**1.** $\dfrac{\partial^2 w}{\partial t^2} = a\dfrac{\partial}{\partial x}\left(w\dfrac{\partial w}{\partial x}\right)$.

1°. Solutions:

$$w(x,t) = \tfrac{1}{2}aA^2t^2 + Bt + Ax + C,$$
$$w(x,t) = \tfrac{1}{12}aA^{-2}(At+B)^4 + Ct + D + x(At+B),$$
$$w(x,t) = \frac{1}{a}\left(\frac{x+A}{t+B}\right)^2,$$
$$w(x,t) = (At+B)\sqrt{Cx+D},$$
$$w(x,t) = \pm\sqrt{A(x+a\lambda t)+B} + a\lambda^2,$$

where $A$, $B$, $C$, $D$, and $\lambda$ are arbitrary constants.

2°. Generalized separable solution quadratic in $x$:

$$w(x,t) = \frac{1}{at^2}x^2 + \left(\frac{C_1}{t^2} + C_2t^3\right)x + \frac{aC_1^2}{4t^2} + \frac{C_3}{t} + C_4t^2 + \frac{1}{2}aC_1C_2t^3 + \frac{1}{54}aC_2^2t^8,$$

where $C_1, \ldots, C_4$ are arbitrary constants.

3°. Solution:

$$w = U(z) + 4aC_1^2t^2 + 4aC_1C_2t, \qquad z = x + aC_1t^2 + aC_2t,$$

where $C_1$ and $C_2$ are arbitrary constants and the function $U(z)$ is determined by the first-order ordinary differential equation $(U - aC_2^2)U'_z - 2C_1U = 8C_1^2z + C_3$.

4°. See also equation 2.2.12 with $f(w) = aw$.

## References


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