7. $y(x) + \lambda \int_{-\infty}^{\infty} e^{-|x-t|} y(t)\,dt = f(x).$

1°. Solution for $\lambda > -\frac{1}{2}$:

$$y(x) = f(x) - \frac{\lambda}{\sqrt{1+2\lambda}} \int_{-\infty}^{\infty} \exp\bigl(-\sqrt{1+2\lambda}\,|x-t|\bigr) f(t)\,dt.$$

2°. If $\lambda \le -\frac{1}{2}$, for the equation to be solvable the conditions

$$\int_{-\infty}^{\infty} f(x)\cos(ax)\,dx = 0, \qquad \int_{-\infty}^{\infty} f(x)\sin(ax)\,dx = 0,$$

where $a = \sqrt{-1-2\lambda}$, must be satisfied. In this case, the solution has the form

$$y(x) = f(x) - \frac{a^2+1}{2a} \int_0^{\infty} \sin(at) f(x+t)\,dt \qquad (-\infty < x < \infty).$$

In the class of solutions not belonging to $L_2(-\infty,\infty)$, the homogeneous equation (with $f(x)\equiv 0$) has a nontrivial solution. In this case, the general solution of the corresponding nonhomogeneous equation with $\lambda \le -\frac{1}{2}$ has the form

$$y(x) = C_1\sin(ax) + C_2\cos(ax) + f(x) - \frac{a^2+1}{4a} \int_{-\infty}^{\infty} \sin(a|x-t|) f(t)\,dt.$$

References

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