



45. $y''_{xx} + (1 - a)f y'_x - a(f^2 + f'_x)y = 0, \quad f = f(x).$

Particular solution: $y_0 = \exp\left(a \int f dx\right).$

Solution:

$$y = y_0 \left(C_1 + C_2 \int \frac{e^{-F}}{y_0^2} dx \right), \quad \text{where } F = (1 - a) \int f dx,$$

C_1 and C_2 are arbitrary constants.

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition*, Chapman & Hall/CRC, Boca Raton, 2003.