7. $f(x)g(y) + h(y) = f(x + y)$.

Here, $f(x)$, $g(y)$, and $h(z)$ are unknown functions.

Solutions:

$$f(x) = C_1 x + C_2, \quad g(x) = 1, \quad h(x) = C_1 x \qquad \text{(first solution);}$$
$$f(x) = C_1 e^{ax} + C_2, \quad g(x) = e^{ax}, \quad h(x) = C_2(1 - e^{ax}) \qquad \text{(second solution),}$$

where a, C_1, and C_2 are arbitrary constants.

Reference

Aczél, J. and Dhombres, J., *Functional Equations in Several Variables*, Cambridge Univ. Press, Cambridge, 1989.

