



5.
$$\frac{\partial^2 u}{\partial t^2} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial u}{\partial x} \right) + u^k f \left(\frac{u}{w} \right), \quad \frac{\partial^2 w}{\partial t^2} = \frac{b}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial w}{\partial x} \right) + w^k g \left(\frac{u}{w} \right).$$

Self-similar solution:

$$u = (C_1 t + C_2)^{\frac{2}{1-k}} y(\xi), \quad w = (C_1 t + C_2)^{\frac{2}{1-k}} z(\xi), \quad \xi = \frac{x}{C_1 t + C_2},$$

where C_1 and C_2 are arbitrary constants, and the functions $y = y(\xi)$ and $z = z(\xi)$ are determined by the system of ordinary differential equations

$$C_1^2 \xi^2 y''_{\xi\xi} + \frac{2C_1^2(k+1)}{k-1} \xi y'_{\xi} + \frac{C_1^2(k+1)}{(k-1)^2} y = \frac{a}{\xi^n} (\xi^n y'_{\xi})'_{\xi} + y^k f \left(\frac{y}{z} \right),$$
$$C_1^2 \xi^2 z''_{\xi\xi} + \frac{2C_1^2(k+1)}{k-1} \xi z'_{\xi} + \frac{C_1^2(k+1)}{(k-1)^2} z = \frac{b}{\xi^n} (\xi^n z'_{\xi})'_{\xi} + z^k g \left(\frac{y}{z} \right).$$