvin degree.

As T_{sky} will have a much lower value than foreseen, the gain modulation factor, r, will be moving away from unity. It is also clear from Eqs. 1 and 2 that the knee frequency for the LFI radiometers would rise to values substantially above those currently sought by the PLANCK team.

In the extreme case, it is simple to consider the consequence of $T_{sky} \to 0$. In this instance, gain modulation factors would drop precipitously from ∼0.89 to ∼0.65 at 30 GHz, and from ∼0.95 to ∼0.84 at 70 GHz. This would translate into substantially elevated f_k values of ∼50 mHz. Even an apparent T_{sky} value of 300 mK would result in r and f_k values in this range. Other than the direct measurement of the sky temperature by the PLANCK LFI in absolute mode, the drop in r values and the tremendous rise in f_k will constitute another indication that the 2.725 K signal does not exist at the L2 point.

Consequently, it is difficult to envision that the PLANCK team will be able to attain the desired image quality if T_{sky} is not at 2.725 K. The spectrometer is not designed to achieve maximal sensitivity in absolute mode, while in difference mode, both its r values and its f_k will be compromised. Destriping algorithms will have to be invoked in a much more central manner than anticipated.

Note that the situation with PLANCK is substantially different from WMAP. With WMAP (see Figure 2), the radiometers do not make use of an absolute reference load, but rather, the two sky horns are constantly and directly being differenced. Thus, the knee frequency for WMAP would be as predicted prior to launch. The WMAP horns are nearly perfectly balanced by the sky itself. Therefore, their performance would not be affected by the real nature of the signal at L2. This is not the case for the PLANCK satellite.

4 Conclusion

The WMAP satellite was designed as a differential spectrometer without absolute calibration. As a result, it is unable to ascertain the absolute magnitude of the microwave signals at the L2 point. The satellite has produced anisotropy maps [39, 40]. Yet, these maps lack the stability required of cosmological signals. Indeed, WMAP appears devoid of any findings relative to cosmology, as previously stated [25]. The only signal of note, and one which was not anticipated [21], is that associated with the dipole [9, 26, 27]. The dipole is important, since it can be used to quantify the motion of objects through the local group. Under the second scenario, this dipole signal implies that there is a Weak Microwave Background (WMB) at the L2 point.

In sharp contrast with WMAP, PLANCK has the advantage of being able to operate in absolute mode. In this configuration, it can directly determine whether or not there is a 2.725 K monopole signal at L2. If the signal is present, as expected by the PLANCK team, and as predicted in the first scenario, then the satellite should be able to acquire simply phenomenal maps of the sky. However, this will not occur. In the absence of a monopole, the PLANCK radiometers will be compromised when operating in difference mode, as their knee frequencies rise. This shall result in the presence of more pronounced image artifacts in the data products, which may not be easily removed through processing, potentially impacting the harvest from PLANCK. Nonetheless, PLANCK should be able to fully characterize the WMB predicted under the second scenario.

At the same time, since the 2.725 K monopole signature does not exist at the L2 point, PLANCK is poised to alter the course of human science. The satellite will help establish that there is no universality [30, 31]. The need to link Planck's equation to the physical world will become evident [30, 31]. It will be realized that the Penzias and Wilson signal did come from the Earth, and that liquids can indeed produce thermal spectra reporting incorrect temperatures. It is likely that a renewed interest will take place in condensed matter physics, particularly related to a more profound understanding of thermal emission, in general, and to the study of thermal processes in liquids, in particular. The consequences for astrophysics will be far reaching, impacting our understanding of stellar structure [59, 60], stellar evolution and cosmology. PLANCK, now, must simply lead the way.

Acknowledgements

The author thanks Dmitri Rabounski for valuable discussions. Luc Robitaille is recognized for figure preparation.

Dedication

This work is dedicated to my brother, Patrice, for his love and encouragement.

Submitted on August 06, 2007
Accepted on August 15, 2007
Published online on September 08, 2007
Revised on September 29, 2007

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On the Nature of the Microwave Background at the Lagrange 2 Point. Part II

Larissa Borissova and Dmitri Rabounski

E-mail: lborissova@yahoo.com; rabounski@yahoo.com

In this work the mathematical methods of General Relativity are used to answer the following questions: if a microwave background originates from the Earth, what would be its density and associated dipole measured at the altitude of a U2 aeroplane (25 km), the COBE satellite (900 km), and the 2nd Lagrange point (1.5 million km, the position of the WMAP and PLANCK satellites)? The first problem is solved via Einstein's equations for the electromagnetic field of the Earth. The second problem is solved using the geodesic equations for light-like particles (photons) which are mediators for electromagnetic radiation. We have determined that a microwave background that originates at the Earth (the Earth microwave background) decreases with altitude so that the density of the energy of such a background at the altitude of the COBE orbit (900 km) is 0.68 times less than that at the altitude of a U2 aeroplane. The density of the energy of the background at the L2 point is only $\sim 10^{-7}$ of the value detected by a U2 aeroplane or at the COBE orbit. The dipole anisotropy of the Earth microwave background, due to the rapid motion of the Earth relative to the source of another field which isn't connected to the Earth but is located in depths of the cosmos, doesn't depend on altitute from the surface of the Earth. Such a dipole will be the same irrespective of the position at which measurements are taken.

1 Problem statement: the space of the Earth and the Earth microwave background

Here we solve two theoretical problems related to the measurement of the microwave background:

(1) What is the density of the Earth microwave background which one will observe at the COBE orbit and at the L2 point?

(2) What is the anisotropy of the Earth microwave background, due to a drift of the whole space of the Earth, which one will observe in the COBE orbit and at the L2 point?

In a sense, the anisotropy we are treating is the sum of the dipole and all other multipoles.

According to General Relativity, the result of an observation depends on the velocity of the observer relative to the object he observes, and also on the properties of the local space (such as the space rotation, gravitation, deformation, curvature, etc.) where the observation is made. Therefore, we are looking for a theoretical solution of the aforementioned problems using the mathematical methods, which are specific to General Relativity.

We solve the first problem using Einstein's equations, manifest in the energy and momentum of a field of distributed matter (an electromagnetic field, for instance), depending on the distance from the field's source, and also on the properties of the space e.g. the space rotation, gravitation, etc.

We solve the second problem using the geodesic equations for light-like particles (photons, which are mediators for microwave radiation, and for any electromagnetic radiation in general). The geodesic equations give a possibility of finding a preferred direction (anisotropy) in such a field due to the presence of a linear drift of the whole reference space of the observer relative to the source of another field, which isn't connected to the observer's space, but moves with respect to it [1, 2]. In the present case, such a linear drift is due to the motion of the observer, in common with the microwave background's source, the Earth, relative to the source of another field such as the common field of a group of galaxies or that of the Universe as a whole (a weak microwave background). Then we compare our theoretical result from General Relativity to the experimental data for the microwave background, obtained in space near the Earth by the COBE satellite, located in a 900 km orbit, and also by the WMAP satellite, located at the L2 point, as far as 1.5 million km from the Earth.

In order to obtain a theoretical result expressed in quantities measurable in practice, we use the mathematical apparatus of chronometric invariants — the projections of four-dimensional quantities on the time line and spatial section of a real observer, which are the physical observable quatities in General Relativity [3, 4].

First, we introduce a space where all the mesurements are taken. Both locations, of the COBE satellite and the L2 point, are connected, by gravitation, to the gravitational field of the Earth, so both observers are connected to the space of the Earth, whose properties (e.g. rotation, gravitation, deformation, etc.) affect the observations. We therefore consider different locations of an observer in the space of the Earth.

We construct the metric for the Earth's space, which is the superposition of the metric of a non-holonomic (self-rotating) space and a gravitating space.

The space of the Earth rotates with a frequency of one revolution per day. By the theory of non-holonomic spaces

[5], a non-holonomic space (space-time) has inclinations between the times lines and the three-dimensional spatial section, cosines of which are represent by the three-dimensional linear velocity of the rotation. The metric of a non-holonomic space (space-time), which rotation is given by a linear velocity v at a given point, is described at this point by

$$ds^2 = c^2 dt^2 + \frac{2v}{c}\, cdt\,(dx+dy+dz) - dx^2 - dy^2 - dz^2. \quad (1)$$

For clarity of further calculation, we change to the cylindrical coordinates r, φ, z, where

$$x = r\cos\varphi\,, \quad y = r\sin\varphi\,, \quad z = z\,, \quad (2)$$

so the metric (1) takes the form

$$\begin{aligned} ds^2 &= c^2 dt^2 + \frac{2v}{c}(\cos\varphi + \sin\varphi)\, cdtdr + \\ &+ \frac{2vr}{c}(\cos\varphi - \sin\varphi)\, cdtd\varphi + \frac{2v}{c}\, cdtdz - \\ &- dr^2 - r^2 d\varphi^2 - dz^2. \end{aligned} \quad (3)$$

The metric of a space, where gravitation is due to a body of a mass M, in quasi-Newtonian approximation and in the cylindrical coordinates, is

$$\begin{aligned} ds^2 = \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 - \left(1 + \frac{2GM}{c^2 r}\right) dr^2 - \\ - r^2 d\varphi^2 - dz^2, \end{aligned} \quad (4)$$

where G is the Newtonian gravitational constant. We consider a satellite which rotates in the metric (4) around the gravitating body. Both observers, located on board the COBE satellite (a 900 km orbit) and the WMAP satellite (the L2 point) respectively, are in a state of weightlessness, which is described by the weightlessness condition

$$\frac{GM}{r} = \omega^2 r^2, \quad (5)$$

where r is the radius of the satellite's orbit, while ω is the angular velocity of the rotation of the observer (in common with the satellite on which he is located) around the gravitating body. So the metric (4) is

$$\begin{aligned} ds^2 = \left(1 - \frac{2GM}{c^2 r} - \frac{\omega^2 r^2}{c^2}\right) c^2 dt^2 - \frac{2\omega r^2}{c}\, cdtd\varphi - \\ - \left(1 + \frac{2GM}{c^2 r}\right) dr^2 - r^2 d\varphi^2 - dz^2, \end{aligned} \quad (6)$$

where $\frac{GM}{r} = \omega^2 r^2$. The weightless state is common to all planets and their satellites. So the Earth's space from the point of an observer located on board the COBE satellite and the WMAP satellite is in the weightless state.

We use the cylindrical coordinates, because such an observer is located on board of a satellite which orbits the Earth.

The metric of the Earth's space at the point of location of such an observer is a superposition of the metric with rotation (3) and the metric with a gravitational field (6), which is

$$\begin{aligned} ds^2 &= \left(1 - \frac{2GM}{c^2 r} - \frac{\omega^2 r^2}{c^2}\right) c^2 dt^2 + \\ &+ \frac{2v(\cos\varphi + \sin\varphi)}{c}\, cdtdr + \\ &+ \frac{2r\left[v(\cos\varphi - \sin\varphi) - \omega r\right]}{c}\, cdtd\varphi + \frac{2v}{c}\, cdtdz - \\ &- \left(1 + \frac{2GM}{c^2 r}\right) dr^2 - r^2 d\varphi^2 - dz^2. \end{aligned} \quad (7)$$

Because the Earth, in common with its space, moves relative to the source of the weak microwave background, this drift should also be taken into account in the metric. This is accomplished by choosing this motion to be in the z-direction and then applying Lorentz' transformations to the z coordinate and time t

$$\tilde{t} = \frac{t + \frac{\mathrm{v}z}{c^2}}{\sqrt{1 - \frac{\mathrm{v}^2}{c^2}}}\,, \qquad \tilde{z} = \frac{z + \mathrm{v}t}{\sqrt{1 - \frac{\mathrm{v}^2}{c^2}}}\,, \quad (8)$$

so the resulting metric of the space of the Earth, where such a drift is taken into account, is

$$\begin{aligned} ds^2 &= \left(1 - \frac{2GM}{c^2 r} - \frac{\omega^2 r^2}{c^2} + \frac{2v\mathrm{v}}{c^2}\right) c^2 dt^2 + \\ &+ \frac{2v(\cos\varphi + \sin\varphi)}{c}\, cdtdr + \\ &+ \frac{2r\left[v(\cos\varphi - \sin\varphi) - \omega r\right]}{c}\, cdtd\varphi + \frac{2v}{c}\, cdt\,dz - \\ &- \left(1 + \frac{2GM}{c^2 r}\right) dr^2 + \frac{2v\mathrm{v}(\cos\varphi + \sin\varphi)}{c^2}\, drdz - r^2 d\varphi^2 + \\ &+ \frac{2r\mathrm{v}\left[v(\cos\varphi - \sin\varphi) - \omega r\right]}{c^2}\, d\varphi dz - \left(1 - \frac{2v\mathrm{v}}{c^2}\right) dz^2, \end{aligned} \quad (9)$$

where we mean $1 - \frac{\mathrm{v}^2}{c^2} \simeq 1$, because the Earth's velocity v relative to the source of the weak microwave background is small to the velocity of light c.

This is the *metric of the real physical space of the Earth*, where we process our observations.

Now we apply this metric to the reference frames of two observers, one of which is located on board the COBE satellite, in an orbit with an altitude of 900 km, while the second observer is located on board of WMAP satellite, at the L2 point, which is far as 1.5 million km from the Earth.

2 The density of the Earth microwave background at the COBE orbit and at the L2 point

Here we answer the question: what is the density of the Earth microwave background that one will observe at the COBE orbit and at the L2 point? Using the main observable characteristics of the space of the Earth, pervaded by an electromagnetic field (the microwave background, for instance), we derive Einstein's equations for the space. Einstein's equations describe the energy and momentum of distributed matter, in this case the microwave background. So we will know precisely, through Einstein's equations, the density of the energy of the Earth microwave background which will be observed at the COBE orbit and at the L2 point.

2.1 The Earth space. Its physical properties manifest in observations of the Earth microwave background

In this particular problem we are interested in the distribution of the Earth microwave background with altitude, giving the difference in the measurement of the background at the COBE orbit and at the L2 point. We therefore neglect terms like $\frac{v\mathrm{v}}{c^2}$, which take into account the drift of the whole space of the Earth. The quantity $\frac{2GM}{c^2 r}$ has its maximum numerical value $\sim 10^{-9}$ at the Earth's surface, and the value substatially decreases with altitude. We therefore neglect the last terms in $g_{11} = -\left(1 + \frac{2GM}{c^2 r}\right)$, but we do not neglect the last terms in $g_{00} = 1 - \frac{2GM}{c^2 r} - \frac{\omega^2 r^2}{c^2}$, because they will be multiplied by c^2 later. In such a case the Earth space metric takes the simplified form

$$ds^2 = \left(1 - \frac{2GM}{c^2 r} - \frac{\omega^2 r^2}{c^2}\right) c^2 dt^2 + \\ + \frac{2v\,(\cos\varphi + \sin\varphi)}{c}\, cdtdr + \\ + \frac{2r\,[v\,(\cos\varphi - \sin\varphi) - \omega r]}{c}\, cdtd\varphi + \frac{2v}{c}\, cdtdz - \\ - dr^2 - r^2 d\varphi^2 - dz^2. \tag{10}$$

We will use this metric to determine the density of the energy of the Earth microwave background at the COBE orbit and at the L2 point. We are loooking for the main observable characteristics of the space. By the theory of physical observable quantities in General Relativity [3, 4], the observable properties of a space are determined within the fixed three-dimensional spatial section of an observer. Those are the quatities invariant within the spatial section (the so-called *chronometric invariants*): the gravitational potential w, the linear velocity of the space rotation v_i, the gravitational inertial force F_i, the angular velocity of the space rotation A_{ik}, the three-dimensional metric tensor h_{ik}, the space deformation D_{ik}, the three-dimensional Christoffel symbols Δ^i_{kn}, and the three-dimensional curvature C_{iklj}. These characteristics can be calculated through the components of the fundamental metric tensor $g_{\alpha\beta}$, which can be easily obtained from a formula for the space metric (see [3, 4] for the details).

The substantially non-zero components of the characteristics of the space of the Earth, calculated though the components $g_{\alpha\beta}$ of the metric (10), are

$$\mathrm{w} = \frac{GM}{r} + \frac{\omega^2 r^2}{2}\,, \tag{11}$$

$$\left.\begin{aligned} v_1 &= -v\,(\cos\varphi + \sin\varphi) \\ v_2 &= -r\,[v\,(\cos\varphi - \sin\varphi) - \omega r] \\ v_3 &= -v \end{aligned}\right\} \tag{12}$$

$$\left.\begin{aligned} &F_1 = (\cos\varphi + \sin\varphi)\, v_t + \omega^2 r - \frac{GM}{r^2} \\ &F_2 = r\,(\cos\varphi - \sin\varphi)\, v_t\,, \qquad F_3 = v_t \end{aligned}\right\} \tag{13}$$

$$\left.\begin{aligned} &A_{12} = \omega r + \frac{1}{2}\big[(\cos\varphi + \sin\varphi)\, v_\varphi - \\ &\qquad\qquad - r\,(\cos\varphi - \sin\varphi)\, v_r\big] \\ &A_{23} = -\frac{v_\varphi}{2}\,, \qquad A_{13} = -\frac{v_r}{2} \end{aligned}\right\} \tag{14}$$

$$\left.\begin{aligned} &h_{11} = h_{33} = 1\,, \quad h_{22} = r^2, \quad h^{11} = h^{33} = 1 \\ &h^{22} = \frac{1}{r^2}\,, \qquad h = r^2, \qquad \frac{\partial \ln\sqrt{h}}{\partial r} = \frac{1}{r} \\ &\Delta^1_{22} = -r\,, \qquad \Delta^2_{12} = \frac{1}{r} \end{aligned}\right\} \tag{15}$$

while all components of the tensor of the space deformation D_{ik} and the space curvature C_{iklj} are zero, in the framework of our assumptions. Here we assume the plane in cylindrical coordinates wherein the space of the Earth rotates: we assume that v doesn't depend from the z-coordinate. This assumption is due to the fact that the Earth, in common with its space, moves relative to a weak (cosmic) microwave background in the direction of its anisotropy. The quantities v_r, v_φ, and v_t denote the partial derivatives of v by the respective coordinates and time.

2.2 Einstein's equations in the Earth space. The density of the energy of distributed matter

Einstein's general covariant equations

$$R_{\alpha\beta} - \frac{1}{2}\, g_{\alpha\beta} R = -\,\kappa\, T_{\alpha\beta} + \lambda\, g_{\alpha\beta}\,, \tag{16}$$

in a reference frame of the fixed spatial section of an observer, are represented by their projections onto the observer's time line and spatial section [3, 4]. We omit the λ-term, the space deformation D_{ik}, and the space curvature, C_{iklj}, because they are zero in the framework of our problem. In such a case the projected Einstein equations, according to Zelmanov [3, 4], are

$$\left.\begin{aligned} &\frac{\partial F^i}{\partial x^i} + \frac{\partial \ln\sqrt{h}}{\partial x^i} F^i - A_{ik}A^{ik} = -\frac{\kappa}{2}\left(\rho c^2 + U\right) \\ &\frac{\partial A^{ik}}{\partial x^k} + \frac{\partial \ln\sqrt{h}}{\partial x^k}\, A^{ik} = -\,\kappa J^i \\ &2A_{ij}A^{\cdot j}_{k\cdot} + \frac{1}{2}\left(\frac{\partial F_i}{\partial x^k} + \frac{\partial F_k}{\partial x^i} - 2\Delta^m_{ik}F_m\right) = \\ &\qquad\qquad = \frac{\kappa}{2}\left(\rho c^2 h_{ik} + 2U_{ik} - U h_{ik}\right) \end{aligned}\right\} \tag{17}$$

$$\left.\begin{aligned}
&-2\omega^2-2\omega\left(\cos\varphi+\sin\varphi\right)\frac{v_\varphi}{r}+2\omega\left(\cos\varphi-\sin\varphi\right)v_r+\left(\cos\varphi+\sin\varphi\right)v_{tr}+\left(\cos\varphi-\sin\varphi\right)\frac{v_{t\varphi}}{r}+\\
&+\left(\cos^2\varphi-\sin^2\varphi\right)\frac{v_r v_\varphi}{r}+\cos\varphi\sin\varphi\left(v_r^2-\frac{v_\varphi^2}{r^2}\right)-v_r^2-\frac{v_\varphi^2}{r^2}=-\,\kappa\rho c^2\\
&\frac{1}{2}\left[\left(\cos\varphi+\sin\varphi\right)\left(\frac{v_r}{r}+\frac{v_{\varphi\varphi}}{r^2}\right)+\left(\cos\varphi-\sin\varphi\right)\left(\frac{v_\varphi}{r^2}-\frac{v_{r\varphi}}{r}\right)\right]=-\,\kappa J^1\\
&\frac{1}{2}\left[\left(\cos\varphi+\sin\varphi\right)\left(\frac{v_\varphi}{r^3}-\frac{v_{r\varphi}}{r^2}\right)-\left(\cos\varphi-\sin\varphi\right)\frac{v_{rr}}{r}\right]=-\,\kappa J^2\\
&\frac{1}{2}\left(v_{rr}+\frac{v_r}{r}+\frac{v_{\varphi\varphi}}{r^2}\right)=-\,\kappa J^3\\
&v_r^2+\frac{v_\varphi^2}{2r^3}+3\omega^2+\frac{2GM}{r^3}+2\omega\left(\cos\varphi+\sin\varphi\right)\frac{v_\varphi}{r}-2\omega\left(\cos\varphi-\sin\varphi\right)v_r+\left(\cos\varphi+\sin\varphi\right)v_{tr}-\\
&-\left(\cos^2\varphi-\sin^2\varphi\right)\frac{v_r v_\varphi}{r}-\cos\varphi\sin\varphi\left(v_r^2-\frac{v_\varphi^2}{r^2}\right)=\kappa U_{11}\\
&\frac{r^2}{2}\left[\frac{v_r v_\varphi}{r^2}+\left(\cos\varphi+\sin\varphi\right)\frac{v_{t\varphi}}{r^2}+\left(\cos\varphi-\sin\varphi\right)\frac{v_{tr}}{r}\right]=\kappa U_{12}\\
&\frac{1}{2}\left[2\omega\frac{v_\varphi}{r}+v_{tr}+\left(\cos\varphi+\sin\varphi\right)\frac{v_\varphi^2}{r^2}-\left(\cos\varphi-\sin\varphi\right)\frac{v_r v_\varphi}{r}\right]=\kappa U_{13}\\
&2\omega^2+2\omega\left(\cos\varphi+\sin\varphi\right)\frac{v_\varphi}{r}-2\omega\left(\cos\varphi-\sin\varphi\right)v_r+\left(\cos\varphi-\sin\varphi\right)\frac{v_{t\varphi}}{r}+\frac{v_r^2}{2}+\frac{v_\varphi^2}{r^2}-\\
&-\left(\cos^2\varphi-\sin^2\varphi\right)\frac{v_r v_\varphi}{r}+\cos\varphi\sin\varphi\left(\frac{v_\varphi^2}{r^2}-v_r^2\right)=\kappa\,\frac{U_{22}}{r^2}\\
&\frac{r^2}{2}\left[\frac{v_{t\varphi}}{r^2}-2\omega\frac{v_r}{r}-\left(\cos\varphi+\sin\varphi\right)\frac{v_r v_\varphi}{r^2}+\left(\cos\varphi-\sin\varphi\right)\frac{v_r^2}{r}\right]=\kappa U_{23}\\
&\frac{v_r^2}{2}+\frac{v_\varphi^2}{2r^2}=\kappa U_{33}
\end{aligned}\right\}\qquad(18)$$

where $\rho=\frac{T_{00}}{g_{00}}$, $J^i=\frac{cT_0^i}{\sqrt{g_{00}}}$, and $U^{ik}=c^2T^{ik}$ are the respective projections of the energy-momentum tensor $T_{\alpha\beta}$ of distributed matter on the right side of the equations: ρ is the density of the energy of the matter field, J^i is the density of the field momentum, and U^{ik} is the stress-tensor of the field.

