3. $\dfrac{\partial w}{\partial t} = \dfrac{\partial^2 w}{\partial x^2} - w(1-w)(a-w).$

FitzHugh–Nagumo equation. This equation arises in genetics, biology, and heat and mass transfer.

Solutions:

$$w(x,t) = \frac{A\exp(z_1) + aB\exp(z_2)}{A\exp(z_1) + B\exp(z_2) + C},$$
$$z_1 = \pm\frac{\sqrt{2}}{2}\,x + \left(\tfrac{1}{2} - a\right)t, \quad z_2 = \pm\frac{\sqrt{2}}{2}\,ax + a\left(\tfrac{1}{2}a - 1\right)t,$$

where A, B, and C are arbitrary constants.

See also: Newell–Whitehead equation (a special case of the Fitzhugh–Nagumo equation).

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