



$$2. \quad \frac{\partial^2 u}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial u}{\partial x} \right) + e^{\lambda u} f(\lambda u - \sigma w), \quad \frac{\partial^2 w}{\partial t^2} = \frac{b}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial w}{\partial x} \right) + e^{\sigma w} g(\lambda u - \sigma w).$$

1°. Solution:

$$u = y(\xi) - \frac{2}{\lambda} \ln(C_1 t + C_2), \quad w = z(\xi) - \frac{2}{\sigma} \ln(C_1 t + C_2), \quad \xi = \frac{x}{C_1 t + C_2},$$

where C_1 and C_2 are arbitrary constants, and the functions $y = y(\xi)$ and $z = z(\xi)$ are determined by the system of ordinary differential equations

$$\begin{aligned} C_1^2 (\xi^2 y'_\xi)'_\xi + 2C_1^2 \lambda^{-1} &= a \xi^{-n} (\xi^n y'_\xi)'_\xi + e^{\lambda y} f(\lambda y - \sigma z), \\ C_1^2 (\xi^2 z'_\xi)'_\xi + 2C_1^2 \sigma^{-1} &= b \xi^{-n} (\xi^n z'_\xi)'_\xi + e^{\sigma z} g(\lambda y - \sigma z). \end{aligned}$$

2°. Solution with $b = a$:

$$u = \theta(x, t), \quad w = \frac{\lambda}{\sigma} \theta(x, t) - \frac{k}{\sigma},$$

where k is a root of the algebraic (transcendental) equation

$$\lambda f(k) = \sigma e^{-k} g(k),$$

and the function $\theta = \theta(x, t)$ satisfies the equation

$$\frac{\partial^2 \theta}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial \theta}{\partial x} \right) + f(k) e^{\lambda \theta}.$$

This equation is solvable at $n = 0$. For its exact solutions, see the "Handbook of Nonlinear Partial Differential Equations" by A. D. Polyanin & V. F. Zaitsev (2004).