We substitute here the formulae obtained for the space of the Earth. In this deduction we take into account the weightlessness condition $\omega^2r^2=\frac{GM}{r}$. (This is because we calculate the equations for a satellite-bound observer.) We also apply the condition $\rho c^2=U$, which is specific to any electromagnetic field; so we mean only an electromagnetic field distributed in the space. As a result, after some algebra, we obtain the projected Einstein equations for the Earth space filled with a background field of matter. The resulting Einstein equations, the system of 10 equations with partial derivatives, are given in formula (18).

(Obvious substitutions such as $\cos^2\varphi-\sin^2\varphi=\cos 2\varphi$ and $\cos\varphi\sin\varphi=\frac{1}{2}\sin 2\varphi$ can be used herein.)

We are looking for a solution of the scalar Enstein equation, the first equation of the system (18). In other words, we are looking for the density of the field's energy, ρ, which originates in the Earth, expressed through the physical properties of the space of the Earth (which decrease with distance from the Earth as well).

As seen, the quantity ρ is expressed through the distribution function of the linear velocity of the space rotation v (see the first equation of the system), which are unknown yet. A great help to us is that fact that we have only an electromagnetic field distributed in the space. This means that with use of the condition $\rho c^2=U$ we equalize ρc^2 and U taken from the Einstein equations (18) so that we get an equation containing the distribution functions of v without the properties of matter (an electromagnetic field, in our case). With such an equation, we find a specific correlation between the distribution functions.

First we calculate is the trace of the stress-tensor of distributed matter

$$U=U_{11}+\frac{U_{22}}{r^2}+U_{33}\qquad(19)$$

which comes from the 5th, 8th, and 10th equations of the

$$\left.\begin{aligned}
&(\cos\varphi-\sin\varphi)\left(\frac{v_{tr\varphi}}{r}-\frac{v_{t\varphi}}{r^2}\right)+\omega\,(\cos\varphi+\sin\varphi)\left(\frac{v_{r\varphi}}{r}-\frac{v_{\varphi}}{r^2}\right)-\omega\,(\cos\varphi-\sin\varphi)\,v_{rr}+2v_r v_{rr}+\frac{v_{\varphi}v_{r\varphi}}{r^2}+\\
&-\frac{v_{\varphi}^2}{r^3}+(\cos\varphi+\sin\varphi)\,v_{trr}-\frac{1}{2}\cos 2\varphi\left(\frac{v_{\varphi}v_{rr}}{r}+\frac{v_r v_{r\varphi}}{r}-\frac{v_r v_{\varphi}}{r^2}\right)+\frac{1}{2}\sin 2\varphi\left(\frac{v_{\varphi}v_{r\varphi}}{r^2}-\frac{v_{\varphi}^2}{r^3}-v_r v_{rr}\right)=0\\
&(\cos\varphi+\sin\varphi)\left(\frac{v_{tr\varphi}}{r^2}-\frac{v_{t\varphi}}{r^3}\right)+(\cos\varphi-\sin\varphi)\left(\frac{v_{t\varphi\varphi}}{r^3}+\frac{v_{tr}}{r^2}\right)+\omega\,(\cos\varphi+\sin\varphi)\left(\frac{v_r}{r^2}+\frac{v_{\varphi\varphi}}{r^3}\right)+\\
&+\omega\,(\cos\varphi-\sin\varphi)\left(\frac{v_{\varphi}}{r^3}-\frac{v_{r\varphi}}{r^2}\right)+\frac{v_{\varphi}v_{\varphi\varphi}}{r^4}+\frac{v_r v_{r\varphi}}{r^2}+\frac{1}{2}\cos 2\varphi\left(\frac{v_{\varphi}^2}{r^4}-\frac{v_r^2}{r^2}-\frac{v_r v_{\varphi\varphi}}{r^3}-\frac{v_{\varphi}v_{r\varphi}}{r^3}\right)+\\
&+\frac{1}{2}\sin 2\varphi\left(\frac{2v_r v_{\varphi}}{r^3}+\frac{v_{\varphi}v_{\varphi\varphi}}{r^4}-\frac{v_r v_{r\varphi}}{r^2}\right)=0
\end{aligned}\right\}\qquad(24)$$

Einstein equations (18). We obtain

$$\begin{aligned}
\kappa U &= 4\omega^2+4\omega\,(\cos\varphi+\sin\varphi)\,\frac{v_{\varphi}}{r}-\\
&-4\omega\,(\cos\varphi-\sin\varphi)\,v_r+2v_r^2+\frac{2v_{\varphi}^2}{r^2}+\\
&+\sin 2\varphi\left(\frac{v_{\varphi}^2}{r^2}-v_r^2\right)-\cos 2\varphi\,\frac{v_r v_{\varphi}}{r}+\\
&+(\cos\varphi+\sin\varphi)\,v_{tr}+(\cos\varphi-\sin\varphi)\,\frac{v_{t\varphi}}{r}\,.
\end{aligned}\qquad(20)$$

Equalizing it to $\kappa\rho c^2$ of the first equation of the Einstein equations (18), we obtain

$$\begin{aligned}
&2\omega^2+2\omega\,(\cos\varphi+\sin\varphi)\,\frac{v_{\varphi}}{r}-2\omega\,(\cos\varphi-\sin\varphi)\,v_r+\\
&+v_r^2+\frac{v_{\varphi}^2}{r^2}+\frac{1}{2}\sin 2\varphi\left(\frac{v_{\varphi}^2}{r^2}-v_r^2\right)-\cos 2\varphi\,\frac{v_r v_{\varphi}}{r}+\\
&+2\,(\cos\varphi+\sin\varphi)\,v_{tr}+2\,(\cos\varphi-\sin\varphi)\,\frac{v_{t\varphi}}{r}=0\,.
\end{aligned}\qquad(21)$$

Thus we have all physically observable components of $T_{\alpha\beta}$ expressed in only the physical observable properties of the space. Substituting the components into the conservation law for the common field of distributed matter in the space, we look for the formulae of the distribution functions of the space rotation velocity v.

The conservation law $\nabla_{\sigma}T^{\alpha\sigma}=0$, expressed in terms of the physical observed quantities*, is [3, 4]

$$\left.\begin{aligned}
&\frac{{}^*\partial\rho}{\partial t}+D\rho+\frac{1}{c^2}\,D_{ij}U^{ij}+\\
&\qquad+\left({}^*\nabla_i-\frac{1}{c^2}\,F_i\right)J^i-\frac{1}{c^2}\,F_iJ^i=0\\
&\frac{{}^*\partial J^k}{\partial t}+2\left(D_i^k+A_{i\cdot}^{\cdot k}\right)J^i+\\
&\qquad+\left({}^*\nabla_i-\frac{1}{c^2}\,F_i\right)U^{ik}-\rho F^k=0
\end{aligned}\right\}\qquad(22)$$

which, under the specific conditions of our problem, become

$$\left.\begin{aligned}
&\frac{\partial J^i}{\partial x^i}+\frac{\partial\ln\sqrt{h}}{\partial x^i}J^i=0\\
&\frac{\partial J^k}{\partial t}+2A_{i\cdot}^{\cdot k}J^i+\frac{\partial U^{ik}}{\partial x^i}+\Delta_{im}^{k}U^{im}+\\
&\qquad+\frac{\partial\ln\sqrt{h}}{\partial x^i}U^{ik}-\rho F^k=0
\end{aligned}\right\}\qquad(23)$$

The first, a scalar equation of conservation, means $\nabla_i J^i=0$, i.e. the flow of the common field of distributed matter is conserved in the space of the Earth. The second, a vector equation of conservation, after substituting the components of J^i and U^{ik} from the Einstein equations (18), and also A_{ik} (14) and Δ_{kn}^{i} (15), give the system (24) of two non-linear differential equations with partial derivatives with respect to v (while the third equation vanishes becoming the identity "zero equals zero").

The exact solution of the system, i.e. a function which when substituted into the equations makes them identities, is

$$v=T(t)\,re^{i\varphi},\qquad(25)$$

where i is the imaginary unit, while T is a function of time (its dimension is $\sec^{-1}$).

Substituting the derivatives

$$\left.\begin{aligned}
&v_r=Te^{i\varphi}\,,\qquad v_{\varphi}=i\,r\,Te^{i\varphi}\,,\quad v_t=\dot{T}re^{i\varphi}\\
&v_{t\varphi}=i\,\dot{T}re^{i\varphi},\quad v_{tr}=Te^{i\varphi}
\end{aligned}\right\}\qquad(26)$$

into (21), we obtain, after transformations,

$$T_t\,(i+1)+\omega T\,(i-1)-\frac{iT^2}{2}+\omega^2=0\,,\qquad(27)$$

where $\dot{T}_t=\frac{\partial T}{\partial t}$. We obtain, for the real part of the equation

$$\dot{T}-\omega T+\omega^2=0\,,\qquad(28)$$

which is a linear differential equation of the first order

$$\dot{T}+f(t)T=g(t)\,,\qquad(29)$$

*The asterisk denotes the chronometrically invariant differential operators, e.g. $\frac{{}^*\partial}{\partial t}=\frac{1}{\sqrt{g_{00}}}\frac{\partial}{\partial t}$ and $\frac{{}^*\partial}{\partial x^i}=\frac{\partial}{\partial x^i}+\frac{1}{c^2}\,v_i\,\frac{{}^*\partial}{\partial t}$; see [3, 4].

whose exact solution is

$$T = e^{-F}\left(T_0 + \int_{t_0=0}^{t} g(t)\,e^{F}\,dt\right), \tag{30}$$

$$F(t) = \int f(t)\,dt\,. \tag{31}$$

Substituting $f = -\omega$, $g = -\omega^2$ and integrating the resulting expression within the limits from t to $t_0 = 0$, we obtain the solution for the real part of the function $T(t)$:

$$T(t) = e^{\omega t}\left(T_0 - \omega\right) + \omega\,, \tag{32}$$

where T_0 is the initial value of T.

The imaginary part of the (27) satisfies the differential equation

$$T_t + \omega T - \frac{1}{2}\,T^2 = 0\,, \tag{33}$$

which is Bernoulli's equation

$$T_t + fT^2 + gT = 0\,, \tag{34}$$

where $f = -\frac{1}{2}$ and $g = \omega$ are constant coefficients. Such a Bernoulli equation has the solution

$$\frac{1}{T} = E(t)\int\frac{f\,dt}{E(t)}\,, \qquad E(t) = e^{\int g\,dt}\,. \tag{35}$$

Integrating this expression, we obtain

$$T(t) = \frac{2\omega}{1 + Ce^{\omega t}}\,, \tag{36}$$

which is the imaginary part of T. Here C is a constant of integration. Assuming the initial value $t_0 = 0$, we obtain

$$C = \frac{2\omega}{T_0} - 1\,, \tag{37}$$

where T_0 is the initial value of T. Because, by definition $v = Tr\,e^{i\varphi}$ (25), T has a dimension of sec^{-1}, we consider T_0 to be the initial frequency of the vibrations of the distributed matter (background).

So we obtain the final formula for the imaginary part of the solution for T:

$$T(t) = \frac{2\omega T_0}{T_0 + \left(2\omega - T_0\right)e^{\omega t}}\,. \tag{38}$$

We therefore write the full solution for T as a complex function, which is

$$T(t) = e^{\omega t}\left(T_0 - \omega\right) + \omega + i\,\frac{2\omega T_0}{\left(2\omega - T_0\right)e^{\omega t} + T_0}\,. \tag{39}$$

We see that the imaginary part of T is zero if $T_0 = 0$. Hence the imaginary part of T originates in the presence of the initial non-zero value of T.

Assuming $T_0 = 0$, we obtain: the full solution for T has only the real solution

$$T = \omega\left(1 - e^{\omega t}\right) \tag{40}$$

when $T_0 = 0$. Substituting this solution into the expression for ρc^2, i.e. the first equation of the system (18), and taking into account the geometrization condition 21 we have obtained for electromagnetic field, we obtain the real component of the density of the energy, which is

$$\rho c^2 = \frac{3\omega}{\kappa}\left(\omega - T\right) = \frac{3\omega^2}{\kappa}\left[1 - \left(1 - e^{\omega t}\right)\right]. \tag{41}$$

This is the final formula for the observable density of the energy $W = \rho c^2$ of distributed matter in the space of the Earth, where the matter is represented by an electromagnetic field which originates in the Earth, with an additional component due to the complete rotation of the Earth's space.

2.3 Calculation of the density of the Earth microwave background at the COBE orbit and at the L2 point

We simplify formula (41) according to the assumptions of our problem. The quantity $\omega = \sqrt{GM_{\oplus}/R^3}$, the frequency of the rotation of the Earth space for an observer existing in the weightless state, takes its maximum numerical value at the equator of the Earth's surface, where $\omega = 1.24\times10^{-3}$ sec^{-1}. Obviously, the numerical value of ω decreases with altitude above the surface of the Earth. Since ω is a small value, we expand $e^{\omega t}$ into the series

$$e^{\omega t} \approx 1 + \omega t + \frac{1}{2}\,\omega^2 t^2 + \ldots \tag{42}$$

where we omit the higher order terms from consideration. As a result, we obtain, for the density of the energy of distributed matter (41) in the space of the Earth (we mean an electromagnetic field originating in the Earth as above),

$$\rho c^2 = \frac{3\omega^2}{\kappa}\,, \tag{43}$$

where $\omega = \sqrt{GM_{\oplus}/R^3}$. (In derivation of this formula we neglected the orders of ω higher than ω^2.) It should be noted that the quantity ω is derived from the weightless condition in the space, depending on the mass of the Earth $M_{\oplus}$, and the distance R from the centre of the Earth.

Because microwave radiation is related to an electromagnetic field, our theoretical result (43) is applicable to a microwave background originating from the Earth.

Now, with formula (43), we calculate the ratio between the density of the energy of the Earth microwave background at the L2 point ($R_{\text{L2}} = 1.5$ million km) and at the COBE orbit ($R_{\text{COBE}} = 6{,}370 + 900 = 7{,}270$ km)

$$\frac{\rho_{\text{L2}}}{\rho_{\text{COBE}}} = \frac{R^3_{\text{COBE}}}{R^3_{\text{L2}}} \simeq 1.1\times10^{-7}. \tag{44}$$

At the altitude of a U2 aeroplane (25 km altitude, which almost coincides with the location at the Earth's surface (within the framework of the precision of our calculation), we have $R_{\text{U2}} = 6{,}370 + 25 = 6{,}395$ km. So, we obtain the ratio between the density of the Earth microwave background at the L2 point, at the COBE orbit, and that at the U2 altitude is

$$\frac{\rho_{\text{L2}}}{\rho_{\text{U2}}} = \frac{R^3_{\text{U2}}}{R^3_{\text{L2}}} \simeq 7.8\times10^{-8}, \quad \frac{\rho_{\text{COBE}}}{\rho_{\text{U2}}} = \frac{R^3_{\text{U2}}}{R^3_{\text{COBE}}} \simeq 0.68. \tag{45}$$

We see, concerning a microwave background field which originates in the Earth (the Earth microwave background), that a measurement of the background by an absolute instrument will give almost the same result at the position of a U2 aeroplane and the COBE satellite. However, at the L2 point (as far as 1.5 million km from the Earth, the point of location of the WMAP satellite and the planned PLANCK satellite), PLANCK, with its ability to function as an absolute instrument, should sense only $\sim10^{-7}$ of the field registered either by the U2 aeroplane or by the COBE satellite.

3 The anisotropy of the Earth microwave background in the COBE orbit and at the L2 point

It is also important to understand what is the anisotropy of the Earth microwave background due to a drift of the whole space of the Earth which would one observe at the COBE orbit and at the L2 point. We solve this problem by using the equations of motion of free light-like particles (photons), which are mediators transferring electromagnetic radiation, including those in the microwave region. When treating the photons which originate in the Earth's field (the Earth microwave background, for instance), the equations of motion should manifest an anisotropy in the directions of motion of the photon due to the presence of a linear drift in the Earth's space as a whole, relative to the source of another field such as the common field of a compact group of galaxies or that of the Universe as a whole [1, 2] (a weak microwave background).

The equations of motion of free particles are the *geodesic equations*.

A light-like free particle, e.g. a free photon, moves along isotropic geodesic trajectories whose four-dimensional equations are [3, 4]

$$\frac{dK^\alpha}{d\sigma} + \Gamma^\alpha_{\mu\nu} K^\mu \frac{dx^\nu}{d\sigma} = 0\,, \tag{46}$$

where $K^\alpha = \frac{\Omega}{c}\frac{dx^\alpha}{d\sigma}$ is the four-dimensional wave vector of the photon (the vector satisfies the condition $K_\alpha K^\alpha = 0$), while Ω is the proper cyclic frequency of the photon. The three-dimensional observable interval equals the interval of observable time $d\sigma = cd\tau$ along isotropic trajectories, so $ds^2 = c^2 d\tau^2 - d\sigma^2 = 0$. In terms of the physical observable quantities, the isotropic geodesic equations are represented by their projections on the time line and spatial section of an observer [1, 2]

$$\left.\begin{aligned} &\frac{d\Omega}{d\tau} - \frac{\Omega}{c^2}\,F_i\,c^i + \frac{\Omega}{c^2}\,D_{ik}\,c^i c^k = 0\,,\\ &\frac{d}{d\tau}\left(\Omega c^i\right) + 2\Omega\left(D^i_k + A^{\cdot i}_{k\cdot}\right)c^k - \\ &\qquad\qquad - \Omega F^i + \Omega\Delta^i_{kn} c^k c^n = 0\,, \end{aligned}\right\} \tag{47}$$

where $c^i = \frac{dx^i}{d\tau}$ is the three-dimensional vector of the observable velocity of light (the square of c^i satisfies $c_k c^k = c^2$ in the fixed spatial section of the observer). The first of the equations (the scalar equation) represents the law of energy for the particle, while the vectorial equation is the three-dimensional equation of its motion.

We apply the isotropic geodesic equations to the space metric (9), which includes a linear drift of the reference space in the z-direction with a velocity v. Because the dipole-fit velocity of the Earth, extracted from the experimentally obtained anisotropy of the microwave background, is only $\mathrm{v} = 365\pm18$ km/sec, we neglect the relativistic square in the metric (9) so that it is

$$\begin{aligned} ds^2 = {} & \left(1 - \frac{2GM}{c^2 r} - \frac{\omega^2 r^2}{c^2} + \frac{2v\mathrm{v}}{c^2}\right)c^2 dt^2 + \\ & + \frac{2v\left(\cos\varphi + \sin\varphi\right)}{c}\,cdtdr + \\ & + \frac{2r\left[v\left(\cos\varphi - \sin\varphi\right) - \omega r\right]}{c}\,cdtd\varphi + \frac{2v}{c}\,cdt\,dz - \\ & - \left(1 + \frac{2GM}{c^2 r}\right)dr^2 + \frac{2v\mathrm{v}\left(\cos\varphi + \sin\varphi\right)}{c^2}\,dr dz - r^2 d\varphi^2 + \\ & + \frac{2r\mathrm{v}\left[v\left(\cos\varphi - \sin\varphi\right) - \omega r\right]}{c^2}\,d\varphi dz - \left(1 - \frac{2v\mathrm{v}}{c^2}\right)dz^2, \end{aligned} \tag{48}$$

We use the metric with the approximation specific to an observer located on board the COBE satellite or the WMAP satellite: the observer exists in the weightless state, so $\omega^2 r^2 = \frac{GM}{r}$; the linear velocity v of the Earth's space rotation doesn't depend on the z-coordinate, the direction of the drift of the whole space. We neglect the terms $\frac{v^2}{c^2}$ and also higher order terms, but retain the term $\frac{v\mathrm{v}}{c^2}$ which takes into account the drift of the whole space of the Earth: the value of v is determined in the weightless state of the observer; it is $\simeq$7.9 km/sec close to the surface of the Earth, and hence we have, near the surface, $\frac{v^2}{c^2} \approx 7\times10^{-10}$ and $\frac{v\mathrm{v}}{c^2} \approx 3\times10^{-8}$. Both values decrease with distance (altitude) from the Earth's surface, but the term $\frac{v\mathrm{v}}{c^2}$ remains two orders higher than $\frac{v^2}{c^2}$. We also neglect $\frac{GM}{c^2 r}$ which is $\approx10^{-9}$ at the Earth's surface.

Due to the fact that the terms $\frac{v\mathrm{v}}{c^2}$ are small corrections in the metric (48), it is easy to show that the exact solution of the conservation equations $v = T(t) r e^{i\varphi}$, obtained earlier in the framework of such a metric without a drift of the whole space (10), satisfies the present metric (48) where the drift is taken into account.

