11. $(a_2x + b_2)y''_{xx} + (a_1x + b_1)y'_x + (a_0x + b_0)y = 0.$

Let the function $\mathcal{J}(a,b;x)$ be an arbitrary solution of the degenerate hypergeometric equation $xy''_{xx} + (b - x)y'_x - ay = 0$ (see 2.10), and the function $Z_\nu(x)$ be an arbitrary solution of the Bessel equation $x^2y''_{xx} + xy'_x + (x^2 - \nu^2)y = 0$ (see 2.13). The results of solving the original equation are presented in Table.

TABLE

Solutions of equation 2.11 for different values of the determining parameters

Solution: $y = e^{kx}w(z)$, where $z = \dfrac{x-\mu}{\lambda}$					
Constraints	k	λ	μ	w	Parameters
$a_2 \neq 0$, $a_1^2 \neq 4a_0a_2$	$\dfrac{\sqrt{D}-a_1}{2a_2}$	$-\dfrac{a_2}{2a_2k+a_1}$	$-\dfrac{b_2}{a_2}$	$\mathcal{J}(a,b;z)$	$a = B(k)/(2a_2k+a_1)$, $b = (a_2b_1-a_1b_2)a_2^{-2}$
$a_2 = 0$, $a_1 \neq 0$	$-\dfrac{a_0}{a_1}$	1	$-\dfrac{2b_2k+b_1}{a_1}$	$\mathcal{J}\left(a,\frac{1}{2};\beta z^2\right)$	$a = B(k)/(2a_1)$, $\beta = -a_1/(2b_2)$
$a_2 \neq 0$, $a_1^2 = 4a_0a_2$	$-\dfrac{a_1}{2a_2}$	a_2	$-\dfrac{b_2}{a_2}$	$z^{\nu/2}Z_\nu\left(\beta\sqrt{z}\right)$	$\nu = 1-(2b_2k+b_1)a_2^{-1}$, $\beta = 2\sqrt{B(k)}$
$a_2 = a_1 = 0$, $a_0 \neq 0$	$-\dfrac{b_1}{2b_2}$	1	$\dfrac{b_1^2-4b_0b_2}{4a_0b_2}$	$z^{1/2}Z_{1/3}\left(\beta z^{3/2}\right)$	$\beta = \dfrac{2}{3}\left(\dfrac{a_0}{b_2}\right)^{1/2}$
Notation: $D = a_1^2-4a_0a_2$, $B(k) = b_2k^2+b_1k+b_0$					

References

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