7. $xy''_{xx} + ay'_x + bxy = 0.$

1°. The solution is expressed in terms of the Bessel functions and modified Bessel functions:

$$y = \begin{cases} x^{\frac{1-a}{2}}\left[C_1 J_\nu\left(\sqrt{b}\,x\right) + C_2 Y_\nu\left(\sqrt{b}\,x\right)\right] & \text{if } b > 0, \\ x^{\frac{1-a}{2}}\left[C_1 I_\nu\left(\sqrt{|b|}\,x\right) + C_2 K_\nu\left(\sqrt{|b|}\,x\right)\right] & \text{if } b < 0, \end{cases}$$

where $\nu = \frac{1}{2}|1-a|$.

2°. For $a = 2n$, where $n = 1, 2, \ldots$, the solution is:

$$y = \begin{cases} C_1\left(\frac{1}{x}\frac{d}{dx}\right)^n \cos\left(x\sqrt{b}\right) + C_2\left(\frac{1}{x}\frac{d}{dx}\right)^n \sin\left(x\sqrt{b}\right) & \text{if } b > 0, \\ C_1\left(\frac{1}{x}\frac{d}{dx}\right)^n \cosh\left(x\sqrt{-b}\right) + C_2\left(\frac{1}{x}\frac{d}{dx}\right)^n \sinh\left(x\sqrt{-b}\right) & \text{if } b < 0. \end{cases}$$

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