Using the solution for $T(t)$ (40), and expanding $e^{\omega t}$ into series $e^{\omega t} \approx 1 + \omega t + \ldots$, we obtain

$$T = -\omega^2 t\,, \tag{49}$$

then

$$v = -\omega^2 tr\, e^{i\varphi}. \tag{50}$$

We assume φ to be small. We calculate the observable characteristics of the Earth space where the drift of the whole space is taken into account, i.e. the space of the metric (48). Using the components of the fundamental metric tensor $g_{\alpha\beta}$ taken from the metric (48), we obtain

$$\left.\begin{aligned} v_1 &= \omega^2 tre^{i\varphi}(\cos\varphi + \sin\varphi)\\ v_2 &= \omega r^2\left[\omega te^{i\varphi}(\cos\varphi - \sin\varphi) + 1\right]\\ v_3 &= \omega^2 rte^{i\varphi}\end{aligned}\right\} \tag{51}$$

$$\left.\begin{aligned} F_1 &= -\omega^2 re^{i\varphi}(\cos\varphi + \sin\varphi) + \omega^2 \mathrm{v}te^{i\varphi}\\ F_2 &= -\omega^2 r^2 e^{i\varphi}(\cos\varphi - \sin\varphi) - i\omega^2 r\mathrm{v}te^{i\varphi}\\ F_3 &= -\omega^2 re^{i\varphi}\end{aligned}\right\} \tag{52}$$

$$\left.\begin{aligned} &A_{12} = \omega r\left[1 + \frac{\omega t}{2}(1-i)\right]\\ &A_{23} = \frac{i\omega^2 tre^{i\varphi}}{2}\,, \qquad A_{13} = \frac{\omega^2 te^{i\varphi}}{2}\end{aligned}\right\} \tag{53}$$

$$\left.\begin{aligned} h_{11} &= 1\,, \quad h_{13} = \frac{\omega^2 \mathrm{v}tr(\cos\varphi + \sin\varphi)\,e^{i\varphi}}{c^2}\\ h_{22} &= r^2, \quad h_{23} = \frac{\omega r^2 \mathrm{v}\left[\omega te^{i\varphi}(\cos\varphi - \sin\varphi) + 1\right]}{c^2}\\ h_{33} &= 1 - \frac{2\omega^2 \mathrm{v}tre^{i\varphi}}{c^2}\\ h &= r^2\left(1 + \frac{2\omega^2 \mathrm{v}tre^{i\varphi}}{c^2}\right)\\ h^{11} &= 1\,, \quad h^{13} = -\frac{\omega^2 \mathrm{v}tr(\cos\varphi + \sin\varphi)\,e^{i\varphi}}{c^2}\\ h^{22} &= \frac{1}{r^2}\,, \quad h^{23} = -\frac{\omega \mathrm{v}\left[\omega te^{i\varphi}(\cos\varphi - \sin\varphi) + 1\right]}{c^2}\\ h^{33} &= 1 + \frac{2\omega^2 \mathrm{v}tre^{i\varphi}}{c^2}\end{aligned}\right\} \tag{54}$$

Because the components h_{13} and h_{23} of the tensor h_{ik} depend on the time coordinate t, we obtain two non-zero components of the tensor of the space deformation D_{ik}

$$\left.\begin{aligned} D_{13} &= \frac{\omega^2 r\mathrm{v}(\cos\varphi + \sin\varphi)\,e^{i\varphi}}{2c^2}\\ D_{23} &= \frac{\omega^2 r^2 \mathrm{v}(\cos\varphi - \sin\varphi)\,e^{i\varphi}}{2c^2}\\ D_{33} &= \frac{\omega^2 r\mathrm{v}e^{i\varphi}}{c^2}\end{aligned}\right\} \tag{55}$$

the scalar $D = h^{ik} D_{ik}$ is

$$D = \frac{\omega^2 r\mathrm{v}e^{i\varphi}}{c^2}. \tag{56}$$

We now calculate the chronometric Christoffel symbols of the second kind

$$\left.\begin{aligned} &\Delta^1_{22} = -r\,, \quad \Delta^1_{23} = \frac{\omega^2 r\mathrm{v}t\,(i-1)}{2c^2} - \frac{\omega r\mathrm{v}}{c^2}\\ &\Delta^1_{33} = \frac{\omega^2 \mathrm{v}te^{i\varphi}}{c^2}\\ &\Delta^2_{12} = \frac{1}{r}\,, \quad \Delta^2_{13} = \frac{\omega^2 \mathrm{v}t\,(1-i)}{2c^2 r} + \frac{\omega \mathrm{v}}{c^2 r}\\ &\Delta^2_{33} = \frac{i\omega^2 \mathrm{v}t\,e^{i\varphi}}{c^2 r}\\ &\Delta^3_{11} = \frac{\omega^2 \mathrm{v}t\,(\cos\varphi + \sin\varphi)\,e^{i\varphi}}{c^2}\\ &\Delta^3_{12} = \frac{\omega^2 r\mathrm{v}t\,(i+1)\,e^{2i\varphi}}{2c^2}\,, \quad \Delta^3_{13} = -\frac{\omega^2 \mathrm{v}te^{i\varphi}}{c^2}\\ &\Delta^3_{22} = \frac{i\omega^2 r^2 \mathrm{v}t\,(\cos\varphi - \sin\varphi)\,e^{i\varphi}}{c^2}\\ &\Delta^3_{23} = \frac{i\omega^2 r\mathrm{v}te^{i\varphi}}{c^2}\end{aligned}\right\} \tag{57}$$

We use the above characteristics of the Earth's space to write the isotropic geodesic equations (47) in component form. We neglect the terms proportional to $\frac{1}{c^2}$ in the equations. Besides, in the framework of our asumptions, the differential with respect to proper time τ, i.e.

$$\frac{d}{d\tau} = \frac{{}^*\partial}{\partial t} + \mathrm{v}^i \frac{{}^*\partial}{\partial x^i}\,, \tag{58}$$

can be removed with the regular partial derivative $\frac{d}{d\tau} = \frac{\partial}{\partial t}$. (The starred derivatives become the regular derivatives, and also the observable velocity of light c^i doesn't depend on the z coordinate in our case where the whole space has a drift in the z direction.)

The vectorial isotropic geodesic equations, written in component notation, are

$$\left.\begin{aligned} &\frac{dc^1}{d\tau} + 2\left(D^1_k + A^{\cdot 1}_{k\cdot}\right)c^k - F^1 + \Delta^1_{22}c^2c^2 +\\ &\qquad + 2\Delta^1_{23}c^2c^3 + \Delta^1_{33}c^3c^3 = 0\\ &\frac{dc^2}{d\tau} + 2\left(D^2_k + A^{\cdot 2}_{k\cdot}\right)c^k - F^2 + 2\Delta^2_{12}c^1c^2 +\\ &\qquad + 2\Delta^2_{13}c^1c^3 + \Delta^2_{33}c^3c^3 = 0\\ &\frac{dc^3}{d\tau} + 2\left(D^3_k + A^{\cdot 3}_{k\cdot}\right)c^k - F^3 + \Delta^3_{11}c^1c^1 + 2\Delta^3_{12}c^1c^2 +\\ &\qquad + 2\Delta^3_{13}c^1c^3 + \Delta^3_{22}c^2c^2 + 2\Delta^3_{23}c^2c^3 = 0\end{aligned}\right\} \tag{59}$$

and after substituting the observable characteristics of the space, take the form (60–62), where dot denotes differentiation with respect to time.

$$\ddot{r} - 2\omega r\left[1 + \frac{\omega t\,(1-i)}{2}\right]\dot{\varphi} - \omega^2 e^{i\varphi}\left[t - \frac{\mathrm{v}r\,(\cos\varphi + \sin\varphi)}{c^2}\right]\dot{z} + \omega^2\left[r\,(\cos\varphi + \sin\varphi) - \mathrm{v}t\right]e^{i\varphi} - \\ - r\dot{\varphi}^2 + \frac{2\omega r\mathrm{v}}{c^2}\left[\frac{\omega t(i-1)}{2} - 1\right]\dot{\varphi}\dot{z} + \frac{\omega^2\mathrm{v}te^{i\varphi}}{c^2}\,\dot{z}^2 = 0\,, \tag{60}$$

$$\ddot{\varphi} + \frac{2\omega}{r}\left[1 + \frac{\omega t\,(1-i)}{2}\right]\dot{r} - \frac{\omega^2 e^{i\varphi}}{r}\left[it - \frac{\mathrm{v}r\,(\cos\varphi - \sin\varphi)}{c^2}\right]\dot{z} + \frac{\omega^2}{r}\left[r\,(\cos\varphi - \sin\varphi) + i\mathrm{v}t\right]e^{i\varphi} + \\ + \frac{2}{r}\,\dot{r}\dot{\varphi} - \frac{2\omega\mathrm{v}}{c^2 r}\left[\frac{\omega t(i-1)}{2} - 1\right]\dot{r}\dot{z} - \frac{i\omega^2\mathrm{v}t}{c^2}\,\dot{z}^2 = 0\,, \tag{61}$$

$$\ddot{z} + \omega^2 e^{i\varphi}\left[t + \frac{\mathrm{v}r\,(\cos\varphi + \sin\varphi)}{c^2}\right]\dot{r} + \omega^2 r e^{i\varphi}\left[it + \frac{\mathrm{v}r\,(\cos\varphi - \sin\varphi)}{c^2}\right]\dot{\varphi} + \frac{2\omega^2 r\mathrm{v}e^{i\varphi}}{c^2}\,\dot{z} + \\ + \omega^2 r e^{i\varphi} + \frac{\omega^2\mathrm{v}te^{i\varphi}\,(\cos\varphi + \sin\varphi)}{c^2}\,\dot{r}^2 + \frac{\omega^2 r\mathrm{v}t\,(i+1)\,e^{2i\varphi}}{c^2}\,\dot{r}\dot{\varphi} + \frac{2\omega^2\mathrm{v}te^{i\varphi}}{c^2}\,\dot{r}\dot{z} + \\ + \frac{i\omega^2 r^2\mathrm{v}t\,(\cos\varphi - \sin\varphi)\,e^{i\varphi}}{c^2}\,\dot{\varphi}^2 + \frac{2i\omega^2 r\mathrm{v}te^{i\varphi}}{c^2}\,\dot{\varphi}\dot{z} = 0, \tag{62}$$

$$\dot{r}^2 + \frac{2\omega^2 r\mathrm{v}t\,(\cos\varphi + \sin\varphi)\,e^{i\varphi}}{c^2}\,\dot{r}\dot{z} + r^2\dot{\varphi}^2 + \frac{2\omega r^2\mathrm{v}\left[\omega te^{i\varphi}\,(\cos\varphi - \sin\varphi) + 1\right]}{c^2}\,\dot{\varphi}\dot{z} + \\ + \left(1 - \frac{2\omega^2 r\mathrm{v}te^{i\varphi}}{c^2}\right)\dot{z}^2 = c^2. \tag{63}$$

The space-time interval ds along isotropic geogesics satisfies the condition $ds^2 = 0$. This condition, in the terms of physical observed quantities, implies constancy of the square of the three-dimensional observable velocity of light $c_i c^i = h_{ik}\, c^i c^k = c^2$ along the trajectory. This condition, for the metric (48), takes the form (63).

A system of the differential equations (60–63) describes the motion of light-like particles completely, in the given space-time of the metric (48).

Earlier in this study we considered only the real part $v = T(t)\, r\, e^{i\varphi}$ of the solution of the conservation equations in an electromagnetic field. Because we study the motion of photons in such an electromagnetic field (in the sample of a microwave background) we only use the real solution in the system of the equations (60–63). After the function $v = T(t)\, r\, e^{i\varphi}$ is substituted into (60–63), we have, after transformations, the formulae (64–67) (see Page 93).

We assume that a light-like signal (photon) of the Earth microwave radiation moves along the radial direction r. Because the space of the Earth at the location of a satellite (the space of the weightless state) rotates with an angular velocity ω which depends upon r, we have $\dot{\varphi} = 0$. Two satellites which measure the Earth microwave background are located at the altitudes $r_1 = 900$ km and $r_2 = 1.5$ million km respectively. Calculation of $\omega^2 = \frac{GM_\oplus}{r^3}$, where $M_\oplus = 6\times 10^{27}$g is the mass of the Earth, gives the values: $\omega_1 = 10^{-3}$ sec^{-1} and $\omega_2 = 3.5\times 10^{-6}$ sec^{-1}. Because both values are small, we use $\cos\varphi \simeq 1 + \omega t$ and $\sin\varphi \simeq \omega t$. Substituting these into the system of equations (64–67), and neglecting the terms of order higher than ω^2 (and also the other higher order terms), we obtain, finally,

$$\ddot{r} - \omega^2\left(t - \frac{r\mathrm{v}}{c^2}\right)\dot{z} + \omega^2\,(r - \mathrm{v}t) + \frac{\omega^2\mathrm{v}t}{c^2}\,\dot{z}^2 = 0\,, \tag{68}$$

$$\ddot{\varphi} + 2\omega\left(1 + \frac{2\omega t}{2}\right)\frac{\dot{r}}{r} + \frac{\omega^2\mathrm{v}}{c^2}\,\dot{z} + 4\omega^2 + 2\omega\frac{\dot{r}}{r} + \\ + \frac{2\omega\mathrm{v}\left(1 + \frac{\omega t}{2}\right)}{c^2 r}\,\dot{r}\dot{z} = 0\,, \tag{69}$$

$$\ddot{z} + \omega^2\left(t + \frac{r\mathrm{v}}{c^2}\right)\dot{r} + \frac{2\omega^2\mathrm{v}r}{c^2}\,\dot{z} + \omega^2 r + \\ + \frac{\omega^2\mathrm{v}t}{c^2}\,\dot{r}^2 + \frac{2\omega^2\mathrm{v}t}{c^2}\,\dot{r}\dot{z} = 0\,, \tag{70}$$

$$\dot{r}^2 + \frac{2\omega^2 r\mathrm{v}t}{c^2}\,\dot{r}\dot{z} + \frac{2\omega^2 r^2\mathrm{v}}{c^2}\,\dot{z} + \\ + \left(1 - \frac{2\omega^2 r\mathrm{v}t}{c^2}\right)\dot{z}^2 = c^2. \tag{71}$$

We do choose the coordinate axes so that the z-axis is directed along the motion of the Earth, in common with its own electromagnetic field, relative to the source of another feld such as the common feld of a compact group of galaxies or that of the Universe as a whole (a weak microwave background). We also assume, for simplicity, that the orbit of the satellite, on board of which an observer is located, lies in the plane orthogonal to the z-direction. In such a case, we have $\dot{z}_0 = 0$. We obtain, assuming $\dot{z}_0 = 0$,

$$\dot{r}_0^2 = c^2, \tag{72}$$

$$\ddot{r} - \omega^2 \left[t\cos\varphi - \frac{r\mathrm{v}\,(1+\cos 2\varphi + \sin 2\varphi)}{c^2} \right] \dot{z} - 2\omega r \left(1 + \frac{\omega t}{2}\right) \dot{\varphi} + 2\omega^2 r\,(1+\cos 2\varphi + \sin 2\varphi) + \\ + \omega^2 \mathrm{v} t\cos\varphi - r\dot{\varphi}^2 - \frac{2\omega r\mathrm{v}\left(\frac{\omega t}{2}+1\right)}{c^2}\,\dot{\varphi}\dot{z} + \frac{\omega^2 \mathrm{v} t\cos\varphi}{c^2}\,\dot{z}^2 = 0\,, \tag{64}$$

$$\ddot{\varphi} + 2\omega\left(1+\frac{\omega t}{2}\right)\frac{\dot{r}}{r} + \frac{\omega^2}{r}\left[t\sin\varphi + \frac{\mathrm{v} r\,(1+\cos 2\varphi - \sin 2\varphi)}{c^2}\right]\dot{z} + 2\omega^2\,(1+\cos 2\varphi - \sin 2\varphi) - \\ - \frac{\omega^2}{r}\,\mathrm{v} t\sin\varphi + \frac{2\dot{r}\dot{\varphi}}{r} + \frac{2\omega\mathrm{v}\left(\frac{\omega t}{2}+1\right)}{c^2 r}\,\dot{r}\dot{z} = 0\,, \tag{65}$$

$$\ddot{z} + \omega^2\left[t\cos\varphi + \frac{r\mathrm{v}\,(1+\cos 2\varphi + \sin 2\varphi)}{c^2}\right]\dot{r} - 2\omega^2 r\left[2t\sin\varphi - \frac{r\mathrm{v}\,(1+\cos 2\varphi - \sin 2\varphi)}{c^2}\right]\dot{\varphi} + \\ + \frac{2\omega^2 r\mathrm{v}\cos\varphi}{c^2}\,\dot{z} + \omega^2 r\cos\varphi + \frac{\omega^2 \mathrm{v} t\,(1+\cos 2\varphi + \sin 2\varphi)}{2c^2}\,\dot{r}^2 + \frac{\omega^2 \mathrm{v} t\,(\cos 2\varphi - \sin 2\varphi)}{c^2}\,\dot{r}\dot{\varphi} + \\ + \frac{2\omega^2 \mathrm{v} t\cos\varphi}{c^2}\,\dot{r}\dot{z} + \frac{2\omega^2 r^2 \mathrm{v} t\,(1-\cos 2\varphi - \sin 2\varphi)}{c^2}\,\dot{\varphi}^2 - \frac{2\omega^2 r\mathrm{v} t\sin\varphi}{c^2}\,\dot{\varphi}\dot{z} = 0\,, \tag{66}$$

$$\dot{r}^2 + \frac{2\omega^2 r\mathrm{v} t\,(1+\cos 2\varphi + \sin 2\varphi)}{c^2}\,\dot{r}\dot{z} + r^2\dot{\varphi}^2 + \frac{2\omega r^2 \mathrm{v}\left[\frac{\omega t}{2}\,(1+\cos 2\varphi - \sin 2\varphi) + 1\right]}{c^2}\,\dot{\varphi}\dot{z} + \\ + \left(1 - \frac{2\omega^2 r\mathrm{v} t\cos\varphi}{c^2}\right)\dot{z}^2 = c^2. \tag{67}$$

hence we assume $\dot{r} \simeq c$. So we have $r \simeq ct$. Substituting these into the equation of motion of a photon in the z-direction (70), and taking the weightless condition into account, we obtain the equation of motion in the z direction for a photon associated with the Earth's electromagnetic field, the Earth microwave background in particular. The equation is

$$\ddot{z} + \frac{2GM_{\oplus}}{c^2 t^2}\left(1 + \frac{\mathrm{v}}{c}\right) = 0\,. \tag{73}$$

Integrating the equation with the conditions $\dot{z}_0 = 0$ and $r \simeq ct$ taken into account, we obtain

$$\dot{z} = \frac{2GM_{\oplus}}{cr}\left(1 + \frac{\mathrm{v}}{c}\right) = \dot{z}' + \Delta z', \tag{74}$$

where the first term shows that such a photon, initially launched in the r-direction in the rotating space (gravitational field) of the Earth, is carried into the z-direction by the rotation of the space of the Earth. The second term shows carriage into the z-direction due to the motion of the Earth in this direction relative to another source such as a local group of galaxies or the whole Universe.

Denoting the first term in this formula as $\dot{z}' = \frac{2GM_{\oplus}}{cr}$ and the second term as $\Delta\dot{z}' = \frac{2GM_{\oplus}\mathrm{v}}{c^2 r}$, we obtain the relative carriage of the three-dimensional vector of the light velocity from the initial r-direction to the z-direction, due to the motion of the Earth, as

$$\frac{\Delta\dot{z}'}{\dot{z}'} = \frac{\mathrm{v}}{c}\,. \tag{75}$$

Such a relative carriage of a photon radiated from the Earth's surface, applied to the field of photons of the Earth microwave background radiated in the radial directions, reveals the anisotropy associated with the dipole component of the background.

Such a relative carriage of a photon, associated with the Earth's electromagnetic field, into the z-direction, doesn't depend on the path travelled by such a photon in the radial direction r from the Earth. This means that the anisotropy associated with the dipole component of the Earth microwave background shouldn't be dependent on altitude: it should be the same be it measured on board a U2 aeroplane (25 km), at the orbit of the COBE satellite (900 km), and at the L2 point (the WMAP satellite and PLANCK satellite, 1.5 million km from the Earth).

4 Comparing the theoretical results to experimental data. Conclusions

We have obtained, from General Relativity, two fundamental results:

- A microwave background which originates in the Earth (the EMB) decreases with altitude, such that the density of the energy of this background at the height of the COBE satellite (900 km) is just 0.68 times less that that at the height of a U2 aeroplane (25 km). The energy of the background at the L2 point (which is up to 1.5 million km from the Earth) is only $\sim 10^{-7}$ that experienced at the location either of a U2 aeroplane or of the COBE satellite;
- The anisotropy of the Earth microwave background,

due to the fast motion of the Earth relative to the source of another field, which isn't connected to the Earth but located in depths of the cosmos, does not depend on the position relative to the Earth's surface. The dipole anisotropy is therefore independent of altitude; the anisotropy will be the same be it measured at the altitude of a U2 aeroplane (25 km), the COBE satellite (900 km), or the WMAP satellite located at the L2 point (1.5 million km).

These purely theoretical conclusions, from General Relativity, cause us to consider an Earth origin of the microwave background, the monopole 2.7 K component of which was discovered in 1965 by Penzias and Wilson, in a ground-based observation [6], while the dipole 3.35 mK component was first observed in 1969 by Conklin, also via a ground-based observation [7], then studied by Henry [8], Corey [9], and also Smoot, Gorenstein, and Muller, who organized a stratosphere observation on board a U2 aeroplane [11]. (See the history of the observations in detail in Lineweaver's paper [10].)

There are many problems in the observation of the microwave background. The monopole component, at low frequencies, is easy to observe at the Earth's surface [6]. The dipole component is best observed at the altitude of a U2 aeroplane [11], at the altitude of 900 km (the COBE satellite) and also at 1.5 million km (the WMAP satellite located at the L2 point) where its anisotropy is clearly indicated [12–17]. Conversely, the monopole observed on Earth and in COBE orbit, has yet to be recorded at the L2 point: the WMAP satellite has only differential instruments on board, which are able to indicate only the anisotropy of the background, not its absolute value.

On the other hand, as shown by Robitaille [18–22], such a phenomenology of the observations has a clear explanation as an Earth microwave background which originates not in a cosmic source, but the oceans of the Earth, which produce microwave signals, in particular, with an apparent temperature of 2.7 K. Besides, as pointed out in [21, 23], the observed anisotropy of the microwave background can be explained as a relativistic effect of the motion of the observer, in common with the source of the background (the Earth), relative to the source of a noise microwave field, which has no specific temperature, and a source of which is located in depths of the cosmos (i.e. the distance from the many sources).

According to our theory, which supports the phenomenology of the Earth microwave background, proposed by Robitaille [18–22], we have four new specific terms, namely:

1. The EMB (the Earth Microwave Background);
2. The EMBM (the monopole associated with the Earth Microwave Background);
3. The EMBD (the dipole associated with the Earth Microwave Background);
4. The EMBA (the anisotropy of the Earth Microwave Background, associated with the dipole).

The PLANCK satellite (which has an absolute instrument on board), will soon be launched to the L2 point, on 31st July 2008, and should find an experimental verification of our theory.

Acknowledgement

We are very grateful to Pierre-Marie Robitaille for consultations and valuable comments. We also are thankful to Delyan Zhelyazov for a few typing mistakes found in the formulae when this issue was sent to print.

Submitted on August 29, 2007 / Accepted on September 06, 2007
First published online on September 08, 2007
Online version corrected on February 11, 2008

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A New Conformal Theory of Semi-Classical Quantum General Relativity

Indranu Suhendro

Department of Physics, Karlstad University, Karlstad 651 88, Sweden

E-mail: spherical_symmetry@yahoo.com

We consider a new four-dimensional formulation of semi-classical quantum general relativity in which the classical space-time manifold, whose intrinsic geometric properties give rise to the effects of gravitation, is allowed to evolve microscopically by means of a conformal function which is assumed to depend on some quantum mechanical wave function. As a result, the theory presented here produces a unified field theory of gravitation and (microscopic) electromagnetism in a somewhat simple, effective manner. In the process, it is seen that electromagnetism is actually an emergent quantum field originating in some kind of stochastic smooth extension (evolution) of the gravitational field in the general theory of relativity.

1 Introduction

We shall show that the introduction of an external parameter, the Planck displacement vector field, that deforms ("maps") the standard general relativistic space-time $\mathbb{S}_1$ into an evolved space-time $\mathbb{S}_2$ yields a theory of general relativity whose space-time structure obeys the semi-classical quantum mechanical law of evolution. In addition, an "already quantized" electromagnetic field arises from our schematic evolution process and automatically appears as an intrinsic geometric object in the space-time $\mathbb{S}_2$. In the process of evolution, it is seen that from the point of view of the classical space-time $\mathbb{S}_1$ alone, an external deformation takes place, since, by definition, the Planck constant does not belong to its structure. In other words, relative to $\mathbb{S}_1$, the Planck constant is an external parameter. However from the global point of view of the universal (enveloping) evolution space $\mathbb{M}_4$, the Planck constant is intrinsic to itself and therefore defines the dynamical evolution of $\mathbb{S}_1$ into $\mathbb{S}_2$. In this sense, a point in $\mathbb{M}_4$ is not strictly single-valued. Rather, a point in $\mathbb{M}_4$ has a "dimension" depending on the Planck length. Therefore, it belongs to both the space-time $\mathbb{S}_1$ and the space-time $\mathbb{S}_2$.

