11. $\displaystyle\int_a^b e^{\lambda|x-t|}y(t)\,dt = f(x), \qquad -\infty < a < b < \infty.$

Solution:

$$y(x) = \frac{1}{2\lambda}\left[f''_{xx}(x) - \lambda^2 f(x)\right].$$

The right-hand side $f(x)$ of the integral equation must satisfy certain relations. The general form of the right-hand side is given by

$$f(x) = F(x) + Ax + B,$$

$$A = \frac{1}{b\lambda - a\lambda - 2}\left[F'_x(a) + F'_x(b) + \lambda F(a) - \lambda F(b)\right], \quad B = -\frac{1}{\lambda}\left[F'_x(a) + \lambda F(a) + Aa\lambda + A\right],$$

where $F(x)$ is an arbitrary bounded, twice differentiable function.

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations*, CRC Press, Boca Raton, 1998.

