



3. $y''_{xx} + ay'_x + by = 0.$

Second-order constant coefficient linear differential equation. In physics this equation is called an **equation of damped vibrations.**

$$\text{Solution: } y = \begin{cases} \exp(-\frac{1}{2}ax) [C_1 \exp(\frac{1}{2}\lambda x) + C_2 \exp(-\frac{1}{2}\lambda x)] & \text{if } \lambda^2 = a^2 - 4b > 0, \\ \exp(-\frac{1}{2}ax) [C_1 \sin(\frac{1}{2}\lambda x) + C_2 \cos(\frac{1}{2}\lambda x)] & \text{if } \lambda^2 = 4b - a^2 > 0, \\ \exp(-\frac{1}{2}ax) (C_1 x + C_2) & \text{if } a^2 = 4b. \end{cases}$$

References

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