



47. $x[f(x^n e^{\alpha y}) + \alpha y g(x^n e^{\alpha y})]y'_x = h(x^n e^{\alpha y}) - n y g(x^n e^{\alpha y}).$

The substitution $t = x^n e^{\alpha y}$ leads to a linear equation with respect to $y = y(t)$:

$$t[nf(t) + \alpha h(t)]y'_t = -ng(t)y + h(t).$$

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition*, Chapman & Hall/CRC, Boca Raton, 2003.