2 Construction of a four-dimensional metric-compatible evolution manifold $\mathbb{M}_4$

We first consider the notion of a four-dimensional, universal enveloping manifold $\mathbb{M}_4$ with coordinates x^μ endowed with a *microscopic* deformation structure represented by an exterior vector field $\phi\,(x^\mu)$ which maps the enveloped space-time manifold $\mathbb{S}_1 \in \mathbb{M}_4$ at a certain initial point P_0 onto a new enveloped space-time manifold $\mathbb{S}_2 \in \mathbb{M}_4$ at a certain point P_1 through the diffeomorphism

$$x^\mu(P_1) = x^\mu(P_0) + l\,\xi^\mu,$$

where $l = \sqrt{\frac{G\hbar}{c^3}} \approx 10^{-33}$ cm is the Planck length expressed in terms of the Newtonian gravitational constant G, the Dirac-Planck constant $\hbar$, and the speed of light in vacuum c, in such a way that

$$\phi^\mu = l\,\xi^\mu$$

$$\lim_{\hbar\to 0} \phi^\mu = 0\,.$$

From its diffeomorphic structure, we therefore see that $\mathbb{M}_4$ is a kind of *strain space*. In general, the space-time $\mathbb{S}_2$ evolves from the space-time $\mathbb{S}_1$ through the non-linear mapping

$$P(\phi) : \mathbb{S}_1 \to \mathbb{S}_2\,.$$

Note that the exterior vector field ϕ can be expressed as $\phi = \phi^\mu h_\mu = \bar{\phi}^\mu g_\mu$ (the Einstein summation convention is employed throughout this work) where h_μ and g_μ are the sets of basis vectors of the space-times $\mathbb{S}_1$ and $\mathbb{S}_2$, respectively (likewise for ξ). We remark that $\mathbb{S}_1$ and $\mathbb{S}_2$ are both endowed with metricity through their immersion in $\mathbb{M}_4$, which we shall now call the *evolution manifold*. Then, the two sets of basis vectors are related by

$$g_\mu = \left(\delta^\nu_\mu + l\,\nabla_\mu\,\xi^\nu\right) h_\nu$$

or, alternatively, by

$$g_\mu = h_\mu + l\left(\bar{\nabla}_\mu\,\bar{\xi}^\nu\right) g_\nu$$

where δ^ν_μ are the components of the Kronecker delta.

At this point, we have defined the two covariant derivatives with respect to the connections ω of $\mathbb{S}_1$ and Γ of $\mathbb{S}_2$ as follows:

$$\nabla_\lambda A^{\alpha\beta\dots}_{\mu\nu\dots} = \partial_\lambda A^{\alpha\beta\dots}_{\mu\nu\dots} + \omega^\alpha_{\sigma\lambda} A^{\sigma\beta\dots}_{\mu\nu\dots} + \omega^\beta_{\sigma\lambda}\omega^{\alpha\sigma\dots}_{\mu\nu\dots} + \dots - \omega^\sigma_{\mu\lambda} A^{\alpha\beta\dots}_{\sigma\nu\dots} - \omega^\sigma_{\nu\lambda} A^{\alpha\beta\dots}_{\mu\sigma\dots} - \dots$$

and

$$\bar{\nabla}_\lambda B^{\alpha\beta\dots}_{\mu\nu\dots} = \partial_\lambda B^{\alpha\beta\dots}_{\mu\nu\dots} + \Gamma^\alpha_{\sigma\lambda} B^{\sigma\beta\dots}_{\mu\nu\dots} + \Gamma^\beta_{\sigma\lambda} B^{\alpha\sigma}_{\mu\nu\dots} + \dots - \Gamma^\sigma_{\mu\lambda} B^{\alpha\beta\dots}_{\sigma\nu\dots} - \Gamma^\sigma_{\nu\lambda} B^{\alpha\beta\dots}_{\mu\sigma\dots} - \dots$$

for arbitrary tensor fields A and B, respectively. Here $\partial_\mu = \partial/\partial x^\mu$, as usual. The two covariant derivatives above are equal only in the limit $\hbar \to 0$.

Furthermore, we assume that the connections ω and Γ are generally asymmetric, and can be decomposed into their symmetric and anti-symmetric parts, respectively, as

$$\omega^{\lambda}_{\mu\nu} = \left(h^{\lambda}, \partial_{\upsilon} h_{\mu}\right) = \omega^{\lambda}_{(\mu\nu)} + \omega^{\lambda}_{[\mu\nu]}$$

and

$$\Gamma^{\lambda}_{\mu\nu} = \left(g^{\lambda}, \partial_{\upsilon} g_{\mu}\right) = \Gamma^{\lambda}_{(\mu\nu)} + \Gamma^{\lambda}_{[\mu\nu]}.$$

Here, by (a, b) we shall mean the inner product between the arbitrary vector fields a and b.

Furthermore, by direct calculation we obtain the relation

$$\partial_{\nu} g_{\mu} = \left(\omega^{\lambda}_{\mu\nu} + l\left(\nabla^{\sigma}_{\mu\xi}\right)\omega^{\lambda}_{\sigma\nu} + l\,\partial_{\nu}\left(\nabla_{\mu}\xi^{\lambda}\right)\right) h_{\lambda}\,.$$

Hence, setting

$$F^{\lambda}_{\mu\nu} = \omega^{\lambda}_{\mu\nu} + l\left(\left(\nabla_{\mu}\xi^{\sigma}\right)\omega^{\lambda}_{\sigma\nu} + \partial_{\nu}\left(\nabla_{\mu}\xi^{\lambda}\right)\right) =$$
$$= \omega^{\lambda}_{\mu\nu} + l\left(\left(\nabla_{\mu}\xi^{\sigma}\right)\omega^{\lambda}_{\sigma\nu} + \partial_{\nu}\partial^{\lambda}_{\mu\xi} + \xi^{\sigma}\partial_{\nu}\omega^{\lambda}_{\sigma\mu} + \left(\partial_{\nu}\xi^{\sigma}\right)\omega^{\lambda}_{\sigma\mu}\right)$$

we may simply write

$$\partial_{\nu} g_{\mu} = F^{\lambda}_{\mu\nu} h_{\lambda}\,.$$

Meanwhile, we also have the following inverse relation:

$$h_{\mu} = \left(\delta^{\nu}_{\mu} - l\,\bar{\nabla}_{\mu}\bar{\xi}^{\nu}\right) g_{\nu}\,.$$

Hence we obtain

$$\partial_{\nu} g_{\mu} = \left(\omega^{\lambda}_{\mu\nu} + l\left(\nabla_{\mu}\xi^{\sigma}\right)\omega^{\lambda}_{\sigma\nu} + l\,\partial_{\nu}\partial^{\lambda}_{\mu\xi} + \right.$$
$$+ l\,\xi^{\sigma}\partial_{\nu}\omega^{\lambda}_{\sigma\mu} + l\left(\partial_{\nu}\xi^{\sigma}\right)\omega^{\lambda}_{\sigma\mu} - l\,\omega^{\sigma}_{\mu\nu}\bar{\nabla}_{\sigma}\bar{\xi}^{\lambda} -$$
$$- l\left(\nabla_{\mu}\xi^{\rho}\right)\omega^{\sigma}_{\rho\nu}\bar{\nabla}_{\sigma}\bar{\xi}^{\lambda} - l\left(\partial_{\upsilon}\partial_{\mu}\xi^{\sigma}\right)\bar{\nabla}_{\sigma}\bar{\xi}^{\lambda} -$$
$$\left. - l\,\xi^{\rho}\left(\partial_{\nu}\omega^{\sigma}_{\rho\mu}\right)\bar{\nabla}_{\sigma}\bar{\xi}^{\lambda} - l\left(\partial_{\nu}\xi^{\rho}\right)\omega^{\sigma}_{\rho\mu}\bar{\nabla}_{\sigma}\bar{\xi}^{\lambda}\right) g_{\lambda}\,.$$

Using the relation $\partial_{\nu} g_{\mu} = \Gamma^{\lambda}_{\mu\nu} g_{\lambda}$ (similarly, $\partial_{\nu} h_{\mu} = \omega^{\lambda}_{\mu\nu} h_{\lambda}$), we obtain the relation between the two connections Γ and ω as follows:

$$\Gamma^{\lambda}_{\mu\nu} = \omega^{\lambda}_{\mu\nu} + l\left(\left(\nabla_{\mu}\xi^{\sigma}\right)\omega^{\lambda}_{\sigma\nu} + \partial_{\nu}\partial_{\mu}\xi^{\lambda} + \right.$$
$$+ \xi^{\sigma}\partial_{\nu}\omega^{\lambda}_{\sigma\mu}\left(\partial_{\nu}\xi^{\sigma}\right)\omega^{\lambda}_{\sigma\mu} - \omega^{\sigma}_{\mu\nu}\bar{\nabla}_{\sigma}\bar{\xi}^{\lambda} - \left(\nabla_{\mu}\xi^{\rho}\right)\omega^{\sigma}_{\rho\nu}\bar{\nabla}_{\sigma}\bar{\xi}^{\lambda} -$$
$$\left. - \left(\partial_{\nu}\partial_{\mu}\xi^{\sigma}\right)\bar{\nabla}_{\sigma}\bar{\xi}^{\lambda} - \xi^{\rho}\left(\partial_{\nu}\omega^{\sigma}_{\rho\mu}\right)\bar{\nabla}_{\sigma}\bar{\xi}^{\lambda} - \left(\partial_{\nu}\xi^{\rho}\right)\omega^{\sigma}_{\rho\mu}\bar{\nabla}_{\sigma}\bar{\xi}^{\lambda}\right)$$

which is a general non-linear relation in the components of the exterior displacement field ξ. We may now write

$$\Gamma^{\lambda}_{\mu\nu} = F^{\lambda}_{\mu\nu} + G^{\lambda}_{\mu\nu}$$

where, recalling the previous definition of $F^{\lambda}_{\mu\nu}$, it can be re-written as

$$F^{\lambda}_{\mu\nu} = \omega^{\lambda}_{\mu\nu} + l\left(\left(\partial_{\nu}\omega^{\lambda}_{\sigma\mu} + \omega^{\rho}_{\mu\sigma}\omega^{\lambda}_{\rho\nu}\right)\xi^{\sigma} + \right.$$
$$\left. + \partial_{\nu}\partial_{\mu}\xi^{\lambda} + \left(\partial_{\mu}\xi^{\sigma}\right)\omega^{\lambda}_{\sigma\nu} + \left(\partial_{\nu}\xi^{\sigma}\right)\omega^{\lambda}_{\sigma\mu}\right)$$

and where

$$G^{\lambda}_{\mu\nu} = -l\left(\omega^{\sigma}_{\mu\nu} + l\left(\left(\nabla_{\mu}\xi^{\rho}\right)\omega^{\sigma}_{\rho\nu}\right.\right.$$
$$\left.\left. + \partial_{\nu}\partial_{\mu}\xi^{\sigma} + \xi^{\rho}\partial_{\nu}\omega^{\sigma}_{\rho\mu} + \left(\partial_{\nu}\xi^{\rho}\right)\omega^{\sigma}_{\rho\mu}\right)\right)\bar{\nabla}_{\sigma}\bar{\xi}^{\lambda}\,.$$

At this point, the intrinsic curvature tensors of the space-times $\mathbb{S}_1$ and $\mathbb{S}_2$ are respectively given by

$$K^{\sigma}{}_{\rho\mu\nu} = 2\left(h^{\sigma}, \partial_{[\mu}\partial_{\nu]}h_{\rho}\right) =$$
$$= \partial_{\mu}\omega^{\sigma}_{\rho\nu} - \partial_{\nu}\omega^{\sigma}_{\rho\mu} + \omega^{\lambda}_{\rho\nu}\omega^{\sigma}_{\lambda\mu} - \omega^{\lambda}_{\rho\mu}\omega^{\sigma}_{\lambda\nu}$$

and

$$R^{\sigma}{}_{\rho\mu\nu} = 2\left(g^{\sigma}, \partial_{[\mu}\partial_{\nu]}g_{\rho}\right) =$$
$$= \partial_{\mu}\Gamma^{\sigma}_{\rho\nu} - \partial_{\nu}\Gamma^{\sigma}_{\rho\mu} + \Gamma^{\lambda}_{\rho\nu}\Gamma^{\sigma}_{\lambda\mu} - \Gamma^{\lambda}_{\rho\mu}\Gamma^{\sigma}_{\lambda\nu}\,.$$

We may also define the following quantities built from the connections $\omega^{\lambda}_{\mu\nu}$ and $\Gamma^{\lambda}_{\mu\nu}$:

$$D^{\sigma}{}_{\rho\mu\nu} = \partial_{\mu}\omega^{\sigma}_{\rho\nu} + \partial_{\nu}\omega^{\sigma}_{\rho\mu} + \omega^{\lambda}_{\rho\nu}\omega^{\sigma}_{\lambda\mu} + \omega^{\lambda}_{\rho\mu}\omega^{\sigma}_{\lambda\nu}$$

and

$$E^{\sigma}{}_{\rho\mu\nu} = \partial_{\mu}\Gamma^{\sigma}_{\rho\nu} + \partial_{\nu}\Gamma^{\sigma}_{\rho\mu} + \Gamma^{\lambda}_{\rho\nu}\Gamma^{\sigma}_{\lambda\mu} + \Gamma^{\lambda}_{\rho\mu}\Gamma^{\sigma}_{\lambda\nu}$$

from which we may define two additional "curvatures" X and P by

$$X^{\sigma}{}_{\rho\mu\nu} = \left(h^{\sigma}, \partial_{\mu}\partial_{\nu}h_{\rho}\right) = \frac{1}{2}\left(K^{\sigma}{}_{\rho\mu\nu} + D^{\sigma}{}_{\rho\mu\nu}\right) =$$
$$= \partial_{\mu}\omega^{\sigma}_{\rho\nu} + \omega^{\lambda}_{\rho\nu}\omega^{\sigma}_{\lambda\mu}$$

and

$$P^{\sigma}{}_{\rho\mu\nu} = \left(g^{\sigma}, \partial_{\mu}\partial_{\nu}g_{\rho}\right) = \frac{1}{2}\left(R^{\sigma}{}_{\rho\mu\nu} + E^{\sigma}{}_{\rho\mu\nu}\right) =$$
$$= \partial_{\mu}\Gamma^{\sigma}_{\rho\nu} + \Gamma^{\lambda}_{\rho\nu}\Gamma^{\sigma}_{\lambda\mu}$$

such that $K^{\sigma}{}_{\rho\mu\nu} = 2\,X^{\sigma}{}_{\rho[\mu\nu]}$ and $R^{\sigma}{}_{\rho\mu\nu} = 2\,P^{\sigma}{}_{\rho[\mu\nu]}$.

Now, we see that

$$F^{\lambda}_{(\mu\nu)} = \omega^{\lambda}_{(\mu\nu)} + l\left(\frac{1}{2}D^{\lambda}{}_{\sigma\mu\nu}\xi^{\sigma} + \partial_{\nu}\partial_{\mu}\xi^{\lambda}\right) +$$
$$+ l\left(\left(\partial_{\mu}\xi^{\sigma}\right)\omega^{\lambda}_{\sigma\nu} + \left(\partial_{\nu}\xi^{\sigma}\right)\omega^{\lambda}_{\sigma\mu}\right)$$

and

$$F^{\lambda}_{[\mu\nu]} = \omega^{\lambda}_{[\mu\nu]} + \frac{1}{2}\,l\,K^{\lambda}{}_{\sigma\mu\nu}\xi^{\sigma}\,.$$

In addition, we also have

$$G^{\lambda}_{(\mu\nu)} = l\left(\omega^{\sigma}_{(\mu\nu)} + l\left(\frac{1}{2}D^{\sigma}{}_{\rho\mu\nu}\xi^{\rho} + \partial_{\nu}\partial_{\mu}\xi^{\sigma}\right)\right)\bar{\nabla}_{\sigma}\bar{\xi}^{\lambda} +$$
$$+ l\left(l\left(\left(\partial_{\mu}\xi^{\rho}\right)\omega^{\sigma}_{\rho\nu} + \left(\partial_{\nu}\xi^{\rho}\right)\omega^{\sigma}_{\rho\mu}\right)\right)\bar{\nabla}_{\sigma}\bar{\xi}^{\lambda}$$

and

$$G^{\lambda}_{[\mu\nu]} = l\left(\omega^{\sigma}_{[\mu\nu]} - \frac{1}{2}\,l\,K^{\sigma}{}_{\rho\mu\nu}\xi^{\rho}\right)\bar{\nabla}_{\sigma}\bar{\xi}^{\lambda}.$$

Now, the metric tensor g of the space-time $\mathbb{S}_1$ and the metric tensor h of the space-time $\mathbb{S}_2$ are respectively given by

$$h_{\mu\nu} = \left(h_{\mu}, h_{\nu}\right)$$

and

$$g_{\mu\nu} = \left(g_{\mu}, g_{\nu}\right)$$

where the following relations hold:

$$h_{\mu\sigma} h^{\nu\sigma} = \delta^{\nu}_{\mu}$$

$$g_{\mu\sigma} g^{\nu\sigma} = \delta^{\nu}_{\mu}$$

In general, the two conditions $h_{\mu\sigma}\, g^{\nu\sigma} \neq \delta^{\nu}_{\mu}$ and $g_{\mu\sigma}\, h^{\nu\sigma} \neq \delta^{\nu}_{\mu}$ must be fulfilled unless $l = 0$ (in the limit $\hbar \to 0$). Furthermore, we have the metricity conditions

$$\nabla_\lambda\, h_{\mu\nu} = 0\,,$$

and

$$\bar{\nabla}_\lambda\, g_{\mu\nu} = 0\,.$$

However, note that in general, $\bar{\nabla}_\lambda\, h_{\mu\nu} \neq 0$ and $\nabla_\lambda g_{\mu\nu} \neq 0$.

Hence, it is straightforward to see that in general, the metric tensor g is related to the metric tensor h by

$$g_{\mu\nu} = h_{\mu\nu} \,+\, 2\,l\,\nabla_{(\mu}\xi_{\nu)} + l^2\,\nabla_\mu \xi^\lambda\,\nabla_\nu \xi_\lambda$$

which in the linear approximation reads

$$g_{\mu\nu} = h_{\mu\nu} + 2l\,\nabla_{(\mu}\xi_{\nu)}\,.$$

The formal structure of our underlying geometric framework clearly implies that the same structure holds in n dimensions as well.

3 The conformal theory

We are now in the position to extract a physical theory of quantum gravity from the geometric framework in the preceding section by considering the following linear conformal mapping:

$$g_\mu = e^{\varphi} h_\mu$$

where the continuously differentiable scalar function $\varphi\,(x^\mu)$ is the generator of the quantum displacement field in the evolution space $\mathbb{M}_4$ and therefore connects the two space-times $\mathbb{S}_1$ and $\mathbb{S}_2$.

Now, for reasons that will be apparent soon, we shall define the generator φ in terms of the canonical quantum mechanical wave function $\psi\,(x^\mu)$ as

$$\varphi = \ln\,(1 + M\psi)^{\frac{1}{2}}$$

where

$$M = \pm\frac{1}{2}\,l\left(i\,\frac{m_0 c}{\hbar}\right)^2.$$

Here m_0 is the rest mass of the electron. Note that the sign $\pm$ signifies the signature of the space-time used.

Now, we also have the following relations:

$$g^\mu = e^{-\varphi} h^\mu,$$

$$h_\mu = e^{-\varphi} g_\mu,$$

$$h^\mu = e^{\varphi} g^\mu,$$

$$(g_\mu, g^\nu) = (h_\mu, h^\nu) = \delta^{\nu}_{\mu}\,,$$

$$(g_\mu, h^\nu) = e^{2\,\varphi}\delta^{\nu}_{\mu}\,,$$

$$(h_\mu, g^\nu) = e^{-2\,\varphi}\delta^{\nu}_{\mu}\,,$$

as well as the conformal transformation

$$g_{\mu\nu} = e^{2\varphi} h_{\mu\nu}\,.$$

Hence

$$g^{\mu\nu} = e^{-2\,\varphi}\, h^{\mu\nu}.$$

We immediately see that

$$g_{\mu\sigma}\, h^{\nu\sigma} = e^{2\varphi}\,\delta^{\nu}_{\mu}\,,$$

$$h_{\mu\sigma}\, g^{\nu\sigma} = e^{-2\varphi}\,\delta^{\nu}_{\mu}\,.$$

At this point, we see that the world-line of the space-time $\mathbb{S}_2$, $s = \int \sqrt{h_{\mu\nu}\, dx^\mu dx^\nu}$, is connected to that of the space-time $\mathbb{S}_1$, $\sigma = \int \sqrt{g_{\mu\nu}\, dx^\mu dx^\nu}$, through

$$ds = e^{2\varphi} d\sigma\,.$$

Furthermore, from the relation

$$g_\mu = \left(\delta^{\nu}_{\mu} + l\,\nabla_\mu \xi^\nu\right) h_\nu = e^{\varphi} h_\mu$$

we obtain the important relation

$$l\,\nabla_\nu \xi_\mu = (e^{\varphi} - 1)\, h_{\mu\nu}\,,$$

which means that

$$\Phi_{\mu\nu} = l\,\nabla_\nu \xi_\mu = \Phi_{\nu\mu}\,,$$

i.e., the quantum displacement gradient tensor field Φ is symmetric. Hence we may simply call Φ the *quantum strain tensor field*. We also see that the components of the quantum displacement field, $\phi^\mu = l\,\xi^\mu$, can now be described by the wave function ψ as

$$\phi_\mu = l\,\partial_\mu \psi$$

i.e.,

$$\psi = \psi_0 + \frac{1}{l}\int \phi_\mu dx^\mu$$

for an arbitrary initial value ψ_0 (which, most conveniently, can be chosen to be 0).

