8. $y(x) + A\int_a^b e^{\lambda|x-t|} y(t)\,dt = f(x)$.

1°. The function $y = y(x)$ obeys the following second-order linear nonhomogeneous ordinary differential equation with constant coefficients:

$$y''_{xx} + \lambda(2A - \lambda)y = f''_{xx}(x) - \lambda^2 f(x). \tag{1}$$

The boundary conditions for (1) have the form

$$\begin{aligned} y'_x(a) + \lambda y(a) &= f'_x(a) + \lambda f(a),\\ y'_x(b) - \lambda y(b) &= f'_x(b) - \lambda f(b). \end{aligned} \tag{2}$$

Equation (1) under the boundary conditions (2) determines the solution of the original integral equation.

2°. For $\lambda(2A - \lambda) < 0$, the general solution of equation (1) is given by

$$y(x) = C_1 \cosh(kx) + C_2 \sinh(kx) + f(x) - \frac{2A\lambda}{k}\int_a^x \sinh[k(x-t)]\, f(t)\,dt, \qquad k = \sqrt{\lambda(\lambda - 2A)}, \tag{3}$$

where C_1 and C_2 are arbitrary constants.

For $\lambda(2A - \lambda) > 0$, the general solution of equation (1) is given by

$$y(x) = C_1 \cos(kx) + C_2 \sin(kx) + f(x) - \frac{2A\lambda}{k}\int_a^x \sin[k(x-t)]\, f(t)\,dt, \qquad k = \sqrt{\lambda(2A - \lambda)}. \tag{4}$$

For $\lambda = 2A$, the general solution of equation (1) is given by

$$y(x) = C_1 + C_2 x + f(x) - 4A^2 \int_a^x (x-t) f(t)\,dt. \tag{5}$$

The constants C_1 and C_2 in solutions (3)–(5) are determined by conditions (2).

3°. In the special case $a = 0$ and $\lambda(2A - \lambda) > 0$, the solution of the integral equation is given by formula (4) with

$$C_1 = \frac{A(kI_c - \lambda I_s)}{(\lambda - A)\sin\mu - k\cos\mu}, \qquad C_2 = -\frac{\lambda}{k}\,\frac{A(kI_c - \lambda I_s)}{(\lambda - A)\sin\mu - k\cos\mu},$$

$$k = \sqrt{\lambda(2A - \lambda)}, \quad \mu = bk, \quad I_s = \int_0^b \sin[k(b-t)] f(t)\,dt, \quad I_c = \int_0^b \cos[k(b-t)] f(t)\,dt.$$

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations*, CRC Press, Boca Raton, 1998.

