9. $F\big(x, y(x), y\big(\sqrt{a^2 - x^2}\,\big)\big) = 0, \qquad 0 \le x \le a.$

On substituting $\sqrt{a^2 - x^2}$ for x, we obtain

$$F\big(\sqrt{a^2 - x^2},\, y\big(\sqrt{a^2 - x^2}\,\big),\, y(x)\big) = 0.$$

On eliminating $y\big(\sqrt{a^2 - x^2}\,\big)$ from this equation and the original one, we arrive at an ordinary algebraic (or transcendental) equation of the form $\Psi\big(x, y(x)\big) = 0$.

In other words, the solution of the original functional equation, $y = y(x)$, is determined parametrically by the system of two algebraic (transcendental) equations

$$F(x,\, y,\, t) = 0, \qquad F\big(\sqrt{a^2 - x^2},\, t,\, y\big) = 0,$$

where t is the parameter.

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations: Exact Solutions (Supplement. Some Functional Equations)* [in Russian], Faktorial, Moscow, 1998.