Furthermore, we note that the integrability condition $\Phi_{\mu\nu} = \Phi_{\nu\mu}$ means that the space-time $\mathbb{S}_1$ must now possess a symmetric, linear connection, i.e.,

$$\omega^{\lambda}_{\mu\nu} = \omega^{\lambda}_{\nu\mu} = \frac{1}{2}\, h^{\sigma\lambda}\left(\partial_\nu h_{\sigma\mu} - \partial_\sigma h_{\mu\nu} + \partial_\mu h_{\nu\sigma}\right),$$

which are just the Christoffel symbols $\{ {}^{\,\lambda}_{\mu\nu} \}$ in the space-time $\mathbb{S}_1$. Hence ω is now none other than the symmetric Levi-Civita (Riemannian) connection. Using the metricity condition $\partial_\lambda g_{\mu\nu} = \Gamma_{\mu\nu\lambda} \,+\, \Gamma_{\nu\mu\lambda}$, i.e.,

$$\partial_\lambda\, g_{\mu\nu} = M h_{\mu\nu}\,\partial_\lambda \psi + (1 + M\,\psi)\,(\omega_{\mu\nu\lambda} \,+\, \omega_{\nu\mu\lambda})\,,$$

we obtain the mixed form

$$\omega_{\lambda\mu\nu} = \frac{1}{2}\,(1 + M\psi)^{-1}\,(\partial_\lambda g_{\mu\nu} - \partial_\mu g_{\nu\lambda} + \partial_\nu g_{\lambda\mu}) - $$
$$- \frac{1}{2}\, M\,(1 + M\psi)^{-1}\,(h_{\mu\nu}\partial_\lambda \psi - h_{\nu\lambda}\partial_\mu \psi + h_{\lambda\mu}\partial_\nu \psi)$$

i.e.,

$$\omega^{\lambda}_{\mu\nu} = \frac{1}{2}\,(1+M\psi)^{-1}\, h^{\lambda\rho}\,(\partial_\rho g_{\mu\nu} - \partial_\mu\, g_{\nu\rho} + \partial_\nu\, g_{\rho\mu}) - \\ -\frac{1}{2}\,M\,(1+M\psi)^{-1}\left(\delta^{\lambda}_{\mu}\partial_\nu\psi + \delta^{\lambda}_{\nu}\partial_\mu\psi - h_{\mu\nu}h^{\lambda\rho}\partial_\rho\psi\right).$$

It may be noted that we have used the customary convention in which $\Gamma_{\lambda\mu\nu} = g_{\lambda\rho}\,\Gamma^{\rho}_{\mu\nu}$ and $\omega_{\lambda\mu\nu} = h_{\lambda\rho}\omega^{\rho}_{\mu\nu}$.

Now we shall see why we have made the particular choice $\varphi = \ln\,(1+M\,\psi)^{\frac{1}{2}}$. In order to explicitly show that it now possess a *stochastic* part, let us rewrite the components of the metric tensor of the space-time $\mathbb{S}_2$ as

$$g_{\mu\nu} = (1+M\psi)\, h_{\mu\nu}\,.$$

Combining this relation with the linearized relation $g_{\mu\nu} = h_{\mu\nu} + 2\,l\,\nabla_{(\mu}\,\xi_{\nu)}$ and contracting the resulting relation, we obtain

$$l D^2\psi = 2\left(e^{2\varphi} - 1\right) = 2M\psi\,,$$

where we have defined the differential operator $D^2 = h^{\mu\nu}\nabla_\mu\nabla_\nu$ such that

$$D^2\psi = h^{\mu\nu}\left(\partial_\mu\partial_\nu\psi - \omega^{\rho}_{\mu\nu}\,\partial_\rho\psi\right).$$

Expressing *M* explicitly, we obtain $D^2\psi = \mp\left(\frac{m_0\,c}{\hbar}\right)^2\psi$, i.e.,

$$\left(D^2 \pm \left(\frac{m_0 c}{\hbar}\right)^2\right)\psi = 0$$

which is precisely the *Klein-Gordon equation in the presence of gravitation*.

We may note that, had we combined the relation $g_{\mu\nu} = (1+M\,\psi)\,h_{\mu\nu}$ with the fully non-linear relation

$$g_{\mu\nu} = h_{\mu\nu} + 2l\,\nabla_{(\mu}\xi_{\nu)} + l^2\,\nabla_\mu\xi^\lambda\nabla_\nu\xi_\lambda\,,$$

we would have obtained the following *non-linear* Klein-Gordon equation:

$$\left(D^2 \pm \left(\frac{m_0\,c}{\hbar}\right)^2\right)\psi = l^2 h^{\rho\sigma}h^{\mu\nu}\,(\nabla_\rho\nabla_\mu\psi)\,(\nabla_\sigma\nabla_\nu\psi)\,.$$

Now, from the general relation between the connections Γ and ω given in Section 2, we obtain the following important relation:

$$\Gamma^{\lambda}_{[\mu\nu]} = -\frac{1}{2}\,l\left(\delta^{\lambda}_{\sigma} - l\,\bar{\nabla}_\sigma\bar{\xi}^\lambda\right)K^{\sigma}{}_{\rho\mu\nu}\xi^\rho,$$

which not only connects the torsion of the space-time $\mathbb{S}_2$ with the curvature of the space-time $\mathbb{S}_1$, but also describes the torsion as an intrinsic (geometric) quantum phenomenon. Note that

$$K^{\sigma}{}_{\rho\mu\nu} = \partial_\mu\begin{Bmatrix}\sigma\\ \rho\nu\end{Bmatrix} - \partial_\nu\begin{Bmatrix}\sigma\\ \rho\mu\end{Bmatrix} + \\ + \begin{Bmatrix}\lambda\\ \rho\nu\end{Bmatrix}\begin{Bmatrix}\sigma\\ \lambda\mu\end{Bmatrix} - \begin{Bmatrix}\lambda\\ \rho\mu\end{Bmatrix}\begin{Bmatrix}\sigma\\ \lambda\nu\end{Bmatrix}$$

are now the components of the Riemann-Christoffel curvature tensor describing the curvature of space-time in the standard general relativity theory.

Furthermore, using the relation between the two sets of basis vectors g_μ and h_μ, it is easy to see that the connection Γ is *semi-symmetric* as

$$\Gamma^{\lambda}_{\mu\nu} = \omega^{\lambda}_{\mu\nu} + \delta^{\lambda}_{\mu}\,\partial_\nu\varphi$$

or, written somewhat more explicitly,

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2}\,h^{\sigma\lambda}\,(\partial_\nu h_{\sigma\mu} - \partial_\sigma h_{\mu\nu} + \partial_\mu h_{\nu\sigma}) + \\ + \frac{1}{2}\,\delta^{\lambda}_{\mu}\,\partial_\nu\big(\ln\,(1+M\psi)\big).$$

We immediately obtain

$$\Gamma^{\lambda}_{(\mu\nu)} = \omega^{\lambda}_{\mu\nu} + \frac{1}{2}\left(\delta^{\lambda}_{\mu}\partial_\nu\varphi + \delta^{\lambda}_{\nu}\,\partial_\mu\varphi\right)$$

and

$$\Gamma^{\lambda}_{[\mu\nu]} = \frac{1}{2}\left(\delta^{\lambda}_{\mu}\partial_\nu\varphi - \delta^{\lambda}_{\nu}\partial_\mu\varphi\right).$$

Additionally, using the relation

$$\omega^{\nu}_{\nu\mu} = \omega^{\nu}_{\mu\nu} = \partial_\mu\left(\ln\sqrt{\det{(h)}}\right) = \\ = \partial_\mu\left(\ln\left(e^{-\varphi}\sqrt{\det{(g)}}\right)\right) = \partial_\mu\left(\ln\sqrt{\det{(g)}}\right) - \partial_\mu\varphi$$

we may now define two *semi-vectors* by the following contractions:

$$\Gamma_\mu = \Gamma^{\nu}_{\nu\mu} = \partial_\mu\left(\ln\sqrt{\det{(h)}}\right) + 4\,\partial_\mu\varphi$$

$$\Delta_\mu = \Gamma^{\nu}_{\mu\nu} = \partial_\mu\left(\ln\sqrt{\det{(h)}}\right) + \partial_\mu\varphi$$

or, written somewhat more explicitly,

$$\Gamma_\mu = \partial_\mu\left(\ln\sqrt{\det{(h)}} + \ln\,(1+M\psi)^2\right)$$

$$\Delta_\mu = \partial_\mu\left(\ln\sqrt{\det{(h)}} + \ln\sqrt{1+M\psi}\right).$$

We now define the *torsion vector* by

$$\tau_\mu = \Gamma^{\nu}_{[\nu\mu]} = \frac{3}{2}\,\partial_\mu\varphi\,.$$

In other words,

$$\tau_\mu = \frac{3}{4}\,\frac{M}{(1+M\psi)}\,\partial_\mu\psi\,.$$

Furthermore, it is easy to show that the curvature tensors of our two space-times $\mathbb{S}_1$ and $\mathbb{S}_2$ are now identical:

$$R^{\sigma}{}_{\rho\mu\nu} = K^{\sigma}{}_{\rho\mu\nu}$$

which is another way of saying that the conformal transformation $g_\mu = e^\varphi\, h_\mu$ leaves the curvature tensor of the space-time $\mathbb{S}_1$ invariant. As an immediate consequence, we obtain the ordinary expression

$$R_{\rho\sigma\mu\nu} = \frac{1}{2}\,(\partial_\mu\partial_\sigma h_{\rho\nu} + \partial_\nu\partial_\rho h_{\sigma\mu} - \partial_\nu\partial_\sigma h_{\rho\mu} - \partial_\mu\partial_\rho h_{\sigma\nu}) + \\ + h_{\alpha\beta}\left(\omega^{\alpha}_{\rho\nu}\omega^{\beta}_{\sigma\mu} - \omega^{\alpha}_{\rho\mu}\,\omega^{\beta}_{\sigma\nu}\right).$$

Hence the following cyclic symmetry in Riemannian geometry:

$$R_{\rho\sigma\mu\nu} + R_{\rho\mu\nu\sigma} + R_{\rho\nu\sigma\mu} = 0$$

is preserved in the presence of torsion. In addition, besides the obvious symmetry $R_{\rho\sigma\mu\nu} = -R_{\rho\sigma\nu\mu}$, we also have the symmetry

$$R_{\rho\sigma\mu\nu} = -R_{\sigma\rho\mu\nu}$$

which is due to the metricity condition of the space-times $\mathbb{S}_1$ and $\mathbb{S}_2$. This implies the vanishing of the so-called Homothetic curvature as

$$H_{\mu\nu} = R^{\sigma}{}_{\sigma\mu\nu} = 0\,.$$

The Weyl tensor is given in the usual manner by

$$C_{\rho\sigma\mu\nu} = R_{\rho\sigma\mu\nu} - \frac{1}{2}\left(h_{\rho\mu}R_{\sigma\nu} + h_{\sigma\nu}R_{\rho\mu} - \right.$$
$$\left. - h_{\rho\nu}R_{\sigma\mu} - h_{\sigma\mu}R_{\rho\nu}\right) - \frac{1}{6}\left(h_{\rho\nu}h_{\sigma\mu} - h_{\rho\mu}h_{\sigma\nu}\right)R\,,$$

where $R_{\mu\nu} = R^{\sigma}{}_{\mu\sigma\nu}$ are the components of the symmetric Ricci tensor and $R = R^{\mu}{}_{\mu}$ is the Ricci scalar.

Now, by means of the conformal relation $g_{\mu\nu} = e^{2\varphi}\, h_{\mu\nu}$ we obtain the expression

$$R_{\rho\sigma\mu\nu} = e^{-2\varphi}\Big(\partial_\mu\partial_\sigma g_{\rho\nu} + \partial_\nu\partial_\rho g_{\sigma\mu} - \partial_\nu\partial_\sigma g_{\rho\mu}\partial_\mu\partial_\rho g_{\sigma\nu} +$$
$$+ g_{\alpha\beta}\left(\Gamma^{\alpha}_{\rho\nu}\Gamma^{\beta}_{\sigma\mu} - \Gamma^{\alpha}_{\rho\mu}\Gamma^{\beta}_{\sigma\nu}\right) + \left(\partial_\mu g_{\sigma\nu} - \partial_\nu g_{\sigma\mu}\right)\partial_\rho\varphi +$$
$$+ \left(\partial_\nu g_{\rho\mu} - \partial_\mu g_{\rho\nu}\right)\partial_\sigma\varphi + \left(\partial_\rho\, g_{\sigma\nu} - \partial_\sigma g_{\rho\nu}\right)\partial_\mu\varphi +$$
$$+ \left(\partial_\sigma g_{\rho\mu} - \partial_\rho g_{\sigma\mu}\right)\partial_\nu\varphi + g_{\sigma\nu}\partial_\mu\partial_\rho\varphi + g_{\rho\mu}\partial_\nu\partial_\sigma\varphi +$$
$$- g_{\rho\nu}\partial_\mu\partial_\sigma\varphi - g_{\mu\sigma}\partial_\nu\partial_\rho\varphi + 2\left(g_{\sigma\mu}\partial_\rho\varphi\,\partial_\nu\varphi + \right.$$
$$\left. + g_{\rho\nu}\partial_\mu\varphi\,\partial_\sigma\varphi - g_{\sigma\nu}\partial_\rho\varphi\,\partial_\mu\varphi - g_{\rho\mu}\partial_\sigma\varphi\,\partial_\nu\varphi\right) +$$
$$+ g_{\alpha\beta}\left(\left(\Gamma^{\alpha}_{\rho\mu}\partial_\nu\varphi - \Gamma^{\alpha}_{\rho\nu}\partial_\mu\varphi\right)\delta^{\beta}_{\sigma} - \left(\Gamma^{\beta}_{\sigma\mu}\partial_\nu\varphi - \Gamma^{\beta}_{\sigma\nu}\partial_\mu\varphi\right)\delta^{\alpha}_{\rho}\right)\Big).$$

Note that despite the fact that the curvature tensor of the space-time $\mathbb{S}_2$ is identical to that of the space-time $\mathbb{S}_1$ and that both curvature tensors share common algebraic symmetries, the Bianchi identity in $\mathbb{S}_2$ is not the same as the ordinary Bianchi identity in the torsion-free space-time $\mathbb{S}_1$. Instead, we have the following *generalized* Bianchi identity:

$$\bar{\nabla}_\lambda R_{\rho\sigma\mu\nu} + \bar{\nabla}_\mu R_{\rho\sigma\nu\lambda} + \bar{\nabla}_\nu R_{\rho\sigma\lambda\mu} =$$
$$= 2\left(\Gamma^{\eta}_{[\mu\nu]}R_{\rho\sigma\eta\lambda} + \Gamma^{\eta}_{[\nu\lambda]}R_{\rho\sigma\eta\mu} + \Gamma^{\eta}_{[\lambda\mu]}\,R_{\rho\sigma\eta\nu}\right).$$

Contracting the above relation, we obtain

$$\bar{\nabla}_\mu\left(R^{\mu\nu} - \frac{1}{2}\,g^{\mu\nu}R\right) = 2\,g^{\rho\nu}\Gamma^{\sigma}_{[\lambda\rho]}\,R^{\lambda}{}_{\sigma} + \Gamma^{\lambda}_{[\rho\sigma]}R^{\rho\sigma\nu}{}_{\lambda}\,.$$

Combining the two generalized Bianchi identities above with the relation $\Gamma^{\lambda}_{[\mu\nu]} = \frac{1}{2}\left(\delta^{\lambda}_{\mu}\,\partial_\nu\varphi - \delta^{\lambda}_{\nu}\partial_\mu\varphi\right)$, as well as recalling the definition of the torsion vector, and taking into account the symmetry of the Ricci tensor, we obtain

$$\bar{\nabla}_\lambda R_{\rho\sigma\mu\nu} + \bar{\nabla}_\mu R_{\rho\sigma\nu\lambda} + \bar{\nabla}_\nu R_{\rho\sigma\lambda\mu} =$$
$$= 2\left(R_{\rho\sigma\mu\nu}\partial_\lambda\varphi + R_{\rho\sigma\nu\lambda}\partial_\mu\varphi + R_{\rho\sigma\lambda\mu}\partial_\nu\,\varphi\right)$$

and

$$\bar{\nabla}_\nu\left(R^{\mu\nu} - \frac{1}{2}\,g^{\mu\nu}R\right) = -\,2\left(R^{\mu\nu} - \frac{1}{2}\,g^{\mu\nu}R\right)\partial_\nu\varphi$$

which, upon recalling the definition of the torsion vector, may be expressed as

$$\bar{\nabla}_\nu\left(R^{\mu\nu} - \frac{1}{2}\,g^{\mu\nu}R\right) = -\,\frac{4}{3}\left(R^{\mu\nu} - \frac{1}{2}\,g^{\mu\nu}R\right)\tau_\nu\,.$$

Apart from the above generalized identities, we may also give the ordinary Bianchi identities as

$$\nabla_\lambda R_{\rho\sigma\mu\nu} + \nabla_\mu R_{\rho\sigma\nu\lambda} + \nabla_\nu R_{\rho\sigma\lambda\mu} = 0$$

and

$$\nabla_\nu\left(R^{\mu\nu} - \frac{1}{2}\,h^{\mu\nu}R\right) = 0\,.$$

4 The electromagnetic sector of the conformal theory. The fundamental equations of motion

Based on the results obtained in the preceding section, let us now take the generator φ as describing the (quantum) electromagnetic field. Then, consequently, the space-time $\mathbb{S}_1$ is understood as being *devoid* of electromagnetic interaction. As we will see, in our present theory, it is the quantum evolution of the gravitational field that gives rise to electromagnetism. In this sense, the electromagnetic field is but an *emergent* quantum phenomenon in the evolution space $\mathbb{M}_4$.

Whereas the space-time $\mathbb{S}_1$ is purely gravitational, the *evolved* space-time $\mathbb{S}_2$ does contain an electromagnetic field. In our present theory, for reasonsthat will be clear soon, we shall define the electromagnetic field $F \in \mathbb{S}_2 \in \mathbb{M}_4$ in terms of the *torsion* of the space-time $\mathbb{S}_2$ by

$$F_{\mu\nu} = 2\,\frac{m_0c^2}{\bar{e}}\,\Gamma^{\lambda}_{[\mu\nu]}u_\lambda\,,$$

where $\bar{e}$ is the (elementary) charge of the electron and

$$u_\mu = g_{\mu\nu}\,\frac{dx^\nu}{ds} = e^{2\varphi}h_{\mu\nu}\frac{dx^\nu}{ds}$$

are the covariant components of the tangent velocity vector field satisfying $u_{mu}\,u^\mu = 1$.

We have seen that the space-time $\mathbb{S}_2$ possesses a manifest quantum structure through its evolution from the purely gravitational space-time $\mathbb{S}_1$. This means that $\bar{e}$ may be defined in terms of the fundamental Planck charge $\hat{e}$ as follows:

$$\bar{e} = N\hat{e} = N\,\sqrt{4\pi\varepsilon_0\hbar c}\,,$$

where N is a positive constant and ε_0 is the permitivity of free space. Further investigation shows that $N = \sqrt{\alpha}$ where $\alpha^{-1} \approx 137$ is the conventional fine structure constant.

Let us now proceed to show that the geodesic equation of motion in the space-time $\mathbb{S}_2$ gives the (generalized) Lorentz equation of motion for the electron. The result of parallel-transferring the velocity vector field u along the world-line (in the direction of motion of the electron) yields

$$\frac{\bar{D}\,u^\mu}{ds} = \left(\bar{\nabla}_\nu u^\mu\right) u^\nu = 0\,,$$

i.e.,

$$\frac{du^\mu}{ds} + \Gamma^\mu_{\rho\sigma} u^\rho u^\sigma = 0\,,$$

where, in general,

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2}\, g^{\sigma\lambda} \left(\partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu} + \partial_\mu g_{\nu\sigma}\right) + \Gamma^\lambda_{[\mu\nu]} - - g^{\lambda\rho} \left(g_{\mu\sigma}\Gamma^\sigma_{[\rho\nu]} + g_{\nu\sigma}\,\Gamma^\sigma_{[\rho\mu]}\right).$$

Recalling our expression for the components of the torsion tensor in the preceding section, we obtain

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2}\, g^{\sigma\lambda}\, \left(\partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu} + \partial_\mu g_{\nu\sigma}\right) + + g_{\mu\nu} g^{\lambda\sigma} \partial_\sigma \varphi - \delta^\lambda_\nu\, \partial_\mu \varphi$$

which is completely equivalent to the previously obtained relation

$$\Gamma^\lambda_{\mu\nu} = \omega^\lambda_{\mu\nu} + \delta^\lambda_\mu\, \partial_\nu \varphi\,.$$

Note that

$$\Delta^\lambda_{\mu\nu} = \frac{1}{2}\, g^{\sigma\lambda} \left(\partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu} + \partial_\mu g_{\nu\sigma}\right)$$

are the Christoffel symbols in the space-time $\mathbb{S}_2$. These are not to be confused with the Christoffel symbols in the space-time $\mathbb{S}_1$ given by $\omega^\lambda_{\mu\nu}$.

Furthermore, we have

$$\frac{du^\mu}{ds} + \Delta^\mu_{\rho\sigma}\, u^\rho u^\sigma = 2 g^{\mu\rho} \Gamma^\lambda_{[\rho\sigma]} u_\lambda u^\sigma.$$

Now, since we have set $F_{\mu\nu} = 2\frac{m_0\,c^2}{\bar{e}}\Gamma^\lambda_{[\mu\nu]} u_\lambda$, we obtain the equation of motion

$$m_0 c^2 \left(\frac{du^\mu}{ds} + \Delta^\mu_{\rho\sigma} u^\rho u^\sigma\right) = \bar{e}\, F^\mu{}_\nu\, u^\nu,$$

which is none other than the Lorentz equation of motion for the electron in the presence of gravitation. Hence, it turns out that the electromagnetic field, which is non-existent in the space-time $\mathbb{S}_1$, is an *intrinsic geometric object* in the space-time $\mathbb{S}_2$. In other words, the space-time structure of $\mathbb{S}_2$ inherently contains both gravitation and electromagnetism.

Now, we see that

$$F_{\mu\nu} = \frac{m_0 c^2}{\bar{e}} \left(u_\mu \partial_\nu \varphi - u_\nu \partial_\mu \varphi\right).$$

In other words,

$$\bar{e}\, F^\mu{}_\nu\, u^\nu = m_0 c^2 \left(u^\mu \frac{d\varphi}{ds} - g^{\mu\nu} \partial_\nu \varphi\right).$$

Consequently, we can rewrite the electron's equation of motion as

$$\frac{du^\mu}{ds} + \Delta^\mu_{\rho\sigma} u^\rho u^\sigma = u^\mu \frac{d\varphi}{ds} - g^{\mu\nu}\, \partial_\nu \varphi\,.$$

We may therefore define an *asymmetric fundamental tensor of the gravoelectromagnetic manifold* $\mathbb{S}_2$ by

$$\tilde{g}_{\mu\nu} = g_{\mu\nu}\, \frac{d\varphi}{ds} - \frac{\bar{e}}{m_0 c^2}\, F_{\mu\nu}$$

satisfying

$$\tilde{g}_{\mu\nu} u^\nu = \partial_\mu \varphi\,.$$

It follows immediately that

$$\left(\delta^\mu_\nu\, \frac{d\varphi}{ds} - \frac{\bar{e}}{m_0 c^2}\, F^\mu{}_\nu\right) u^\nu = g^{\mu\nu}\, \partial_\nu \varphi$$

which, when expressed in terms of the wave function ψ, gives the *Schrödinger-like* equation

$$u_\mu \frac{d\psi}{ds} = \frac{1}{M} \left(\partial_\mu \varphi + \frac{\bar{e}}{m_0 c^2} F_{\mu\nu} u^\nu\right) \psi\,.$$

We may now proceed to show that the electromagnetic current density given by the covariant expression

$$j^\mu = -\frac{c}{4\,\pi} \bar{\nabla}_\nu F^{\mu\nu}$$

is conserved in the present theory.

