Systems of Ordinary Differential Equations > Nonlinear Systems of Three and More Equations

4. $x'_t = cF_2 - bF_3, \quad y'_t = aF_3 - cF_1, \quad z'_t = bF_1 - aF_2, \quad \text{where} \quad F_n = F_n(x, y, z, t).$

1°. First integral:

$$ax + by + cz = C_1,$$

where C is an arbitrary constant.

2°. Suppose the function F_n is independent of t: $F_n = F_n(x, y, z)$. Then, on eliminating t and z from the first two equations of the system (with the above integral), one arrives at the first-order equation

$$\frac{dy}{dx} = \frac{aF_3(x, y, z) - cF_1(x, y, z)}{cF_2(x, y, z) - bF_3(x, y, z)}, \quad \text{where} \quad z = \frac{1}{c}(C_1 - ax - by).$$