



11.  $M(f(x), f(y)) = f(M(x, y))$ .

Here,  $M(x, y) = \varphi^{-1}\left(\frac{\varphi(x) + \varphi(y)}{2}\right)$  is the quasi-arithmetic mean for a continuous strictly monotonic function  $\varphi$ , and  $\varphi^{-1}$  is the inverse of  $\varphi$ .

Solution:

$$f(x) = \varphi^{-1}(a\varphi(x) + b),$$

where  $a$  and  $b$  are arbitrary constants.

### Reference

Aczél, J. and Dhombres, J., *Functional Equations in Several Variables*, Cambridge Univ. Press, Cambridge, 1989.