Let us first call the following expression for the covariant components of the electromagnetic field tensor in terms of the covariant components of the canonical electromagnetic four-potential A:

$$F_{\mu\nu} = \bar{\nabla}_\nu A_\mu - \bar{\nabla}_\mu A_\nu$$

such that $\bar{e}\, \bar{\nabla}_\nu A_\mu = m_0 c^2 u_\mu \partial_\nu \varphi$, i.e.,

$$m_0 c^2 \partial_\mu \varphi = \bar{e}\, u^\nu \bar{\nabla}_\mu A_\nu$$

which directly gives the equation of motion

$$m_0 c^2 \frac{d\varphi}{ds} = \bar{e}\, u^\mu u^\nu \bar{\nabla}_\mu A_\nu\,.$$

Hence, we obtain the following equation of state:

$$m_0 c^2\, \frac{d\psi}{ds} = 2\,\bar{e}\, \frac{(1 + M\psi)}{M}\, u^\mu u^\nu \bar{\nabla}_\mu A_\nu\,.$$

Another alternative expression for the electromagnetic field tensor is given by

$$F_{\mu\nu} = \partial_\nu A_\mu \; - \; \partial_\mu A_\nu 2\Gamma^\lambda_{[\mu\nu]} A_\lambda \; = = \; \partial_\nu A_\mu - \partial_\mu A_\nu + A_\nu\, \partial_\mu \varphi - A_\mu \partial_\nu \varphi\,.$$

In the particular case in which the field-lines of the electromagnetic four-potential propagate in the direction of the electron's motion, we have

$$F_{\mu\nu} = \Lambda\, \frac{\bar{e}}{\left(1 - \frac{\beta^2}{c^2}\right)}\, \left(\partial_\nu u_\mu - \partial_\mu u_\nu\right)$$

where Λ is a proportionality constant and $\beta = \pm\, \bar{e}\, \sqrt{\frac{\Lambda}{m_0}}$. Then, we may define a vortical velocity field, i.e., a spin field, through the vorticity tensor which is given by

$$\omega_{\mu\nu} = \frac{1}{2} \left(\partial_\nu u_\mu - \partial_\mu u_\nu\right)$$

and hence

$$F_{\mu\nu} = 2\Lambda \frac{\bar{e}}{\left(1-\frac{\beta^2}{c^2}\right)}\,\omega_{\mu\nu}\,,$$

which describes an electrically charged spinning region in the *space-time continuum* $\mathbb{S}_2$.

Furthermore, we have the following generalized identity for the electromagnetic field tensor:

$$\bar{\nabla}_\lambda F_{\mu\nu} + \bar{\nabla}_\mu F_{\nu\lambda} + \bar{\nabla}_\nu F_{\lambda\mu} = \\ = 2\left(\Gamma^\sigma_{[\mu\nu]}F_{\sigma\lambda} + \Gamma^\sigma_{[\nu\lambda]}F_{\sigma\mu} + \Gamma^\sigma_{[\lambda\mu]}F_{\sigma\nu}\right)$$

which, in the present theory, takes the particular form

$$\bar{\nabla}_\lambda F_{\mu\nu} + \bar{\nabla}_\mu F_{\nu\lambda} + \bar{\nabla}_\nu F_{\lambda\mu} = \\ = 2\left(F_{\mu\nu}\partial_\lambda\varphi + F_{\nu\lambda}\partial_\mu\varphi + F_{\lambda\mu}\partial_\nu\varphi\right).$$

Contracting, we have

$$\bar{\nabla}_\mu\, j^\mu = -\frac{c}{4\pi}\,\bar{\nabla}_\mu\left(\Gamma^\mu_{[\rho\sigma]}F^{\rho\sigma}\right).$$

We therefore expect that the expression in the brackets indeed vanishes. For this purpose, we may set

$$j^\mu = -\frac{c}{4\pi}\,\Gamma^\mu_{[\rho\sigma]}\,F^{\rho\sigma}$$

and hence, again, using the relation

$$\Gamma^\lambda_{[\mu\nu]} = \frac{1}{2}\left(\delta^\lambda_\mu\partial_\nu\varphi - \delta^\lambda_\nu\partial_\mu\varphi\right),$$

we immediately see that

$$\bar{\nabla}_\mu j^\mu - \frac{c}{4\pi}\left(\partial_\nu\varphi\,\bar{\nabla}_\mu F^{\mu\nu} + F^{\mu\nu}\left(\partial_\nu\partial_\mu\varphi - \Gamma^\lambda_{[\mu\nu]}\partial_\lambda\varphi\right)\right) = \\ = -\,j^\mu\,\partial_\mu\varphi - \frac{c}{4\pi}\,\Gamma^\lambda_{[\mu\nu]}F^{\mu\nu}\,\partial_\lambda\varphi$$

i.e.,

$$\bar{\nabla}_\mu\, j^\mu = 0\,.$$

At this point, we may note the following: the fact that our theory employs torsion, from which the electromagnetic field is extracted, and at the same time guarantees electromagnetic charge conservation (in the form of the above continuity equation) in a natural manner is a remarkable property.

Now, let us call the relation

$$\Gamma^\lambda_{[\mu\nu]} = -\frac{1}{2}\,l\,\left(\delta^\lambda_\sigma - l\,\bar{\nabla}_\sigma\bar{\xi}^\lambda\right)R^\sigma{}_{\rho\mu\nu}\,\xi^\rho$$

obtained in Section 3 of this work (in which $R^\sigma{}_{\rho\mu\nu} = K^\sigma{}_{\rho\mu\nu}$). This can simply be written as

$$\Gamma^\lambda_{[\mu\nu]} = -\frac{1}{2}\,l\,e^{-\varphi}R^\lambda{}_{\rho\mu\nu}\,\xi^\rho$$

i.e.,

$$\Gamma^\lambda_{[\mu\nu]} = -\frac{1}{2}\,l\,e^{-\varphi}R^\lambda{}_{\rho\mu\nu}\,g^{\rho\sigma}\partial_\sigma\psi\,.$$

Hence, we obtain the elegant result

$$F_{\mu\nu} = -\,l\,\frac{m_0c^2}{\bar{e}}\,e^{-\varphi}R^\lambda{}_{\rho\mu\nu}\,u_\lambda\,g^{\rho\sigma}\partial_\sigma\psi$$

i.e.,

$$F_{\mu\nu} = -\frac{l}{\bar{e}}\frac{m_0c^2}{\sqrt{1+M\psi}}\,R^\lambda{}_{\rho\mu\nu}\,u_\lambda\,g^{\rho\sigma}\partial_\sigma\psi$$

or, in terms of the components of the (dimensionless) microscopic displacement field ξ,

$$F_{\mu\nu} = -\,l\,\frac{m_0c^2}{\bar{e}}\,e^{-\varphi}R^\lambda{}_{\rho\mu\nu}\,u_\lambda\,g^{\rho\sigma}\xi_\sigma$$

which further reveals *how the electromagnetic field originates in the gravitational field in the space-time* $\mathbb{S}_2$ *as a quantum field*. Hence, at last, we see a complete picture of the electromagnetic field as an emergent phenomenon. This completes the long-cherished hypothesis that the electromagnetic field itself is caused by a *massive* charged particle, i.e., when $m_0 = 0$ neither gravity nor electromagnetism can exist. Finally, with this result at hand, we obtain the following equation of motion for the electron in the gravitational field:

$$\frac{du^\mu}{ds} + \Delta^\mu_{\rho\sigma}u^\rho u^\sigma = -\,l\,e^{-\varphi}R^{\rho\sigma\mu}{}_\nu\,u_\rho\,\xi_\sigma\,u^\nu$$

i.e.,

$$\frac{du^\mu}{ds} + \Delta^\mu_{\rho\sigma}u^\rho u^\sigma = -\frac{l}{\sqrt{1+M\psi}}\,R^{\rho\sigma\mu}{}_\nu\,u_\rho\,u^\nu\,\partial_\sigma\psi\,.$$

In addition, we note that the torsion tensor is now seen to be given by

$$\tau_\mu = -\frac{1}{2}\,l\,e^{-\varphi}R_{\mu\nu}\,\xi^\nu$$

or, alternatively,

$$\tau_\mu = -\frac{1}{2}\,l\,e^{-\varphi}R_{\mu\nu}\,g^{\nu\lambda}\,\partial_\lambda\psi\,.$$

In other words,

$$\tau_\mu = -\frac{1}{2}\frac{l}{\sqrt{1+M\,\psi}}\,R_{\mu\nu}\,g^{\nu\lambda}\,\partial_\lambda\psi\,.$$

Hence, the second generalized Bianchi identity finally takes the somewhat more transparent form

$$\bar{\nabla}_\nu\left(R^{\mu\nu} - \frac{1}{2}\,g^{\mu\nu}R\right) = \\ = -\frac{2}{3}\,l\,e^{-\varphi}\left(R^{\mu\nu}R_{\nu\rho} - \frac{1}{2}\,R\,R^\mu{}_\rho\right)g^{\rho\sigma}\,\partial_\sigma\psi$$

i.e.,

$$\bar{\nabla}_\nu\left(R^{\mu\nu} - \frac{1}{2}\,g^{\mu\nu}R\right) = \\ = -\frac{2}{3}\frac{l}{\sqrt{1+M\psi}}\left(R^{\mu\nu}R_{\nu\rho} - \frac{1}{2}\,R\,R^\mu{}_\rho\right)g^{\rho\sigma}\,\partial_\sigma\psi\,.$$

5 Final remarks

The present theory, in its current form, is still in an elementary state of development. However, as we have seen, the emergence of the electromagnetic field from the quantum evolution of the gravitational field is a remarkable achievement which deserves special attention. On another occasion, we shall expect to expound the structure of the generalized Einstein's equation in the present theory with a generally non-conservative energy-momentum tensor given by

$$T_{\mu\nu} = \pm \frac{c^4}{8\pi G} \left(R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu} R \right)$$

which, like in the case of self-creation cosmology, seems to allow us to attribute the creation and annihilation of matter directly to the scalar generator of the quantum evolution process, and hence the wave function alone, as

$$\bar{\nabla}_\nu T^{\mu\nu} = -\frac{2}{3} \frac{l}{\sqrt{1 + M\psi}}\, T^{\mu\nu} R_{\nu\rho}\, g^{\rho\sigma}\, \partial_\sigma \psi \neq 0\,.$$

6 Acknowledgements

The author is dearly indebted to Dmitri Rabounski and Stephen J. Crothers for their continuous support and sincere assistance.

Submitted on August 23, 2007
Accepted on September 24, 2007

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Joint Wave-Particle Properties of the Individual Photon

Bo Lehnert

Alfvén Laboratory, Royal Institute of Technology, S-10044 Stockholm, Sweden
E-mail: Bo.Lehnert@ee.kth.se

Two-slit experiments performed earlier by Tsuchiya et al. and recently by Afshar et al. demonstrate the joint wave-particle properties of the single indiviual photon, and agree with Einstein's argument against Complementarity. These results cannot be explained by conventional theory in which Maxwell's equations serve as a guiding line and basis. On the other hand a revised quantum electrodynamic theory based on a nonzero electric field divergence in the vacuum yields results which appear to be consistent with the experiments. A model of the individual photon is thus deduced from the theory, in the form of a wave packet behaving as a single entity and having simultaneous wave and particle properties.

1 Introduction

Ever since the earlier epoch of natural science, the wave-particle duality of light has appeared as something of an enigma. In Bohr's principle of Complementarity, this duality has been a cornerstone in the interpretation of quantum mechanics. Thereby the wavelike and particlelike properties are conceived to be complementary, in the sense that they are mutually exclusive, and no experiment can reveal both at once. This formulation of quantum mechanics has been successful in many applications and is widely accepted by physicists, but it is full of apparent paradoxes which made Einstein deeply uncomfortable [1].

During the latest decades additional investigations on the nature of light have been made, among which the two-slit experiments by Tsuchiya et al. [2] and Afshar et al. [3] deserve particular attention. These investigations verify that there is a joint wave-particle duality of the individual photon, thus being in agreement with Einstein's argument against Complementarity.

In this paper part of the results by Tsuchiya et al. and Afshar et al. are reviewed and compared with a revised quantum electrodynamic theory by the author. The latter theory is based on a vacuum state that is not merely an empty space but includes the electromagnetic fluctuations of the zero point energy and a corresponding nonzero electric charge density associated with a nonzero electric field divergence. A short description of the theory is presented, whereas its detailed deductions are given elsewhere [4–7].

2 The two-slit experiments

A photon-counting imaging system has earlier been elaborated by Tsuchiya et al. [2] and incorporates the ability to detect individual photons, spatial resolution, and the capability of real-time imaging and subsequent image analysis. Two parallel slits of size $50\,\mu$m $\times$ 4 mm at a spacing of $250\,\mu$m were arranged to pass light through an interference filter at a wavelength of 253.7 nm. The full size of the obtained image on the monitor screen of the experiment was 11.4 mm at the input plane. Since the purpose of the investigation was to demonstrate the interference property of a single photon itself, the spacing of individual photons was made much longer than their coherence time, so that interference between individual photons could be prevented. For this reason, neutral density filters were used to realize a very low light level, where the counting rates were of the order of 100 per second.

As the measurements started, bright very small dots appeared at random positions on the monitor screen. After 10 seconds had elapsed, a photon-counting image was seen on the screen, containing 10^3 events, but its overall shape was not yet clearly defined. After 10 minutes, however, the total accumulated counts were 6×10^4, and an interference pattern formed by the dots was clearly detected. The diameter of each dot was of the order of 6×10^{-3} of the screen size, and the fringe distance about 5×10^{-2} of it. The effect of closing one of the double slits was finally observed. Then the interference pattern did not appear, but a diffraction pattern was observed.

As concluded by Tsuchiya et al., these results cannot be explained by mutually exclusive wave and particle descriptions of the photon, but give a clear indication of the wave-particle duality of the single individual photon [2].

These important results appear not to have attracted the wide interest which they ought to deserve. However, as long as 22 years later, Afshar et al. [3] conducted a two-slit experiment based on a different methodology but with a similar outcome and conclusions. In this investigation there was a simultaneous determination of the wave and particle aspects of light in a “welcher-weg” experiment, beyond the limitations set by Bohr's principle of Complementarity. The experiment included a pair of pinholes with diameters of 40 nm and center-to-center separation of $250\,\mu$m, with light from a diode laser of the wavelength 638 nm. These parameter values were thus not too far from those of the experiments by Tsuchiya et al. In addition, six thin wires of 127 μm diameter were placed at a distance of 0.55 m from the pinholes, and at the minima of the observed interference pattern. When this pattern was present, the disturbance to the incoming beam by the wire grid was minimal. On the other hand, when the interference pattern was absent, the wire grid obstructed the beam. Also here the investigation was conducted in the low photon flux regime, to preclude loss of which-way information due to the intrinsic indistinguishability of coherent multi-photon systems. When the flux was 3×10^4 photons per second, the average separation between successive photons was estimated to about 10 km. The experiments were performed in four ways, i.e. with both pinholes open in absence of the wire grid, with both pinholes open in presence of the wire grid, and with either pinhole open in presence of the same grid.

From the measured data the which-way information and the visibility of an interference pattern could then be determined within the same experimental setup. The which-way information thus indicates through which pinhole the particlelike photon has passed. At the same time the interference indicates that the same wavelike photon must have *sampled* both pinholes so that an interference pattern could be formed. These derived properties of the individual photon refer back to the same space-time event, i.e. to the moment when the single photon passed the plane of the pinholes.

Consequently, also these experimental results force us to agree vith Einstein's argument against Complementarity [3].

3 Shortcomings of conventional theory

In conventional quantum electrodynamics (QED), Maxwell's equations have served as a guiding line and basis when there is a vacuum state with a vanishing electric charge density and a zero electric field divergence [8]. According to Schiff [8] and Heitler [9] the Poynting vector then defines the momentum of the pure radiation field, expressed by sets of quantized plane waves. As pointed out by Feynman [10], there are nevertheless unsolved problems which lead to difficulties with Maxwell's equations that are not removed by and not directly associated with quantum mechanics. Consequently, QED will also become subject to the shortcomings of the conventional field theory.

To be more specific in connection with a theoretical model of the individual photon, we start here with the following general physical requirements to be fulfilled:

- The model should have the form of a wave or a wave packet of preserved and limited geometrical shape, propagating with undamped motion in a defined direction of three-space. This leads to an analysis in a cylindrical frame (r, φ, z) with z in the direction of propagation;
- The obtained general solutions for the field quantities should extend all over space, and no artificial boundaries would have to be introduced in the vacuum;
- The integrated total field energy should remain finite;
- The solutions should result in an angular momentum (spin) of the photon as a propagating boson particle.

Maxwell's equations in the vacuum state yield solutions for any field quantity Q having the normal mode form

$$Q = \hat{Q}(r)\exp\left[i\left(-\omega t + \bar{m}\varphi + kz\right)\right] \qquad (1)$$

in cylindrical geometry where ω is the frequency and k and $\bar{m}$ are the wave numbers with respect to the z and φ directions. We further introduce

$$K_0^2 = \left(\frac{\omega}{c}\right)^2 - k^2. \qquad (2)$$

When $K_0^2 > 0$ the phase velocity becomes larger and the group velocity smaller than the velocity c of light. The general solution then has field components in terms of Bessel functions $Z_{\bar{m}}(K_0 r)$ of the first and second kind, where the r-dependence of every component is of the form $Z_{\bar{m}}/r$ or $Z_{\bar{m}+1}$ [11]. Application to a photon model then leads to the following results:

- Already the purely axisymmetric case $\bar{m} = 0$ results in a Poynting vector which yields zero spin;
- The spin also vanishes when $K_0 = 0$ and the phase and group velocities both are equal to c;
- There is no clearly defined spatial limitation of the solutions;
- With no material boundaries such as walls, the total integrated field energy becomes divergent.

Consequently, conventional theory based on Maxwell's equations in the vacuum state does not lead to a physically relevant model for the individual photon.

4 Photon physics in revised quantum electrodynamics

An extended electromagnetic theory applied to the vacuum state and aiming beyond Maxwell's equations serves as a guiding line and basis of the present theoretical approach [4–7]. In four-dimensional representation the theory has the following form

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right) A_\mu = \mu_0 J_\mu\,, \qquad \mu = 1,2,3,4\,, \qquad (3)$$

where A_μ are the electromagnetic potentials. As deduced from the requirement of Lorentz invariance, the four-current density of the right-hand member of equation (3) becomes

$$J_\mu = (\mathbf{j}, ic\bar{\rho}) = \varepsilon_0(\operatorname{div}\mathbf{E})\,(\mathbf{C}, ic)\,, \qquad \mathbf{C}^2 = c^2 \qquad (4)$$

with c as the velocity of light, $\mathbf{E}$ denoting the electric field strength, and SI units being adopted. Further $\mathbf{B} = \operatorname{curl}\mathbf{A}$ is the magnetic field strength derived from the three-space magnetic vector potential $\mathbf{A}$. In equation (4) the velocity vector $\mathbf{C}$ has the modulus c. Maxwell's equations in the vacuum are recovered when $\operatorname{div} \mathbf{E} = 0$, whereas $\operatorname{div} \mathbf{E} \neq 0$ leads to a space-charge current density (4) in the vacuum. The corresponding three-space part $\mathbf{j} = \varepsilon_0 (\operatorname{div} \mathbf{E})\, \mathbf{C}$ appears in addition to the displacement current.

The revised basic field equations of dynamic states in a three-dimensional representation are now given by the wave equation

$$\left(\frac{\partial^2}{\partial t^2} - c^2\,\nabla^2\right)\mathbf{E} + \left(c^2\,\nabla + \mathbf{C}\,\frac{\partial}{\partial t}\right)(\operatorname{div}\mathbf{E}) = 0 \qquad (5)$$

for the electric field, and the equation

$$\operatorname{curl}\mathbf{E} = -\,\frac{\partial \mathbf{B}}{\partial t} \qquad (6)$$

of electromagnetic induction. The characteristic features of the field equations (3)–(6) are as follows:

- The theory is based on the pure radiation field in the vacuum state, including contributions from a nonzero electric charge density;
- The associated nonzero electric field divergence introduces an additional degree of freedom, leading to new solutions and new physical phenomena. This also becomes important in situations where this divergence appears to be small;
- The theory is both Lorentz and gauge invariant;
- The velocity of light is no longer a scalar c but a vector $\mathbf{C}$ with the modulus c.

To become complete, the theory has to be quantized. In absence as well as in presence of source terms, such as the right-hand member of equation (3), the quantized field equations are generally equivalent to the original field equations in which all field quantities are replaced by their expectation values, as shown by Heitler [9]. As a first step and a simplification, the general solutions of the field equations will therefore first be determined, and relevant quantum conditions will afterwards be imposed on these solutions. This is justified by the expectation values due to Heitler. The present theory may therefore not be too far from the truth, in the sense that it represents the most probable states in a first approximation to a rigorous quantum-theoretical deduction.

4.1 Application to a model of the individual photon

The theory of equations (3)–(6) is now applied to the model of an individual photon in the axisymmetric case where $\partial/\partial\varphi = 0$ in a cylindrical frame (r, φ, z) with z along the direction of propagation. Screw-shaped modes where $\partial/\partial\varphi \neq 0$ end in several respects up with similar results, but become more involved and have been described elsewhere [6, 7].

The velocity vector of equation (4) is in this axisymmetric case given by

$$\mathbf{C} = c\,(0, \cos\alpha, \sin\alpha) \qquad (7)$$

where α is a constant angle, and $\cos\alpha$ and $\sin\alpha$ could in principle have either sign but are here limited to positive values for the sake of simplicity. The form (7) can be shown to imply that the electromagnetic energy has one part which propagates in the z-direction, and another part which circulates in the φ-direction around the axis of symmetry and becomes associated with the spin [6, 7]. Normal modes of the form (1) with $\bar{m} = 0$ then result in general solutions for the components of $\mathbf{E}$ and $\mathbf{B}$, being given in terms of differential expressions of a generating function

$$F = G_0 R(\rho) \exp\big[i(-\omega t + kz)\big]. \qquad (8)$$

(Here G_0 is an amplitude factor, $\rho = r/r_0$, and r_0 represents a characteristic radius of the geometrical configuration in question.) The corresponding dispersion relation becomes

$$\omega = kv\,, \qquad v = c\,(\sin\alpha) \qquad (9)$$

thus resulting in axial phase and group velocities, both being equal to $v < c$. Not to get into conflict with the experiments by Michelson and Morley, the condition $0 < \cos\alpha \ll 1$ has to be imposed on the parameter $\cos\alpha$. As an example, $\cos\alpha \leqslant 10^{-4}$ would make the velocity v differ from c by less than the eight decimal in the value of c. As a consequence of the dispersion relation (9) with $v < c$ and of the detailed deductions, the total integrated field energy mc^2 further becomes equivalent to a total mass m and a rest mass

$$m_0 = m\,\sqrt{1 - (v/c)^2} = m\,(\cos\alpha). \qquad (10)$$

This rest mass is associated with the angular momentum which only becomes nonzero for a nonzero electric field divergence. When $\operatorname{div}\mathbf{E}$, $\cos\alpha$, and m_0 vanish, we are thus back to the conventional case of Section 3 with its spinless and physically irrelevant basis for a photon model. Even if the electric field divergence at a first glance appears to be a small quantity, it thus has a profound effect on the physics of an individual photon model.

