**3.** $\displaystyle Ay(x) + \frac{B}{\pi}\int_{-1}^{1} \frac{y(t)\,dt}{t-x} = f(x), \qquad -1 < x < 1.$

In the equation and its solutions, singular integrals are understood in the sense of the Cauchy principal value. Without loss of generality we may assume that $A^2 + B^2 = 1$.

1°. The solution bounded at the endpoints:

$$y(x) = Af(x) - \frac{B}{\pi}\int_{-1}^{1} \frac{g(x)}{g(t)}\frac{f(t)\,dt}{t-x}, \qquad g(x) = (1+x)^{\alpha}(1-x)^{1-\alpha}, \tag{1}$$

where $\alpha$ is the solution of the trigonometric equation

$$A + B\cot(\pi\alpha) = 0 \tag{2}$$

on the interval $0 < \alpha < 1$. This solution $y(x)$ exists if and only if $\displaystyle\int_{-1}^{1} \frac{f(t)}{g(t)}\,dt = 0$.

2°. The solution bounded at the endpoint $x = 1$ and unbounded at the endpoint $x = -1$:

$$y(x) = Af(x) - \frac{B}{\pi}\int_{-1}^{1} \frac{g(x)}{g(t)}\frac{f(t)\,dt}{t-x}, \qquad g(x) = (1+x)^{\alpha}(1-x)^{-\alpha}, \tag{3}$$

where $\alpha$ is the solution of the trigonometric equation (2) on the interval $-1 < \alpha < 0$.

3°. The solution unbounded at the endpoints:

$$y(x) = Af(x) - \frac{B}{\pi}\int_{-1}^{1} \frac{g(x)}{g(t)}\frac{f(t)\,dt}{t-x} + Cg(x), \qquad g(x) = (1+x)^{\alpha}(1-x)^{-1-\alpha},$$

where $C$ is an arbitrary constant and $\alpha$ is the solution of the trigonometric equation (2) on the interval $-1 < \alpha < 0$.

## References

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