6. $y''''_{xxxx} + ax^n y''_{xx} + b(ax^n - b)y = 0.$

1°. Particular solutions with $b > 0$: $y_1 = \cos\left(x\sqrt{b}\right)$, $y_2 = \sin\left(x\sqrt{b}\right)$.

2°. Particular solutions with $b < 0$: $y_1 = \exp\left(-x\sqrt{-b}\right)$, $y_2 = \exp\left(x\sqrt{-b}\right)$.

3°. The substitution $w = y''_{xx} + by$ leads to a second-order linear equation: $w''_{xx} + (ax^n - b)w = 0$.

References

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