From the obtained general solutions it has further been shown that the total integrated charge and magnetic moment vanish, whereas the total integrated mass m and angular momentum s remain nonzero.

From the solutions of the normal wave modes, a wave packet has to be formed. In accordance with experimental experience, such a packet should have a narrow line width. Its spectrum of wave numbers k should then be piled up around a main wave number k_0 and a corresponding wavelength $\lambda_0 = 2\pi/k_0$. The effective axial length $2z_0$ of the packet is then much larger than λ_0.

To close the system, two relevant quantum conditions have further to be imposed. The first concerns the total integrated field energy, in the sense that $mc^2 = h\nu_0$ according to Einstein and Planck, where the frequency $\nu_0 \cong c/\lambda_0$, for $\cos\alpha \ll 1$. The second condition is imposed on the total integrated angular momentum which should become equal to $s = h/2\pi$ for the photon to behave as a boson particle.

From combination with the wave packet solutions, the imposed quantum conditions result in expressions for an effective transverse diameter $2\hat{r}$ of the wave packet. In respect to the radial part R of the generating function (8), there are two alternatives which are both given by

$$2\hat{r} = \frac{\varepsilon\,\lambda_0}{\pi\,(\cos\alpha)} \tag{11}$$

and become specified as follows:

- When $\varepsilon = 1$ expression (11) stands for a part $R(\rho)$ which is convergent at the origin $\rho = 0$. This results in an effective photon diameter being only moderately small, but still becoming large as compared to atomic dimensions;
- When $\varepsilon \ll 1$ there are solutions for a part $R(\rho)$ which is divergent at $\rho = 0$. Then finite field quantities can still be obtained within a whole range of small ε, in the limit of a shrinking characteristic radius $r_0 = c_0\,\varepsilon$ where c_0 is a positive constant having the dimension of length. This alternative results in an an effective photon diameter which can become very small, such as to realize a state of "needle radiation" first proposed by Einstein. Then the diameter (11) can become comparable to atomic dimensions.

It is thus seen that the requirements on a photon model can be fulfilled by the present revised theory. Its wave packet solutions have joint wave-particle properties. In some respects this appears to be similar to the earlier wave-particle duality outlined by de Broglie, where there is a "pilot wave" propagating along the axis, on which wave a "particle-like" part is "surfing". However, such a subdivision is not necessary in the present case where the wave packet behaves as one single entity, having wave and particle properties at the same time.

Attention is finally called to a comparison between the definition of the momentum of the pure radiation field in terms of the Poynting vector on one hand, and that given by the expression $\mathbf{p} = -i\hbar\nabla$ in the deduction of the Schrödinger equation for a particle with mass on the other [5]. For normal modes the axial component of $\mathbf{p}$ becomes $p_z = \hbar k$ as expected. However, in the transverse direction of a photon model being spatially limited and having a finite effective diameter (11), there would arise a nonzero transverse momentum p_r as well, but this appears to be physically unacceptable for a photon model.

4.2 The present photon model and its relation to two-slit experiments

The limits of the effective photon diameter (11) can be estimated by assuming an upper limit of $2\hat{r}$ when $\varepsilon = 1$ and $\cos\alpha = 10^{-4}$, and a lower limit of $2\hat{r}$ when $\varepsilon = \cos\alpha$. Then the effective diameter would be in the range of the values $\lambda_0/\pi \leqslant 2\hat{r} \leqslant 10^4\lambda_0/\pi$, but the lower limit could even be lower when $\varepsilon < \cos\alpha$ for strongly pronounced needle radiation. From this first order estimate, and from the features of the theory, the following points should be noticed:

- The diameter of the dot-shaped marks on the monitor screen of the experiment by Tsuchiya et al. is of the order of 6×10^{-3} of the screen size, i.e. about 10^{-4} m. With the wave length $\lambda_0 = 253.7$ nm, the effective photon diameter would then be in the range of the values $7\times10^{-4} \geqslant 2\hat{r} \geqslant 7\times10^{-8}$ m. This range covers the observed size of the dots;
- The width of the parallel slits in the experiments by Tsuchiya et al. is 5×10^{-5} m and their separation distance is 25×10^{-5} m. The corresponding pinhole diameters and their center-to-center separation in the experiments by Afshar et al. are 4×10^{-5} m and 25×10^{-5} m, respectively, and the wavelength is $\lambda_0 = 638$ nm. In the latter experiments the effective diameter is estimated to be in the range $2\times10^{-7} \leqslant 2\hat{r} \leqslant 2\times10^{-3}$ m. In both experiments the estimated ranges of $2\hat{r}$ are thus seen to cover the slit widths and separation distances;
- A large variation of a small $\cos\alpha$ has only a limited effect on the phase and group velocities of equation (9). Also a considerable variation of a small ε does not influence the general deductions of the theory [4, 6, 7] even if it ends up with a substantial change of the diameter (11). This leads to the somewhat speculative question whether the state of the compound parameter $\varepsilon/\cos\alpha$ could adopt different values during the propagation of the wave packet. This could then be related to "photon oscillations" as proposed for a model with a nonzero rest mass, in analogy with neutrino oscillations [4, 7];
- As compared to the slit widths and the separation distances, the obtained ranges of $2\hat{r}$ become consistent with the statement by Afshar et al. that the same wave-like photon can *sample* both pinholes to form an interference pattern;
- Interference between cylindrical waves should take place in a similar way as between plane waves. In particular, this becomes obvious at the minima of the interference pattern where full cancellation takes place;
- Due to the requirement of a narrow line width, the wave packet length $2z_0$ by far exceeds the wave length λ_0 and the effective diameter $2\hat{r}$. Therefore the packet forms a very long and narrow wave train;
- Causality raises the question how the photon can "know" to form the interference pattern on the monitor screen already when it passes the slits. An answer may be provided by the front part of the elongated packet

which may serve as a "precursor", thereby also representing the quantum mechanical wave nature of the packet. Alternatively, there may exist a counterpart to the precursor phenomenon earlier discussed by Stratton [11] for conventional electromagnetic waves.

5 Conclusions

The two-slit experiments by Tsuchiya et al. and by Afshar et al. demonstrate the joint wave-particle properties of the individual photon, and agree with Einstein's argument against Complementarity. These experiments cannot be explained by conventional theory. The present revised theory appears on the other hand to become consistent with the experiments.

Submitted on September 17, 2007
Accepted on September 24, 2007

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A New Derivation of Biquaternion Schrödinger Equation and Plausible Implications

Vic Christianto* and Florentin Smarandache†

*Sciprint.org — a Free Scientific Electronic Preprint Server, http://www.sciprint.org
E-mail: admin@sciprint.org

†Department of Mathematics, University of New Mexico, Gallup, NM 87301, USA
E-mail: smarand@unm.edu

In the preceding article we argue that biquaternionic extension of Klein-Gordon equation has solution containing imaginary part, which differs appreciably from known solution of KGE. In the present article we discuss some possible interpretation of this imaginary part of the solution of biquaternionic KGE (BQKGE); thereafter we offer a new derivation of biquaternion Schrödinger equation using this method. Further observation is of course recommended in order to refute or verify this proposition.

1 Introduction

There were some attempts in literature to generalise Schrödinger equation using quaternion and biquaternion numbers. Because quaternion number use in Quantum Mechanics has often been described [1, 2, 3, 4], we only mention in this paper the use of biquaternion number. Sapogin [5] was the first to introduce biquaternion to extend Schrödinger equation, while Kravchenko [4] use biquaternion number to describe neat link between Schrödinger equation and Riccati equation.

In the present article we discuss a new derivation of biquaternion Schrödinger equation using a method used in the preceding paper. Because the previous method has been used for Klein-Gordon equation [1], now it seems natural to extend it to Schrödinger equation. This biquaternion effect may be useful in particular to explore new effects in the context of low-energy reaction (LENR) [6]. Nonetheless, further observation is of course recommended in order to refute or verify this proposition.

2 Some interpretations of preceding result of biquaternionic KGE

In our preceding paper [1], we argue that it is possible to write biquaternionic extension of Klein-Gordon equation as follows

$$\left[\left(\frac{\partial^2}{\partial t^2}-\nabla^2\right)+i\left(\frac{\partial^2}{\partial t^2}-\nabla^2\right)\right]\varphi(x,t)=-m^2\varphi(x,t). \quad (1)$$

Or this equation can be rewritten as

$$\left(\diamondsuit\bar{\diamondsuit}+m^2\right)\varphi(x,t)=0 \quad (2)$$

provided we use this definition

$$\diamondsuit=\nabla^q+i\nabla^q=\left(-i\frac{\partial}{\partial t}+e_1\frac{\partial}{\partial x}+e_2\frac{\partial}{\partial y}+e_3\frac{\partial}{\partial z}\right)+ +i\left(-i\frac{\partial}{\partial T}+e_1\frac{\partial}{\partial X}+e_2\frac{\partial}{\partial Y}+e_3\frac{\partial}{\partial Z}\right) \quad (3)$$

where e_1, e_2, e_3 are *quaternion imaginary units* obeying (with ordinary quaternion symbols: $e_1=i$, $e_2=j$, $e_3=k$)

$$i^2=j^2=k^2=-1\,,\quad ij=-ji=k\,,\qquad jk=-kj=i\,,\quad ki=-ik=j\,, \quad (4)$$

and quaternion *Nabla operator* is defined as [7]

$$\nabla^q=-i\frac{\partial}{\partial t}+e_1\frac{\partial}{\partial x}+e_2\frac{\partial}{\partial y}+e_3\frac{\partial}{\partial z}\,. \quad (5)$$

Note that equation (3) and (5) included partial time-differentiation.

It is worth nothing here that equation (2) yields solution containing imaginary part, which differs appreciably from known solution of KGE:

$$y(x,t)=\left(\frac{1}{4}-\frac{i}{4}\right)m^2t^2+\text{constant}. \quad (6)$$

Some possible alternative interpretations of this *imaginary part* of the solution of biquaternionic KGE (BQKGE) are:

(a) The imaginary part implies that there is exponential term of the wave solution, which is quite similar to the Ginzburg-Landau extension of London phenomenology [8]

$$\psi(r)=|\psi(r)|\,e^{i\varphi(r)}\,, \quad (7)$$

because (6) can be rewritten (approximately) as:

$$y(x,t)=\frac{e^i}{4}\,m^2t^2; \quad (8)$$

(b) The aforementioned exponential term of the solution (8) can be interpreted as signature of vortices solution. Interestingly Navier-Stokes equation which implies vorticity equation can also be rewritten in terms of Yukawa equation [3];

(c) The imaginary part implies that there is spiral wave, which suggests spiralling motion of meson or other particles. Interestingly it has been argued that one can explain electron phenomena by assuming spiralling electrons [9]. Alternatively this spiralling wave may already be known in the form of Bierkeland flow. For meson observation, this could be interpreted as another form of meson, which may be called "supersymmetric-meson" [1];

(d) The imaginary part of solution of BQKGE also implies that it consists of standard solution of KGE [1], and its alteration because of imaginary differential operator. That would mean the resulting wave is composed of two complementary waves;

(e) Considering some recent proposals suggesting that neutrino can have *imaginary mass* [10], the aforementioned imaginary part of solution of BQKGE can also imply that the (supersymmetric-) meson may be composed of neutrino(s). This new proposition may require new thinking both on the nature of neutrino and also supersymmetric-meson [11].

While some of these propositions remain to be seen, in deriving the preceding BQKGE we follow Dirac's phrase that "*One can generalize his physics by generalizing his mathematics*". More specifically, we focus on using a "theorem" from this principle, i.e.: "*One can generalize his mathematics by generalizing his (differential) operator*".

3 Extended biquaternion Schrödinger equation

One can expect to use the same method described above to generalize the standard Schrödinger equation [12]

$$\left[-\frac{\hbar^2}{2m}\,\Delta u + V(x)\right] u = Eu\,, \tag{9}$$

or, in simplified form, [12, p.11]:

$$(-\Delta + w_k) f_k = 0\,, \qquad k = 0, 1, 2, 3. \tag{10}$$

In order to generalize equation (9) to biquaternion version (BQSE), we use first quaternion Nabla operator (5), and by noticing that $\Delta \equiv \nabla\nabla$, we get

$$-\frac{\hbar^2}{2m}\left(\nabla^q \bar{\nabla}^q + \frac{\partial^2}{\partial t^2}\right) u + \big(V(x) - E\big)\, u = 0\,. \tag{11}$$

Note that we shall introduce the second term in order to 'neutralize' the partial time-differentiation of $\nabla^q \bar{\nabla}^q$ operator.

To get biquaternion form of equation (11) we can use our definition in equation (3) rather than (5), so we get

$$-\frac{\hbar^2}{2m}\left(\diamondsuit\bar{\diamondsuit} + \frac{\partial^2}{\partial t^2} - i\,\frac{\partial^2}{\partial T^2}\right) u + \big(V(x) - E\big)\, u = 0\,. \tag{12}$$

This is an alternative version of *biquaternionic* Schrödinger equation, compared to Sapogin's [5] or Kravchenko's [4] method. We also note here that the route to *quaternionize* Schrödinger equation here is rather different from what is described by Horwitz [13, p. 6]

$$\tilde{H}\,\psi = \psi\, e_1\, E\,, \tag{13}$$

or

$$\tilde{H}\,\psi\, q = \psi\, q\left(q^{-1} e_1\, q\right) E\,, \tag{14}$$

where the quaternion number q, can be expressed as follows (see [13, p. 6] and [4])

$$q = q_0 + \sum_{i=1}^{3} q_i\, e_i\,. \tag{15}$$

Nonetheless, further observation is of course recommended in order to refute or verify this proposition (12).

4 Numerical solution of biquaternion Schrödinger equation

It can be shown that numerical solution (using Maxima [14]) of biquaternionic extension of Schrödinger equation yields different result compared to the standard Schrödinger equation, as follows. For clarity, all solutions were computed in 1-D only.

For standard Schrödinger equation [12], one can rewrite equation (9) as follows:

(a) For $V(x) > E$:

$$-\frac{\hbar^2}{2m}\,\Delta u + a \cdot u = 0\,; \tag{16}$$

(b) For $V(x) < E$:

$$-\frac{\hbar^2}{2m}\,\Delta u - a \cdot u = 0\,. \tag{17}$$

Numerical solution of equation (16) and (17) is given (by assuming $\hbar = 1$ and $m = 1/2$ for convenience)

(%i44) -'diff (y, x, 2) + a*y;
(%o44) $a \cdot y - \frac{d^2}{d^2 x} y$

(a) For $V(x) > E$:

(%i46) ode2 (%o44, y, x);
(%o46) $y = k_1 \cdot \exp(\sqrt{a \cdot x}) + k_2 \cdot \exp(-\sqrt{ax})$

(b) For $V(x) < E$:

(%i45) ode2 (%o44, y, x);
(%o45) $y = k_1 \cdot \sinh(\sqrt{a \cdot x}) + k_2 \cdot \cosh(\sqrt{a \cdot x})$

In the meantime, numerical solution of equation (12), is given (by assuming $\hbar = 1$ and $m = 1/2$ for convenience)

(a) For $V(x) > E$:

(%i38) (%i+1)*'diff (y, x, 2) + a*y;
(%o38) $(i+1)\,\frac{d^2}{d^2 x} y + a \cdot y$
(%i39) ode2 (%o38, y, x);
(%o39) $y = k_1 \cdot \sin(\sqrt{\frac{a}{i+1}} \cdot x) + k_2 \cdot \cos(\sqrt{\frac{a}{i+1}} \cdot x)$

(b) For $V(x) < E$:

(%i40) (%i+1)*'diff (y, x, 2) - a*y;
(%o40) $(i+1)\,\frac{d^2}{d^2 x} y - a \cdot y$
(%i41) ode2 (%o40, y, x);
(%o41)$y = k_1 \cdot \sin(\sqrt{-\frac{a}{i+1}} \cdot x) + k_2 \cdot \cos(\sqrt{-\frac{a}{i+1}} \cdot x)$

Therefore, we conclude that numerical solution of biquaternionic extension of Schrödinger equation yields different result compared to the solution of standard Schrödinger equation. Nonetheless, we recommend further observation in order to refute or verify this proposition/numerical solution of biquaternion extension of spatial-differential operator of Schrödinger equation.

As side remark, it is interesting to note here that if we *introduce imaginary number* in equation (16) and equation (17), the numerical solutions will be quite different compared to solution of equation (16) and (17), as follows

$$-\frac{i\hbar^2}{2m}\,\Delta u + au = 0\,, \tag{18}$$

where $V(x) > E$, or

$$-\frac{i\hbar^2}{2m}\,\Delta u - au = 0\,, \tag{19}$$

where $V(x) < E$.

Numerical solution of equation (18) and (19) is given (by assuming $\hbar = 1$ and $m = 1/2$ for convenience)

(a) For $V(x) > E$:

(%i47) -%i*'diff (y, x, 2) + a*y;
(%o47) $a \cdot y - i\frac{d^2}{d^2x}y$
(%i48) ode2 (%o47, y, x);
(%o48) $y = k_1 \cdot \sin(\sqrt{ia \cdot x}) + k_2 \cdot \cos(\sqrt{ia \cdot x})$

(b) For $V(x) < E$:

(%i50) -%i*'diff (y, x, 2) - a*y;
(%o50) $-a \cdot y - i\frac{d^2}{d^2x}y$
(%i51) ode2 (%o50, y, x);
(%o51) $y = k_1 \cdot \sin(-\sqrt{ia \cdot x}) + k_2 \cdot \cos(-\sqrt{ia \cdot x})$

It shall be clear therefore that using different sign for differential operator yields quite different results.

Acknowledgement

Special thanks to Prof. Diego Rapoport who mentioned Sprössig's interesting paper [3]. VC would like to dedicate this article for RFF.

Submitted on September 24, 2007
Accepted on September 26, 2007

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Thirty Unsolved Problems in the Physics of Elementary Particles

Vic Christianto* and Florentin Smarandache†

*Sciprint.org — a Free Scientific Electronic Preprint Server, http://www.sciprint.org
E-mail: admin@sciprint.org
†Department of Mathematics, University of New Mexico, Gallup, NM 87301, USA
E-mail: smarand@unm.edu

Unlike what some physicists and graduate students used to think, that physics science has come to the point that the only improvement needed is merely like adding more numbers in decimal place for the masses of elementary particles or gravitational constant, there is a number of unsolved problems in this field that may require that the whole theory shall be reassessed. In the present article we discuss thirty of those unsolved problems and their likely implications. In the first section we will discuss some well-known problems in cosmology and particle physics, and then other unsolved problems will be discussed in next section.

1 Unsolved problems related to cosmology

In the present article we discuss some unsolved problems in the physics of elementary particles, and their likely implications. In the first section we will discuss some well-known problems in cosmology and particle physics, and then other unsolved problems will be discussed in next section. Some of these problems were inspired by and expanded from Ginzburg's paper [1]. The problems are:

1. The problem of the three origins. According to Marcelo Gleiser (Darthmouth College) there are three unsolved questions which are likely to play significant role in 21st-century science: the origin of the universe, the origin of life, and the origin of mind;
2. The problem of symmetry and antimatter observation. This could be one of the biggest puzzle in cosmology: If it's true according to theoretical physics (Dirac equation etc.) that there should be equal amounts of matter and antimatter in the universe, then why our observation only display vast amounts of matter and very little antimatter?
3. The problem of dark matter in cosmology model. Do we need to introduce dark matter to describe galaxy rotation curves? Or do we need a revised method in our cosmology model? Is it possible to develop a new theory of galaxy rotation which agrees with observations but without invoking dark matter? For example of such a new theory without dark matter, see Moffat and Brownstein [2, 3];
4. Cosmological constant problem. This problem represents one of the major unresolved issues in contemporary physics. It is presumed that a presently unknown symmetry operates in such a way to enable a vanishingly small constant while remaining consistent with all accepted field theoretic principles [4];
5. Antimatter hydrogen observation. Is it possible to find isolated antimatter hydrogen (antihydrogen) in astrophysics (stellar or galaxies) observation? Is there antihydrogen star in our galaxy?

Now we are going to discuss other seemingly interesting problems in the physics of elementary particles, in particular those questions which may be related to the New Energy science.

2 Unsolved problems in the physics of elementary particles

We discuss first unsolved problems in the Standard Model of elementary particles. Despite the fact that Standard Model apparently comply with most experimental data up to this day, the majority of particle physicists feel that SM is not a complete framework. E. Goldfain has listed some of the most cited reasons for this belief [5], as follows:

6. The neutrino mass problem. Some recent discovery indicates that neutrino oscillates which implies that neutrino has mass, while QM theories since Pauli predict that neutrino should have no mass [6]. Furthermore it is not yet clear that neutrino (oscillation) phenomena correspond to Dirac or Majorana neutrino [7];
7. SM does not include the contribution of gravity and gravitational corrections to both quantum field theory and renormalization group (RG) equations;
8. SM does not fix the large number of parameters that enter the theory (in particular the spectra of masses, gauge couplings, and fermion mixing angles). Some physicists have also expressed their objections that in the QCD scheme the number of quarks have increased to more than 30 particles, therefore they assert that QCD-quark model cease to be a useful model for elementary particles;

9. SM has a gauge hierarchy problem, which requires fine tuning. Another known fine-tuning problem in SM is "strong CP problem" [8, p. 18];
10. SM postulates that the origin of electroweak symmetry breaking is the Higgs mechanism. Unfortunately Higgs particle has never been found; therefore recently some physicists feel they ought to introduce more speculative theories in order to save their Higgs mechanism [9];
11. SM does not clarify the origin of its gauge group $SU(3)\times SU(2)\times U(1)$ and why quarks and lepton occur as representations of this group;
12. SM does not explain why (only) the electroweak interactions are chiral (parity-violating) [8, p. 16];
13. Charge quantization problem. SM does not explain another fundamental fact in nature, i.e. why all particles have charges which are multiples of $e/3$ [8, p. 16].

