



3. Fredholm Integral Equations of the First Kind

3-1. Integral equations whose kernels contain power-law functions

1. $\int_a^b |x-t| y(t) dt = f(x).$
2. $\int_0^a \frac{y(t)}{\sqrt{|x-t|}} dt = f(x).$
3. $\int_a^b \frac{y(t)}{|x-t|^k} dt = f(x).$
4. $\int_0^b \frac{y(t)}{|x^\lambda - t^\lambda|^k} dt = f(x).$
5. $\int_{-\infty}^{\infty} \frac{y(t)}{|x-t|^{1-\lambda}} dt = f(x).$
6. $\int_{-\infty}^{\infty} \frac{\text{sign}(x-t)}{|x-t|^{1-\lambda}} y(t) dt = f(x).$
7. $\int_{-\infty}^{\infty} \frac{a + b \text{sign}(x-t)}{|x-t|^{1-\lambda}} y(t) dt = f(x).$
8. $\int_0^{\infty} \frac{y(x+t) - y(x-t)}{t} dt = f(x).$
9. $\int_{-\infty}^{\infty} \frac{y(t) dt}{t-x} = f(x).$
10. $\int_a^b \frac{y(t) dt}{t-x} = f(x).$

3-2. Integral equations whose kernels contain exponential or logarithmic functions

11. $\int_a^b e^{\lambda|x-t|} y(t) dt = f(x).$
12. $\int_a^b \ln|x-t| y(t) dt = f(x).$ Carleman's equation.
13. $\int_a^b (\ln|x-t| + \beta) y(t) dt = f(x).$
14. $\int_{-a}^a \left(\ln \frac{A}{|x-t|} \right) y(t) dt = f(x).$
15. $\int_0^a \ln \left| \frac{x+t}{x-t} \right| y(t) dt = f(x).$

3-3. Integral equations whose kernels contain trigonometric functions

16.
$$\int_0^{\infty} \cos(xt)y(t) dt = f(x).$$

17.
$$\int_0^{\infty} \sin(xt)y(t) dt = f(x).$$

18.
$$\int_0^{\pi/2} y(\xi) dt = f(x), \quad \xi = x \sin t. \quad \text{Schl\"omilch equation.}$$

19.
$$\int_0^{2\pi} \cot\left(\frac{t-x}{2}\right)y(t) dt = f(x).$$

3-4. Other Integral equations

20.
$$\int_0^{\infty} tJ_{\nu}(xt)y(t) dt = f(x).$$

21.
$$\int_{-\infty}^{\infty} K_0(|x-t|)y(t) dt = f(x).$$

22.
$$\int_{-\infty}^{\infty} K(x-t)y(t) dt = f(x).$$

23.
$$\int_0^{\infty} K(x-t)y(t) dt = f(x). \quad \text{Wiener-Hopf equation of the first kind.}$$

The EqWorld website presents extensive information on solutions to various classes of ordinary differential equations, partial differential equations, integral equations, functional equations, and other mathematical equations.