2. Second-Order Linear Ordinary Differential Equations

2.1. Ordinary Differential Equations Involving Power Functions

1. $y''_{xx} + ay = 0.$ *Equation of free oscillations.*
2. $y''_{xx} - ax^n y = 0.$
3. $y''_{xx} + ay'_x + by = 0.$ *Second-order constant coefficient linear equation.*
4. $y''_{xx} + ay'_x + (bx + c)y = 0.$
5. $y''_{xx} + (ax + b)y'_x + (\alpha x^2 + \beta x + \gamma)y = 0.$
6. $xy''_{xx} + ay'_x + by = 0.$
7. $xy''_{xx} + ay'_x + bxy = 0.$
8. $xy''_{xx} + ny'_x + bx^{1-2n}y = 0.$
9. $xy''_{xx} + ay'_x + bx^n y = 0.$
10. $xy''_{xx} + (b - x)y'_x - ay = 0.$ *Degenerate hypergeometric equation.*
11. $(a_2x + b_2)y''_{xx} + (a_1x + b_1)y'_x + (a_0x + b_0)y = 0.$
12. $x^2y''_{xx} + axy'_x + by = 0.$ *Euler equation.*
13. $x^2y''_{xx} + xy'_x + (x^2 - \nu^2)y = 0.$ *Bessel equation.*
14. $x^2y''_{xx} + xy'_x - (x^2 + \nu^2)y = 0.$ *Modified Bessel equation.*
15. $x^2y''_{xx} + axy'_x + (bx^n + c)y = 0, \quad n \neq 0.$
16. $x^2y''_{xx} + axy'_x + x^n(bx^n + c)y = 0.$
17. $x^2y''_{xx} + (ax + b)y'_x + cy = 0.$
18. $(1 - x^2)y''_{xx} - 2xy'_x + n(n + 1)y = 0, \quad n = 0, 1, 2, \ldots.$ *Legendre equation.*
19. $(1 - x^2)y''_{xx} - 2xy'_x + \nu(\nu + 1)y = 0.$ *Legendre equation.*
20. $(ax^2 + b)y''_{xx} + axy'_x + cy = 0.$
21. $(1 - x^2)y''_{xx} + (ax + b)y'_x + cy = 0.$
22. $x(x - 1)y''_{xx} + [(\alpha + \beta + 1)x - \gamma]y'_x + \alpha\beta y = 0.$ *Gaussian hypergeometric equation.*
23. $(1 - x^2)^2y''_{xx} - 2x(1 - x^2)y'_x + [\nu(\nu + 1)(1 - x^2) - \mu^2]y = 0.$ *Legendre equation.*
24. $(x - a)^2(x - b)^2y''_{xx} - cy = 0, \quad a \neq b.$
25. $(ax^2 + bx + c)^2y''_{xx} + Ay = 0.$
26. $x^2(ax^n - 1)y''_{xx} + x(apx^n + q)y'_x + (arx^n + s)y = 0.$

2.2. Ordinary Differential Equations Involving Exponential and Other Functions

27. $y''_{xx} + ae^{\lambda x}y = 0, \quad \lambda \neq 0.$

28. $y''_{xx} + (ae^{x} - b)y = 0.$

29. $y''_{xx} - (ae^{2\lambda x} + be^{\lambda x} + c)y = 0.$

30. $y''_{xx} + ay'_x + be^{2ax}y = 0.$

31. $y''_{xx} - ay'_x + be^{2ax}y = 0.$

32. $y''_{xx} + ay'_x + (be^{\lambda x} + c)y = 0.$

33. $y''_{xx} - (a - 2q\cosh 2x)y = 0.$ *Modified Mathieu equation.*

34. $y''_{xx} + (a - 2q\cos 2x)y = 0.$ *Mathieu equation.*

35. $y''_{xx} + a\tan x\, y'_x + by = 0.$

2.3. Ordinary Differential Equations Involving Arbitrary Functions

36. $y''_{xx} + fy'_x + a(f - a)y = 0.$

37. $y''_{xx} + xfy'_x - fy = 0.$

38. $xy''_{xx} + (xf + a)y'_x + (a - 1)fy = 0.$

39. $xy''_{xx} + [(ax + 1)f + ax - 1]y'_x + a^2xfy = 0.$

40. $xy''_{xx} + [(ax^2 + bx)f + 2]y'_x + bfy = 0.$

41. $x^2y''_{xx} + xfy'_x + a(f - a - 1)y = 0.$

42. $y''_{xx} + (f + ae^{\lambda x})y'_x + ae^{\lambda x}(f + \lambda)y = 0.$

43. $y''_{xx} - (f^2 + f'_x)y = 0.$

44. $y''_{xx} + 2fy'_x + (f^2 + f'_x)y = 0.$

45. $y''_{xx} + (1 - a)fy'_x - a(f^2 + f'_x)y = 0.$

46. $y''_{xx} + fy'_x + (fg - g^2 + g'_x)y = 0.$

47. $fy''_{xx} - af'_xy'_x - bf^{2a+1}y = 0.$

48. $f^2y''_{xx} + f(f'_x + a)y'_x + by = 0.$

49. $y''_{xx} - f'_xy'_x + a^2e^{2f}y = 0.$

50. $y''_{xx} - f'_xy'_x - a^2e^{2f}y = 0.$