Other than the known problems with SM as described above, there are other quite fundamental problems related to the physics of elementary particles and mathematical physics in general, for instance [10]:

14. Is there dynamical explanation of quark confinement problem? This problem corresponds to the fact that quarks cannot be isolated. See also homepage by Clay Institute on this problem;
15. What is the dynamical mechanism behind Koide's mixing matrix of the lepton mass formula [11]?
16. Does neutrino mass correspond to the Koide mixing matrix [12]?
17. Does Dirac's new electron theory in 1951 reconcile the quantum mechanical view with the classical electrodynamics view of the electron [13]?
18. Is it possible to explain anomalous ultraviolet hydrogen spectrum?
19. Is there quaternion-type symmetry to describe neutrino masses?
20. Is it possible to describe neutrino oscillation dynamics with Bogoliubov-deGennes theory, in lieu of using standard Schrödinger-type wave equation [6]?
21. Solar neutrino problem — i.e. the seeming deficit of observed solar neutrinos [14]. The Sun through fusion, send us neutrinos, and the Earth through fission, antineutrinos. But observation in SuperKamiokande etc. discovers that the observed solar neutrinos are not as expected. In SuperKamiokande Lab, it is found that the number of electron neutrinos which is observed is 0.46 that which is expected [15]. One proposed explanation for the lack of electron neutrinos is that they may have oscillated into muon neutrinos;
22. Neutrino geology problem. Is it possible to observe terrestrial neutrino? The flux of terrestrial neutrino is a direct reflection of the rate of radioactive decays in the Earth and so of the associated energy production, which is presumably the main source of Earth's heat [14];
23. Is it possible to explain the origin of electroweak symmetry breaking without the Higgs mechanism or Higgs particles? For an example of such alternative theory to derive boson masses of electroweak interaction without introducing Higgs particles, see E. Goldfain [16];
24. Is it possible to write quaternionic formulation [17] of quantum Hall effect? If yes, then how?
25. Orthopositronium problem [18]. What is the dynamics behind orthopositronium observation?
26. Is it possible to conceive New Energy generation method from orthopositronium-based reaction? If yes, then how?
27. Muonium problem. Muonium is atom consisting of muon and electron, discovered by a team led by Vernon Hughes in 1960 [19]. What is the dynamics behind muonium observation?
28. Is it possible to conceive New Energy generation method from muonium-based reaction? If yes, then how?
29. Antihydrogen problem [20]. Is it possible to conceive New Energy generation method from antihydrogen-based reaction? If yes, then how?
30. Unmatter problem [21]. Would unmatter be more useful to conceiving New Energy than antimatter? If yes, then how?

It is our hope that perhaps some of these questions may be found interesting to motivate further study of elementary particles.

Acknowledgment

VC would like to dedicate this article for RFF.

Submitted on September 24, 2007
Accepted on September 26, 2007

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LETTERS TO PROGRESS IN PHYSICS

Charles Kenneth Thornhill (1917–2007)

Jeremy Dunning-Davies

Department of Physics, University of Hull, Hull, England

E-mail: J.Dunning-Davies@hull.ac.uk

Dr. Charles Kenneth Thornhill, who died recently, was a proud, gritty Yorkshireman who, throughout his long life, genuinely remained true to himself. This led him into conflicts within the scientific community. The jury is still out on whether he was correct or not in his ideas but, be that as it may, all can learn a tremendous amount from the courage of this man in standing up for what he truly believed.

Dr. Charles Kenneth Thornhill

Dr. Charles Kenneth Thornhill was born in Sheffield on 25th November 1917. To the very end he remained fiercely proud of being a Yorkshireman. Indeed, throughout his life, he faced all problems, both personal and academic, with that gritty fortitude many associate with people from Yorkshire.

His secondary education was undertaken at the King Edward VII School in Sheffield. In 1936 he was awarded an Open (Jodrell) Scholarship for Mathematics at Queen's College, Oxford. This scholarship was worth 110 a year, a considerable amount in those days. He completed his undergraduate studies at the beginning of the Second World War and spent that war devoting his considerable mathematical talent to the aid of the war effort. During the War and in subsequent years, he worked in a variety of fields with a bias towards unsteady gasdynamics. These included external, internal, intermediate and terminal ballistics; heat transfer and erosion in gun-barrels; gasdynamics and effects of explosions; theories of damage; detonation and combustion; thermodynamics of solids and liquids under extreme conditions, etc. As a result of the war work, he was awarded the American Presidential Medal of Freedom. This was an award of which he was, quite properly, inordinately proud. The actual citation was as follows:

> *Mr. C. Kenneth Thornhill, United Kingdom, during the period of active hostilities in World War II, performed meritorious service in the field of scientific research. As a mathematician working in the field of gun erosion, he brought to the United States a comprehensive knowledge of the subject, and working in close co-operation with American scientists concerned with the study of erosion in gun barrels, he aided and stimulated the work in improving the performance of guns.*

After the war, he spent the remainder of his working life working at Fort Halstead for the Ministry of Defence.

Throughout his time at the Ministry of Defence, he had kept abreast of developments in the areas of theoretical physics that fascinated him, — those areas popularly associated with the names relativity and cosmology. One way he achieved this was through his membership of the Royal Astronomical Association. However, on his retirement in 1977 — incidentally, according to him, retirement was the job he recommended to everyone — he was able to devote his time and intellect to considering those deep problems which continue to concern so many. Also, relating to that transitional time, he commented that, up to retirement, he had worked for man but afterwards he had worked for mankind. His main interests were in the physical properties of the ether and the construction of a non-singular ethereal cosmology. Unfortunately, because of his disbelief in relativity, many refused to even listen to his views. One undoubted reason for this was his insistence on referring to the aether by that very name. It is quite likely that if he'd been willing to compromise and use words such as "vacuum" he might have had more success with publication in the better-known journals. However, some journal editors are courageous and genuinely believe in letting the scientific community at large judge the worth of peoples' work.

It is seen immediately that some of these articles make truly substantial contributions to science. Not all are incredibly long but all result from enormous thought and mathematical effort, effort in which Kenneth Thornhill's geometrical knowledge and skill are well to the fore. It is also immediately clear that here was a man who was prepared to think for himself and not allow himself to be absolutely bound by what appeared in books, whether the books in question be academic tomes or mere popular offerings.

In his life, Kenneth Thornhill was ostracised by many in the scientific establishment as some sort of "enfant terrible". In truth, many of these people really feared his intellect. That is not to say that all his thoughts were correct. The jury should still be out on many of his ideas but, to do that, the members of the jury must have read his offerings and done so with open scientific minds. Kenneth Thornhill left us all a truly enormous legacy and that is that he showed us all that it is vitally important to be true to yourself. He never pandered to the establishment rather he stuck with what he genuinely believed.

Kenneth Thornhill died peacefully on 30th June 2007 and is survived by four children, eight grandchildren and two great grandchildren. To the end he was enormously proud of all fourteen and to them must be extended our heartfelt sympathy. To the scientific community at large must be extended the hope that its members will learn the true meaning of scientific integrity from this gritty Yorkshireman. As one who was privileged to know him, albeit mainly through lengthy, enjoyable telephone conversations, I feel his scientific integrity alone will result in the words:

"Well done, thou good and faithful servant."

Submitted on August 07, 2007
Accepted on August 23, 2007

LETTERS TO PROGRESS IN PHYSICS

Max Karl Ernst Ludwig Planck: (1858 –1947)

Pierre-Marie Robitaille

Dept. of Radiology, The Ohio Sate University, 130 Means Hall, 1654 Upham Drive, Columbus, Ohio, 43221, USA

E-mail: robitaille.1@osu.edu

October 4th, 2007 marks the 60th anniversary of Planck's death. Planck was not only the father of Quantum Theory. He was also a man of profound moral and ethical values, with far reaching philosophical views. Though he lived a life of public acclaim for his discovery of the Blackbody radiation formula which bares his name, his personal life was beset with tragedy. Yet, Planck never lost his deep faith and belief in a personal God. He was admired by Einstein, not so much for his contributions to physics, but rather, for the ideals which he embodied as a person. In this work, a brief synopsis is provided on Planck, his life, and his philosophical writings. It is hoped that this will serve as an invitation to revisit the philosophical works of the man who, more than any other, helped set the course of early 20th century physics.

"Many kinds of men devote themselves to science, and not all for the sake of science herself. There are some who come into her temple because it offers them the opportunity to display their particular talents. To this class of men science is a kind of sport in the practice of which they exult, just as an athlete exults in the exercise of his muscular prowess. There is another class of men who come into the temple to make an offering of their brain pulp in the hope of securing a profitable return. These men are scientists only by the chance of some circumstance which offered itself when making a choice of career. If the attending circumstance had been different, they might have become politicians or captains of business. Should an angel of God descend and drive from the temple of science all those who belong to the categories I have mentioned, I fear the temple would be nearly emptied. But a few worshippers would still remain — some from former times and some from ours. To these latter belongs our Planck. And that is why we love him. . .

. . . (Planck's) work has given one of the most powerful of all impulses to the progress of science. His ideas will be effective as long as physical science lasts. And I hope that the example which his personal life affords will not be less effective with later generations of scientists."

Albert Einstein, 1932

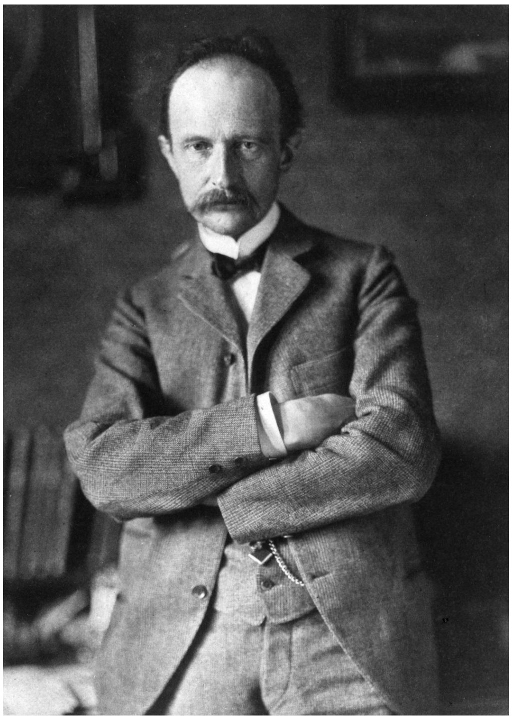

Fig. 1: Max Planck in his earlier years. AIP Emilio Segre Visual Archives, W. F. Meggers Collection. Reproduced through permission.

Biography

Max Planck, the father of quantum theory, was born on the 23rd of April 1858 in the town of Kiel, Germany [1–5]. His father had been a professor of law in the same town, while his paternal grandfather and great grandfather had been leading Lutheran theologians at the University of Göttingen. In 1867, when Planck reached the age of nine, his father received an academic appointment at the University of Munich and the Planck family relocated to this city. In Munich, he would attend the Maximillian Gymnasium and there gained his first love for Physics and Mathematics. In 1874, while still only 16, he enrolled at the University of Munich to study Physics. Beginning in 1877, he would spend one year at the University of Berlin where he was taught by Gustav Robert Kirchhoff and Hermann von Helmholtz, both of whom had been eminent physicists of the period. He was impressed with both of these men, but had little regard for the quality of their lectures. During his studies, Planck took an early interest in thermodynamics and immersed himself in Rudolf Clausius' work on the subject. He would receive his doctorate in physics in 1879 from the University of Munich at the age of 21. His thesis was focused on the second law of thermodynamics. In 1885, through the influence of his father, Max Planck received an appointment as an associate professor of physics at the University of Kiel. Later, he would present a paper on thermodynamics that would result in an appointment for him at the University of Berlin upon the death of Kirchoff in 1889. Kirchhoff had been the chair of theoretical physics in Berlin and Planck would become the only theoretical physicist on the faculty. He would hold this chair until his retirement in 1927, having become a full professor in 1892 [1–5].

In 1913, Planck would offer Albert Einstein a professorship in Berlin. The two of them, along with Planck's student, Professor Max von Laue, would remain close personal friends and scientific colleagues even after Einstein departed for Princeton. Rosenthal-Schneider [6] describes Planck as gentle, reserved, unpretentious, noble-minded and warm-hearted. He deeply loved mountain-climbing and music. He might well have been a concert pianist rather than a theoretical physicist, but he believed that he would do better as an average physicist than as an average pianist [1, 6].

While in Berlin, Planck would turn his attention to the emission of heat and light from solids. From these studies, his famous equation would emerge and quantum theory, through "the discovery of the elementary quantum of action", would be born [7]. Planck recognized the far reaching impact of his discovery:

"*(The essence of Quantum Physics)... consists in the fact that it introduces a new and universal constant, namely the elementary Quantum of Action. It was this constant which, like a new and mysterious messenger from the real world, insisted on turning up in every kind of measurement, and continued to claim a place for itself. On the other hand, it seemed so incompatible with the traditional view of the universe provided by Physics that it eventually destroyed the framework of this older view. For a time it seemed that a complete collapse of classical Physics was not beyond the bounds of possibility; gradually, however, it appeared, as had been confidently expected by all who believed in the steady advance of science, that the introduction of Quantum Theory led not to the destruction of Physics, but to a somewhat profound reconstruction, in the course of which the whole science was rendered more universal. For if the Quantum of Action is assumed to be infinitely small, Quantum Physics become merged with classical Physics...*" [8, p. 22–23].

Planck also believed that his equation could be applied to all objects independent of the phases of matter:

"*According to the Kirchhoff law this radiant energy is independent of the nature of the radiating substance and therefore has a universal significance*" [9, p. 18].

Planck's personal life would take a tragic turn after his discovery of the quantum in 1900 [7]. In 1909, he would lose his wife of 22 years. His oldest son, Karl, would be killed in action at Verdun in 1916. In 1917, his daughter Margerite would die in childbirth. In 1919, his second daughter Emma would suffer the same fate. In the meantime, though the First World War had just ended, Planck would win the 1918 Nobel Prize in Physics [2–5].

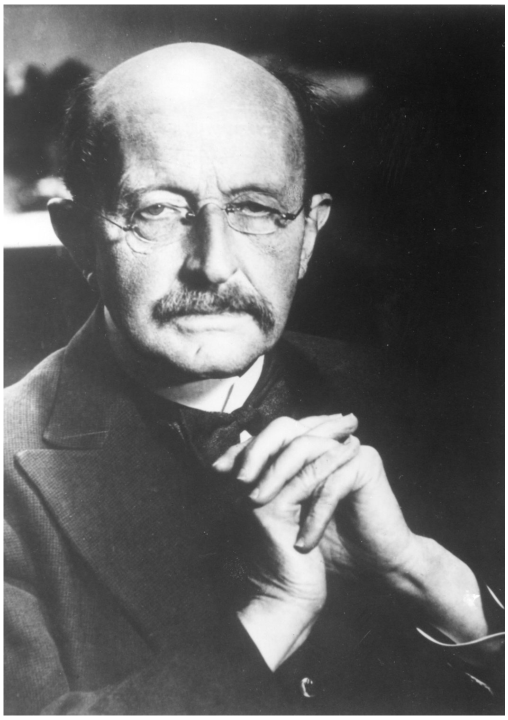

Fig. 2: Max Planck, the 1930's. AIP Emilio Segre Visual Archives. Reproduced through permission.

Unfortunately however, Planck's misfortunes continued. His home would be demolished in an ally air raid in 1944. Planck would later acknowledge gifts of food shipped from Australia by his former student Iles Rosenthal-Schneider [6]. Beginning in the early 1930's, Planck had expressed strong private and public views against the Nazi regime. Little did he realize at that time the price that he, and indeed much of the free world, would have to pay for the curse of this regime. Thus, in January 1945, his son Erwin was charged with an attempt on Hitler's life. Erwin was his only remaining child from his first marriage. Once his son was charged, Planck and von Laue tried to intervene before Heinrich Himmler, the second most powerful man in Germany [4]. But upon his arrival to Berlin, Hitler himself ordered the execution and immediate hanging of Planck's son. It is said that this execution robbed Planck of much of his will to live. Then in August 1945, the atomic bomb would be dropped on Hiroshima. Planck would express concern for the fate of mankind over these developments [6]. Eventually, Planck would be taken by the allies to Göttingen to live with his niece. He was accompanied by his second wife and their son. He would die in Göttingen on October 4, 1947 [1–5].

Philosophy of life

As Planck began to age, he devoted much of his time to philosophical works [1, 8, 9, 10]. These centered on the search for an absolute truth and on other philosophical aspects of Physics and Religion. Planck viewed science as the primary means of extracting the absolute. Planck believed that it was possible to move from the relative to the absolute. He thought that the Theory of Relativity itself promoted the absolute by quantifying in absolute terms the speed of light in a vacuum and the amount of energy within an object at rest ($E = mc^2$).

Planck saw the physical world as an objective reality and its exploration as a search for truth. Philosophers have often questioned physical reality, but men like Einstein and Planck viewed the physical world as real and the pursuit of science as forever intertwined with the search for truth [6]. These men saw the search for truth as elevating humanity. In Planck's words:

"*Science enhances the moral values of life, because it furthers a love of truth and reverence — love of truth displaying itself in the constant endeavor to arrive at more exact knowledge of the world of mind and matter around us, and reverence, because every advance in knowledge brings us face to face with the mystery of our own being*" [9, p. 122].

Thus, Planck had a deep love and respect for truthfulness. He regarded it as a central human virtue and as the most important quality of the scientist:

"*But truthfulness, this noblest of all human virtues, is authoritative even here over a well-defined domain, within which its moral commandment acquires an absolute meaning, independent of all specific viewpoints. This is probing to one's own self, before one's own conscience. Under no circumstances can there be in this domain the slightest moral compromise, the slightest moral justification for the smallest deviation. He who violates this commandment, perhaps in the endeavor to gain some momentary worldly advantage, by deliberately and knowingly shutting his eyes to the proper evaluation of the true situation, is like a spendthrift who thoughtlessly squanders away his wealth, and who must inevitably suffer, sooner or later, the grave consequences of his foolhardiness*" [1, p. 79].

He saw his quest for truth and the absolute as a never ending struggle from which he could take no rest:

"*We cannot rest and sit down lest we rust and decay. Health is maintained only through work. And as it is with all life so it is with science. We are always struggling from the relative to the absolute*" [9, p. 151].

As he continued his works in search of truth and the absolute, Planck was guided by his undying scientific faith:

"*Anyone who has taken part in the building up of science is well aware from personal experience that every endeavor in this direction is guided by an unpretentious but essential principle. This principle is faith — a faith which looks ahead*" [10, p. 121].

At the same time, Planck recognized that one could never arrive at the absolute truth. This did not deter him:

"*What will be the ultimate goal? ... research in general has a twofold aim — the effective domination of the world of senses, and the complete understanding of the real world; and that both these aims are in principle unattainable. But it would be a mistake to be discouraged on this account. Both our theoretical and practical tangible results are too great to warrant discouragement; and every day adds to them. Indeed, there is perhaps some justification for seeing in the very fact that this goal is unattainable, and the struggle unending, a blessing for the human mind in its search after knowledge. For it is in this way that its two noblest impulses — enthusiasm and reverence — are preserved and inspired anew*" [8, p. 61].

For Planck, the understanding of physical laws would occupy his entire adult life. He would write:

"*The laws of Physics have no consideration for the human senses; they depend on the facts, and not upon the obviousness of the facts*" [8, p. 73].

When he formulated his now famous Law of Thermal Radiation [7], he must have encountered tremendous opposition for what he was proposing went well beyond the senses:

"*An important scientific innovation rarely makes its way by gradually winning over and converting its opponents: it rarely happens that Saul become Paul. What does happen is that its opponents gradually die out and that the growing generation is familiarized with the idea from the beginning: another instance of the fact that the future lies with youth*" [10, p. 97].

One can but imagine the courage and scientific faith he must have held, but Planck himself summarizes well for us:

"*... in science as elsewhere fortune favors the brave*" [10, p. 112].

According to Thomas Braun "*Planck was a man of deeply religious outlook. His scientist's faith in the lawfulness of nature was inseparable from his faith in God*" [6, p. 23]. Planck believed that "*man needs science for knowledge and religion for his actions in daily life*" [6, p. 106]. For Planck: "*religion and natural science are fighting a joint battle in an incessant, never relaxing crusade against scepticism and against dogmatism, against disbelief and against superstition...*" [1, p. 186–187].

Yet, Planck made a clear distinction between science and religion stating that:

"*Religion belongs to that realm that is inviolable before the laws of causation and therefore closed to science*" [9, p. 121].

Planck seemed to marvel at the mystery of scientific discovery in a manner that most clearly conveys his religious philosophy:

"*In fact, how pitifully small, how powerless we human beings must appear to ourselves if we stop to think that the planet Earth on which we live our lives is just a minute, in-*

finitesimal mote of dust; on the other hand how peculiar it must seem that we, tiny creatures on a tiny planet, are nevertheless capable of knowing though not the essence at least the existence and the dimensions of the basic building blocks of the entire great Cosmos!" [1, p. 174].

Perhaps there is no more suitable way of closing a work on Max Planck than to recall the memorial address delivered by Professor Max von Laue at the Albani Church in Göttingen on October 7, 1947 [1, p. 7–10]. Max von Laue was a colleague of Max Planck at the University of Berlin. In 1914, he had received the Nobel Prize in Physics for his study of the diffraction of X-rays by crystals.

My Fellow Mourners:

We stand at the bier of a man who lived to be almost four-score-and-ten. Ninety years are a long life, and these particular ninety years were extraordinarily rich in experiences. Max Planck would remember, even in his old age, the sight of the Prussian and Austrian troops marching into his native town of Kiel. The birth and meteoric ascent of the German Empire occurred during his lifetime, and so did its total eclipse and ghastly disaster. These events had a most profound effect on Planck in his person, too. His eldest son, Karl, died in action at Verdun in 1916. In the Second World War, his house went up in flames during an air raid. His library, collected throughout a whole long lifetime, disappeared, no one knows where, and the most terrible blow of all fell when his second son, Erwin, lost his life in the rule of terror in January, 1945. While on a lecture tour, Max Planck, himself, was an eye-witness of the destruction of Kassel, and was buried in an air raid shelter for several hours. In the middle of May, 1945, the Americans sent a car to his estate of Rogatz on the Elbe, then a theatre of war, to take him to Göttingen. Now we are taking him to his final resting-place.

In the field of science, too, Planck's lifetime was an epoch of deep-reaching changes. The physical science of our days shows an aspect totally different from that of 1875, when Planck began to devote himself to it — and Max Planck is entitled to the lion's share in the credit for these changes. And what a wonderous story his life was! Just think — boy of seventeen, just graduated from high school, he decided to take up a science which even its most authoritative representative who he could consult, described as one of mighty meager prospects. As a student, he chose a certain branch of science, for which even its neighbor sciences had but little regard — and even within this particular branch a highly specialized field, in which literally nobody at all had any interest whatever. His first scientific papers were not read by Helmholtz, Kirchhoff and Clausius, the very men who would have found it easiest to appreciate them. Yet, he continued on his way, obeying an inner call, until he came face to face with a problem which many others had tried and failed to solve, a problem for which the very path taken by him turned out to have been the best preparation. Thus, he was able to recognize and formulate, from measurements of radiations, the law which today bears and immortalizes his name for all times. He announced it before the Berlin Physical Society on October 19, 1900. To be sure, the theoretical substantiation of it made it necessary for him to reconsider his views and to fall back on methods of the atom theory, which he had been wont to regard with certain doubts, And beyond that, he had to venture a hypothesis, the audacity of which was not clear at first, to its full extent, to anybody, not even him. But on December 14, 1900, again before the German Physical Society, he was able to present the theoretic deduction of the law of radiation. This was the birthday of quantum theory. This achievement will perpetuate his name forever.

Max von Laue, 1947

First Published Online on June 23, 2001
on http://www.thermalphysics.org
Revised March 14, 2002
Submitted on September 11, 2007
Accepted on September 17, 2007
Published online on October 04, 2007

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Department of Mathematics and Science, University of New Mexico,
200 College Road, Gallup, NM 87301, USA

Printed in the United States of America

Issue 2007, Volume 4
US $ 20.00