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1.
$$a_1x + b_1y = c_1$$
, $a_2x + b_2y = c_2$.

System of two linear algebraic equations.

Three different cases are possible depending on the values of the equation coefficients a_k, b_k, c_k .

1°. If $\Delta = a_1b_2 - a_2b_1 \neq 0$, the system has a unique solution:

$$x = \frac{c_1b_2 - c_2b_1}{a_1b_2 - a_2b_1}, \quad y = \frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}.$$

2°. If $\Delta = a_1b_2 - a_2b_1 = 0$ and $a_1c_2 - a_2c_1 = 0$ (in this case, the equation coefficients are proportional), the system has infinitely many solutions given by

$$x = t$$
, $y = \frac{c_1 - a_1 t}{b_1}$ $(b_1 \neq 0)$,

where t is an arbitrary number.

3°. If $\Delta = a_1b_2 - a_2b_1 = 0$ and $a_1c_2 - a_2c_1 \neq 0$, the system has no solutions.

System of Two Linear Algebraic Equations

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