10. $y(x) + \int_a^x (x-t)^n f\bigl(t, y(t)\bigr)\,dt = g(x), \qquad n = 1, 2, \dots$

Differentiating the equation $n+1$ times with respect to x, we obtain an $(n+1)$st-order nonlinear ordinary differential equation for $y = y(x)$:

$$y_x^{(n+1)} + n!\, f(x, y) - g_x^{(n+1)}(x) = 0.$$

This equation under the initial conditions

$$y(a) = g(a), \quad y'_x(a) = g'_x(a), \quad \dots, \quad y_x^{(n)}(a) = g_x^{(n)}(a),$$

defines the solution of the original integral equation.

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations*, CRC Press, Boca Raton, 1998.

