5. $y(x) + \int_a^b g(x)y(x)y(t)\,dt = f(x).$

A solution:

$$y(x) = \frac{f(x)}{1 + \lambda g(x)},$$

where λ is a root of the algebraic (or transcendental) equation

$$\lambda - \int_a^b \frac{f(t)\,dt}{1 + \lambda g(t)} = 0.$$

Different roots generate different solutions of the integral equation.

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations*, CRC Press, Boca Raton, 1998.

