



21. $(1 - x^2)y''_{xx} + (ax + b)y'_x + cy = 0.$

1°. The substitution $2z = 1 + x$ leads to the hypergeometric equation 2.22:

$$z(1 - z)y''_{zz} + [az + \frac{1}{2}(b - a)]y'_z + cy = 0.$$

2°. For $a = -2m - 3$, $b = 0$, and $c = \lambda$, the Gegenbauer functions are solutions of the equation.

References

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